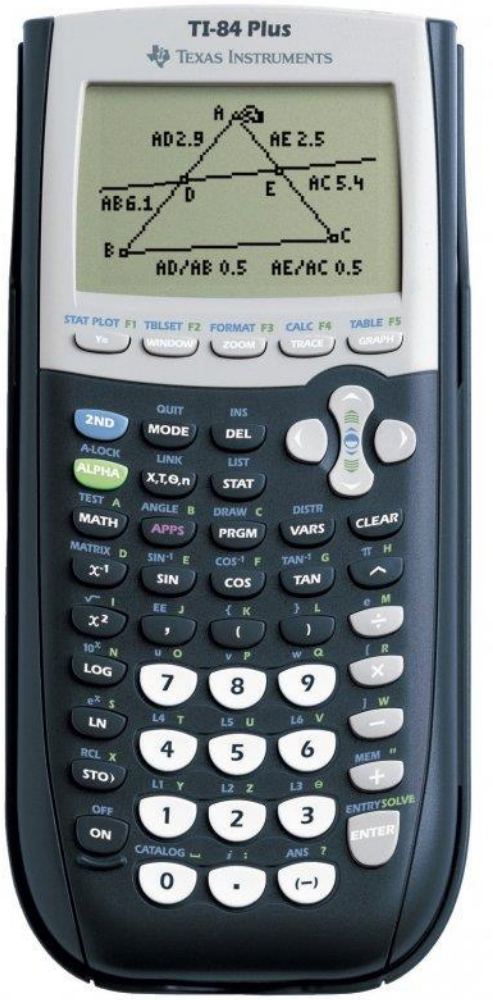
OPERATIONS WITH
THE GRAPHING CALCULATOR
TI-84



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1. General Information

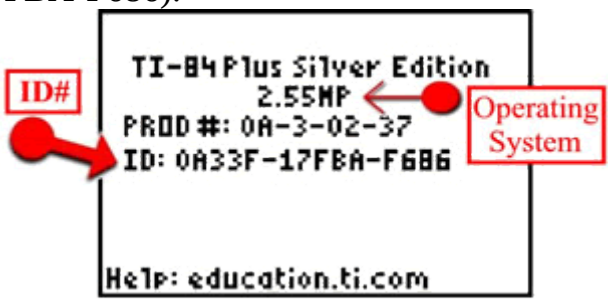
Identify your calculator

Be sure that you know how to identify your calculator
(or a calculator that you have been assigned).

The TI-83+/84+ and the TI-83+/84+SE have hard-wired Identification Numbers.

Go to (2nd) Mem and choose 1: About**.** The number listed after ID: will be the calculators unique Identification Number. Be careful when reading the 8's and the B's. The 8's are blocky and the B's are more rounded on the right hand side, as you can see in the Identification number listed below: (ID: 0A33F-17FBA-F686).





Be sure to record this Identification Number in a secure location.

If your calculator is lost or stolen, you will most likely be asked to identify the calculator by this number.

The Basics: Characteristics of the TI-84+



HOME SCREEN: This is where most calculations will take place. To access the home screen at any time- press 2nd QUIT.

CONTRAST: If your screen is visually too light or too dark, adjust the contrast by pressing 2nd and Cursor UP to darken or DOWN to lighten.

KEYPAD: The keys on the TI-83 and TI-84 have many functions. To access the options above the key use either the 2nd or ALPHA key.

THE EQUALS SIGN: The TI-83 & TI-84 do not have your traditional = key. The ENTER key on the bottom right corner means =.

Clearing an Entry or Error

1. If you have entered the wrong number or letter, set the cursor on the error and enter the correct information

2. If you need to delete a number or letter, set the cursor on the error, and press the **DEL** key, located next to the cursor.

3. To erase the entire line, press CLEAR, located underneath the cursor, once.

4. To clear the whole screen, press CLEAR twice.

Example:

$3 - 6 + 12 =$

Let's say your calculator looks like this:



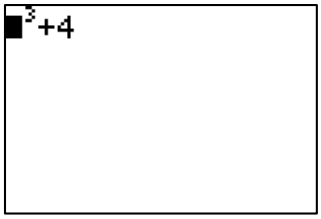
To fix the problem, press LEFT CURSOR until it blinks over the + sign. Then, move cursor to the place of the mistake and simply press – to correct the problem.

Inserting a Character

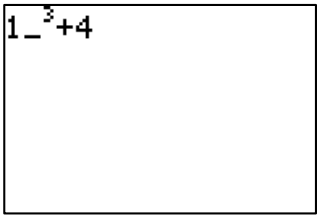
To insert a number or letter, set the cursor on the character to the right and then press **2ND** and **INS**. You may enter as many characters at that point as you wish without pressing **INS** again.

Example:

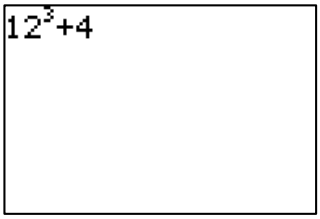
Enter $2^3 + 4$
Change it to read $21^2 + 4$

Step 1. 

Move cursor back to "2". Press **2ND** and **INC**.

Step 2. 

Type “1”. After that, the number “2” became to blink.

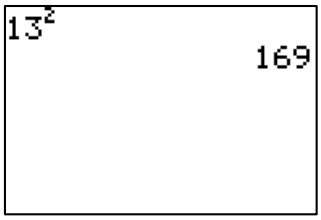
Step 3. 

Move cursor to the end of the phrase.

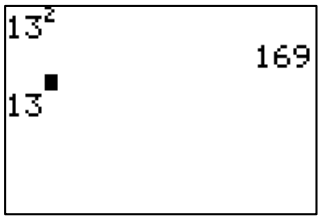
Recalling Previous Entries

Sometimes, it may be necessary to recall a previous entry, or modify a calculation. By pressing **2ND** and **ENTER**, you can access and edit prior actions.

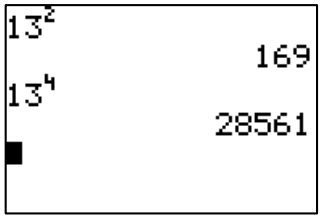
Example:



Suppose you just calculated 13^2, and you wish to find 13^4 but you need to keep the present value also.



Press **2ND** and **ENTER** and move cursor to the ² position. The number “2” became to blink.

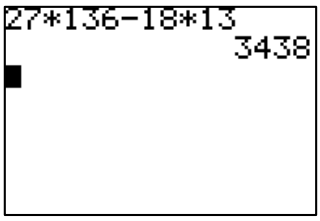


Press **^** and **4** and press the side bottom **›** of the cursor and **ENTER**.

Entering Expressions

Expressions are usually entered as they appear in print. The calculator is programmed to follow the order of operations. The answer will appear on the right side of the screen

Example:



Evaluate $27a - 18b$, for $a = 136$ and $b = 13$

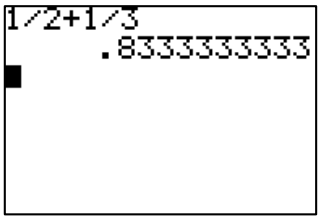
Press **27 × 136 − 18 × 13** and **ENTER**

Adding and Subtracting Fractions

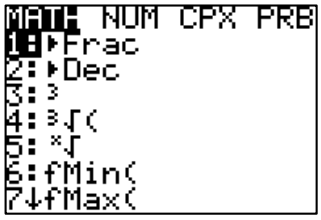
The TI-84 can perform operations with fractions. Often, the answer is expressed in decimal form, but it can easily be transformed back into a fraction.

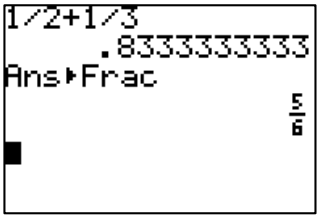
Example:

$\frac{1}{2} + \frac{1}{3} =$



Press: **1 ÷ 2 + 1 ÷ 3 ENTER**





In order to change this answer back into a fraction:
Press **MATH** key and select **1:▷Frac**.
Press **ENTER** two times to get fractional equivalent.

Power and the ^ Keys

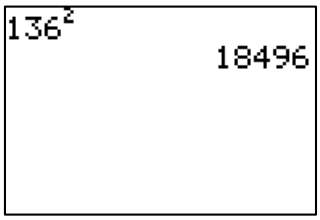
To enter an exponential expression you must enter the base and then:

-- Press the key x^2, if you operate with the second (square) power, then hit **ENTER**.

-- Press the key ^ and then type the power at the window, if you operate with the power, . higher than the second power. After typing the power press the keys **›** and **ENTER**.

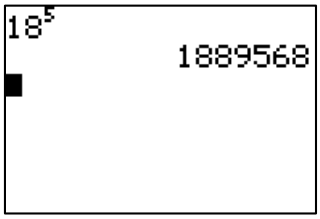
Examples:

Find 136^2



Press **136** x^2 and **ENTER**

Find 18^5



Press **18 ^ 5 › ENTER**

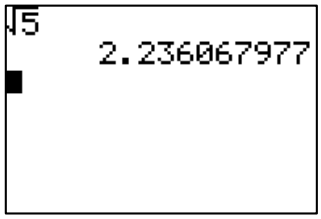
Using the Root (√) Key

To approximate square roots on the graphing calculator, you must place the √ symbol before you enter the number. This is different than many scientific calculators. The √ symbol located above the x^2 key and can be accessed by pressing 2ND and x^2.

Remember: The key √ meaning the root of the second power (square root) only.

Example:

Find $\sqrt{5}$



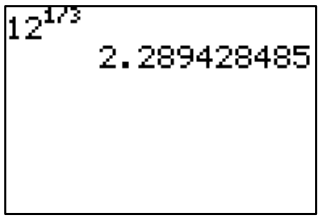
Press **2ND √ 5) ENTER**

Find the root from the number with the power greater than second

Remember the transformation formula: $\sqrt[n]{a} = a^{\frac{1}{n}}, a \geq 0$

Example:

Find $\sqrt[3]{12} = 12^{\frac{1}{3}}$



Press: **12 ^ 1 ÷ 3** the side bottom **›** and **ENTER**

Using the Correct Negative Sign

On the calculator TI-84, you may notice 2 negative signs.

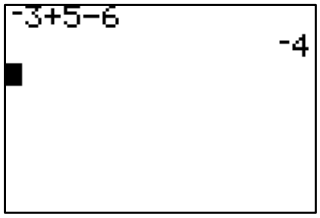
The **–** sign, located in the right column, it is the subtraction sign. You use it for mathematical operations like 7 – 6 and 34 – 45.

The **(-)** sign, located to the left of the enter key, is the negative sign. It goes in front of a number to negate it. -4 would be entered by pressing (-) then 4.

This may take some time to get used to. If your calculator shows **ERR: SYNTAX** after you press enter, you have probably used the wrong sign! This error gives you two options:
1) **QUIT**
2) **GO TO**.
QUIT brings you to the home screen whereas **GO TO** brings you directly to the error.

EXAMPLE

-3 + 5 − 6 = should be entered as



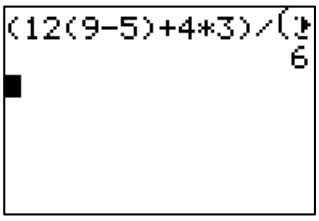
(-) 3 + 5 – 6 ENTER

Using Parentheses

On the TI-84, grouping symbols, like the fraction bar, must be replaced by parentheses. This is true for both numerical and algebraic expressions.

Example:

$$\frac{12(9-5)+4\cdot 3}{3^2+1}=$$



Press: **(12 (9 – 5) + 4 × 3 ÷ (3 ^ 2 › + 1) ENTER**

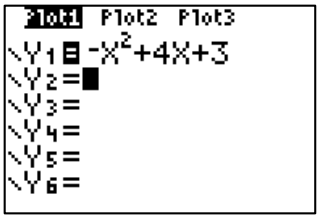
Attention: Your typing message cannot fit at the calculator's screen completely and it mark by the black triangle ▸ , mean the mathematic phrase still continued.

Evaluating Expressions Using the Table

The TI-84 calculator have a table feature that enables the calculator to evaluate a variable expression for different *x* values.

Example:

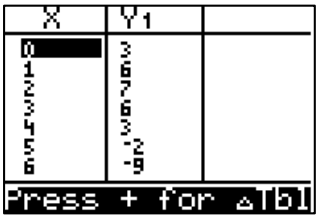
Find the values of the function $f(x) = -x^2 + 4x + 3$ for $x = \{0, 1, 2, 3\}$



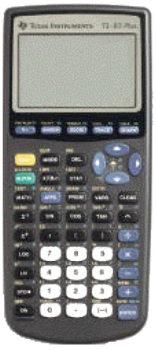
Step 1. Press **Y=** for enter the equation. The variable *x* is typing by pressing the X, T, θ, n key.
Press: **(-)** $\boldsymbol{X, T, \theta, n}$ $\boldsymbol{x^2}$ **+ 4** $\boldsymbol{X, T, \theta, n}$ **+ 3**.



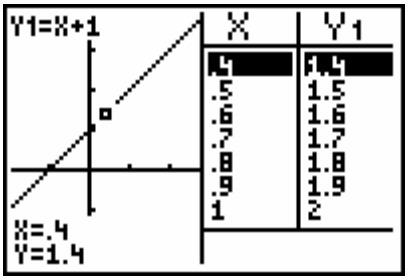
Step 2. Check the tabulator. Press **2ND** and **TBLSET**. You must be sure that all value are on the right conditions, as on the picture. If it is wrong, use the navigator key and fix it. As you will finish, press, **ENTER**.

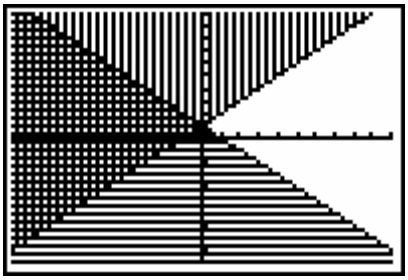


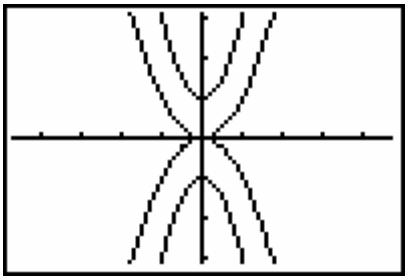
Step 3. Press **2ND** and **TABLE** and select values from X and Y from the given set.

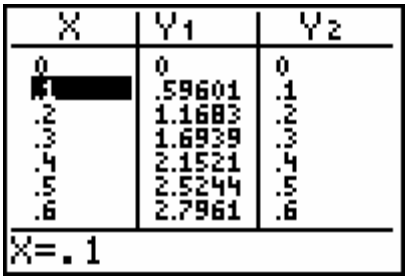


Basic Use of the TI-84+





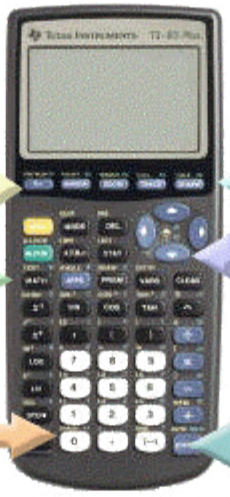




Know Your Keys

To controll Screen
BRIGHTNESS
2nd - up/down arrow

DEFAULTS:
1. MEM (2nd +)
2. #7 Reset
3. #2 Defaults



2nd Yellow (Cursor is an up arrow.)

ALPHA green (Cursor is an A.)

Graphing Keys

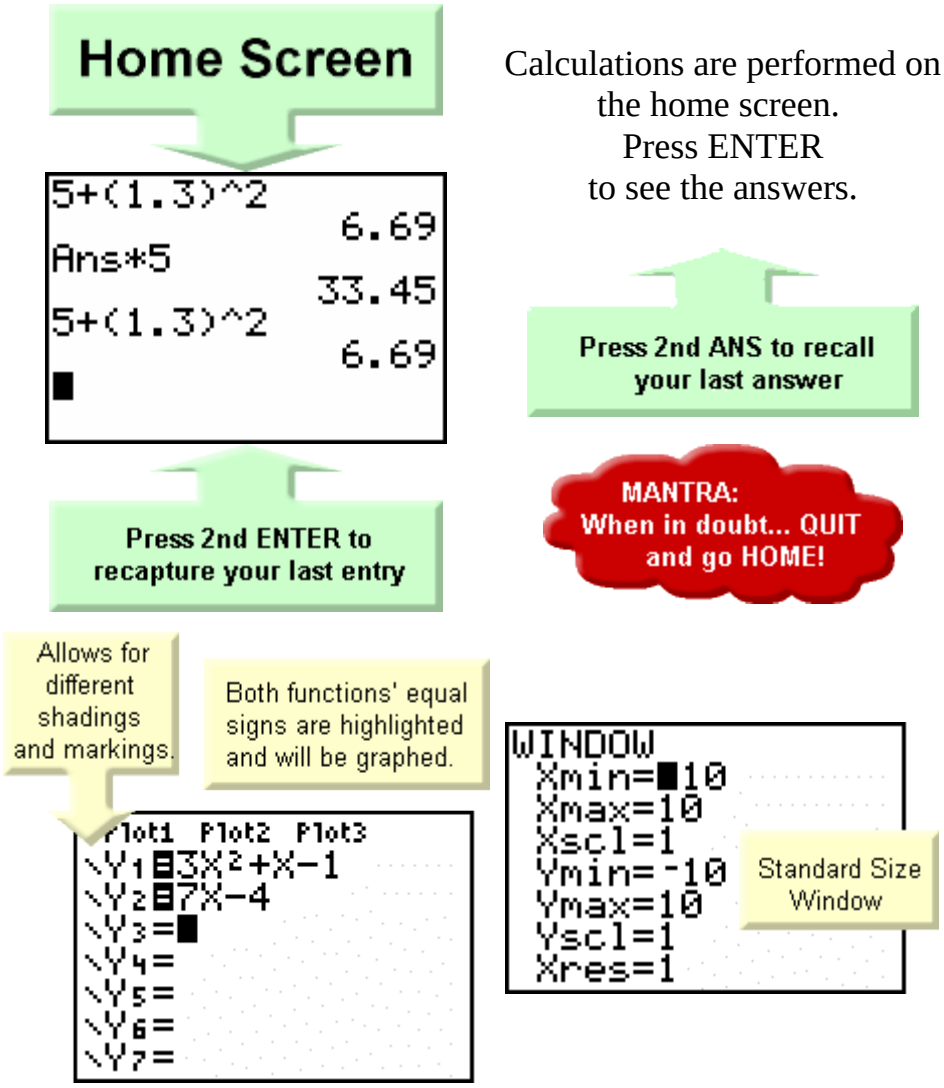
Arrows move cursor

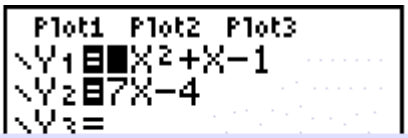
Catalog - alpha listing of items

ENTER key similar to = on other calculators

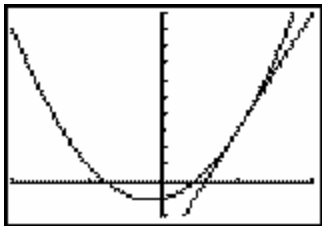
MODE G - T
Graph - Table

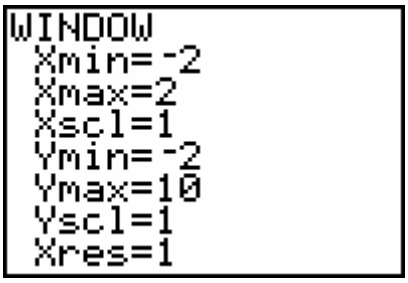
Tilt lid for easy viewing


Home Screen
5+(1.3)^2
6.69
Ans*5
33.45
5+(1.3)^2
6.69
Calculations are performed on the home screen.
Press ENTER to see the answers.
Press 2nd ANS to recall your last answer
MANTRA: When in doubt... QUIT and go HOME!
Press 2nd ENTER to recapture your last entry
Allows for different shadings and markings.
Both functions' equal signs are highlighted and will be graphed.
Plot1 Plot2 Plot3
\Y1=3X^2+X-1
\Y2=7X-4
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
WINDOW
Xmin=-10
Xmax=10
Xscl=1
Ymin=-10
Ymax=10
Yscl=1
Xres=1
Standard Size Window

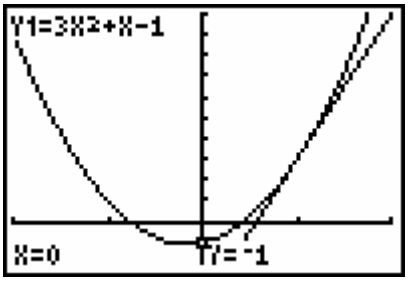


GRAPHING

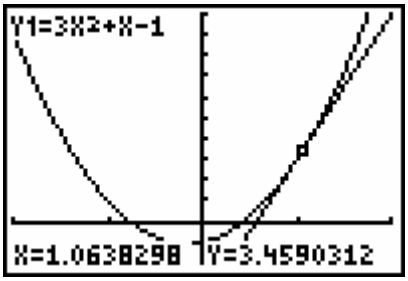




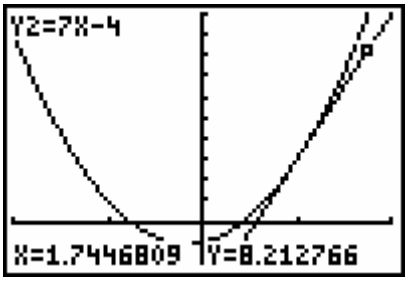
Pressing GRAPH will display the two highlighted functions entered in "y=".



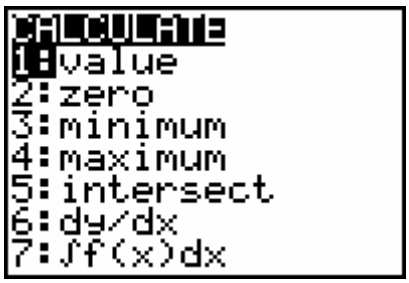
Pressing TRACE will engage the "spider" on your first graph. The equation upon which the spider is riding appears in the upper left corner.



The left/right ARROW keys will move the spider along the graph while showing you your coordinates.

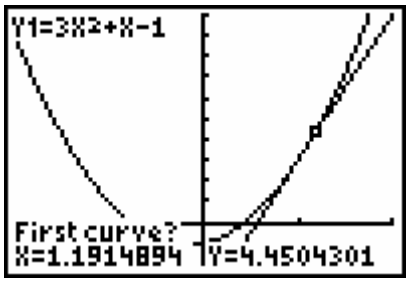


Pressing the UP (or DOWN) arrow will move the spider to the second curve. Notice the change in the equation.

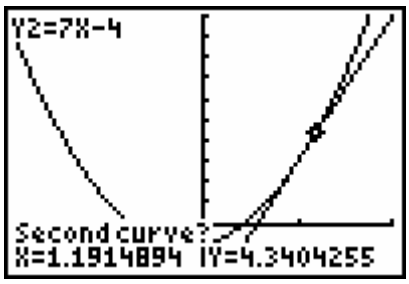


Intersection Points

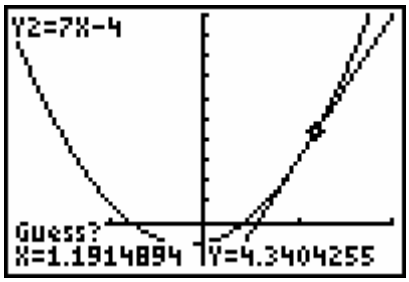
Use the CALC menu (2nd TRACE). Choose option #5



When asked for "First Curve?", move the spider along the curve to the approximate point of intersection. Press ENTER.

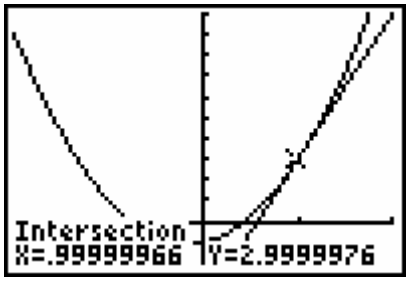


When asked for "Second curve?", move the spider close to the intersection. Press ENTER.



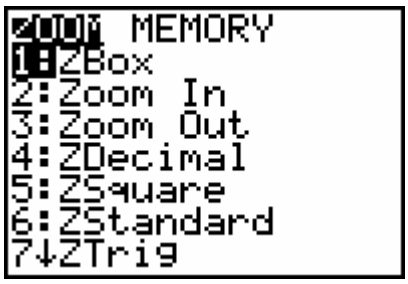
When asked for "Guess", just press ENTER.

Read the coordinates of the intersection at the bottom of the screen.



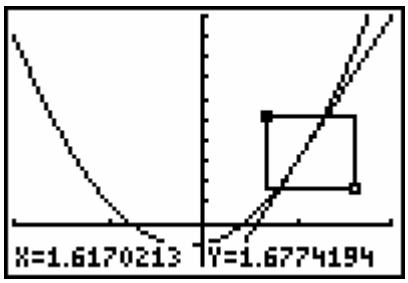
ANSWER

Zooming

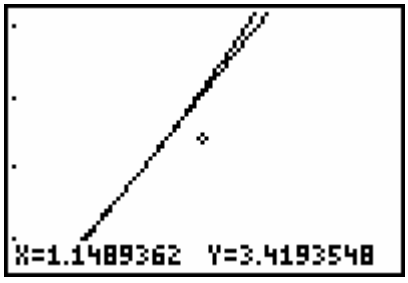


Press ZOOM to see your options.

Option 6: Standard is the safety valve. Zstandard will always return you to the standard window settings of 10 by 10.



Option 1: Zbox (Zoom Box) Use arrows to move the + to the upper left hand corner of your box. Hit ENTER to anchor this corner. Use arrows to move to the lower right hand corner of the box. Hit ENTER.



You will now see an enlargement of the area inside of the box.

If you get stuck, choose Zstandard to return to a normal window.

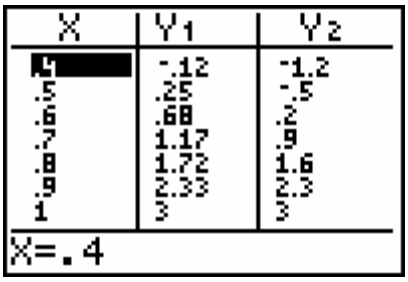
Option 2: Zoom In
Move the + near the section of the graph to be enlarged.
Press ZOOM #2. Nothing will happen until you press ENTER.

TABLES of function values



Press **2nd TblSet.**
You can choose the value to start your table.
You can choose the increment value within the table
(delta Tbl= .1 in this example)

Y-values for the first function listed under Y1

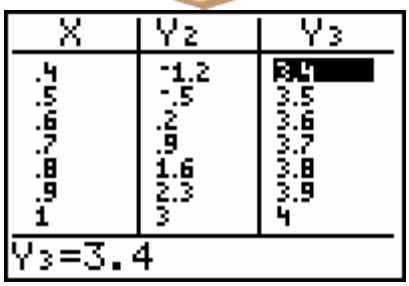


When in AUTO setting, the table values will be "automatically" generated.

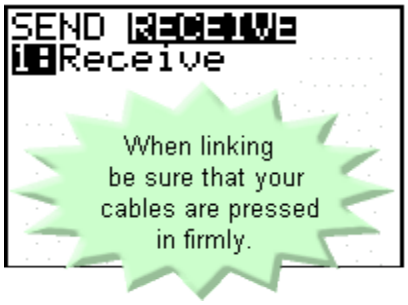
When in ASK setting, you will be able to enter the x-values you wish to investigate.

You can scroll up and down within the table. Use your ARROW keys.

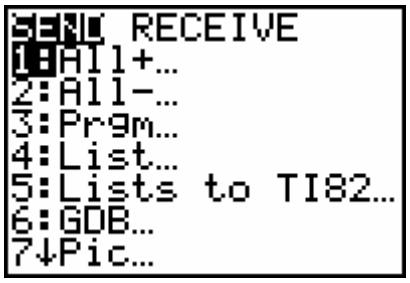
Arrowing to the right will show tables of successive functions.



Sharing Programs/APPS via Link



To RECEIVE a program, press 2nd LINK. Arrow to the right to RECEIVE. Press ENTER. It should read "waiting..."



To SEND a program, press 2nd LINK. Arrow to #3 Prgm or #C APPS, hit ENTER. Arrow down to the program(s)/APP(s) you wish to send. Hit ENTER when you reach the program. A small diamond will appear in front of the name. Arrow to the right - TRANSMIT.

To RUN a program, press PRGM. Select the name of the program. Hit ENTER. The name appears on the home screen. Press ENTER.
To EXIT a program at any time, hit ON and choose QUIT.

The TI-83+/84+ use 5 batteries: 4 AAA alkaline batteries and one lithium battery. The lithium battery provides auxiliary power to retain memory while you replace the AAA batteries. When the battery voltage level drops below a usable level, the calculator displays a warning message when you turn it on.

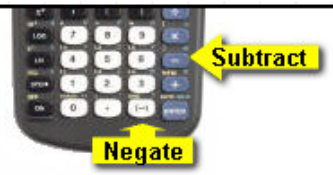
When replacing batteries:

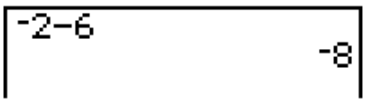
- Do NOT remove both types of batteries (AAA and lithium) at the same time.
- Do NOT mix new and old batteries. Do NOT mix brands of batteries. Install batteries according to polarity (+ and -). Always turn off the calculator when replacing batteries.
- Please, dispose of batteries properly.

Basic Use Skills

Things to keep in mind when using your graphing calculator to solve number problems:

1. Do not confuse the subtraction key with the negation key.





When you look at the subtraction symbol on the screen, it will appear to be slightly wider and lower set than the negation symbol.

2. There are two basic ways to square a value.



The caret symbol (^) can be used to raise a value to **any** power.

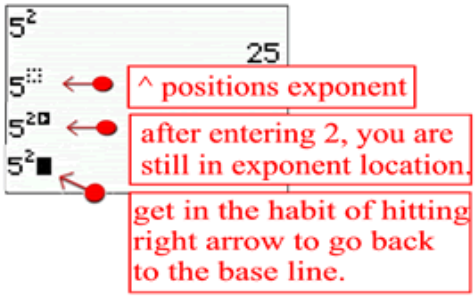
Classic Mode:



Squaring can be accomplished by using the "squared" key or by raising the value to the power of 2 with the use of the ^ (caret) symbol.

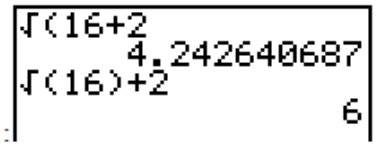
MathPrint mode: the ^ (caret) will move you up to the exponent position. Type the 2. Now, you will REMAIN up in the exponent position until you hit the right arrow to return to the base line.

Simply hitting **ENTER** after the 2 will engage the squaring process, BUT... get in the habit of using the right arrow after entering exponents, as you will need this skill on future problems.

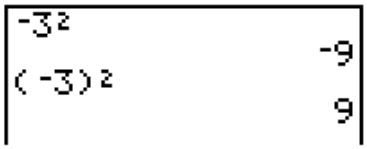


3. There may be a need for parentheses.

Classic Mode:

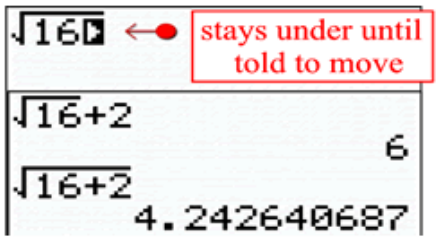


Be careful with parentheses. Some functions, such as square root, automatically start a left parenthesis for you. YOU must be sure to tell the calculator when to close the parenthesis.

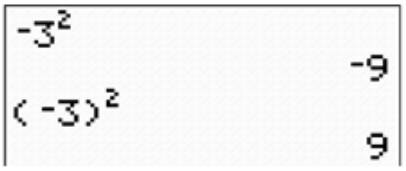


Parentheses are also important when evaluating expressions. Remember that in the order of operations, if there are no parentheses present, powers will be done BEFORE the assigning of negative status.

MathPrint Mode:



Numbers will stay UNDER the radical sign until you hit **ENTER**, or until you hit the right arrow to move out. Get in the habit of moving out from under the radical sign.

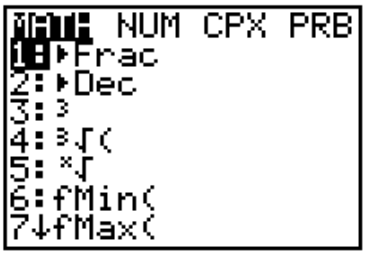


Same as the Classic Mode.

4. There are several ways to deal with the radical symbol.

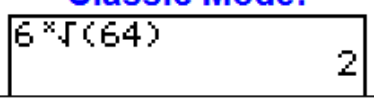


The square root symbol is above the "squared" key.



The cube root and higher roots are found under the **MATH** key (in both modes).

Classic Mode:



When finding a higher root, **enter the root number first**, then use **MATH #5**, then type the value.

MathPrint Mode:



When finding a higher root, **enter the root number first**. When you use **MATH #5**, the root will be transported to its correct position and the root symbol will appear. Remember, after entering the number under the root symbol, hit right arrow to move out from under the symbol.

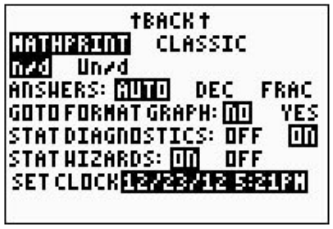
Mode Key

Hit the MODE key:

Classic Mode:



MathPrint mode adds a second page:



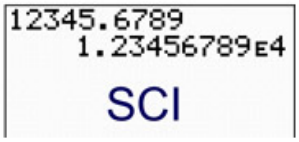
The shaded choices are set by default (meaning they are turned on unless you change them). Notice that the MathPrint mode (if available) is turned on by default. You can change back to Classic mode, but you will need to move to the statement CLASSIC, and press ENTER to highlight (engage) that choice.

Available Options:

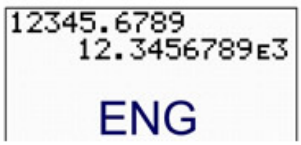
Notation modes only affect the way answers are displayed on the Home screen.
Numeric values can be displayed with up to 10 digits and a two-digit exponent.



NORMAL notation displays digits to the left and right of the decimal point, as entered.
It is the usual way we write numbers. In NORMAL mode, if the number is beyond 10 digits, or the absolute value is less than 0.001, the calculator will express the answer in scientific notation.



SCI (scientific) notation displays one digit to the left of the decimal point, with the remaining significant digits to the right of the decimal with the appropriate power of 10 displayed to the right of **E**.



ENG (engineering) notation displays the value with one, two, or three digits to the left of the decimal point, with the power of 10 exponent expressed in multiples of three.

FLOAT (means floating decimal point) displays up to 10 digits plus the sign and the decimal point.

0 1 2 3 4 5 6 7 8 9 (fixed decimal point) specifies the number of digits (from 0 to 9) to display to the right of the decimal point. Move the cursor to the desired number of decimal points and hit ENTER.

RADIAN: interprets all angle values as radians, with answers displayed in radians.

DEGREE: interprets all angle values as degrees, with answers displayed in degrees.

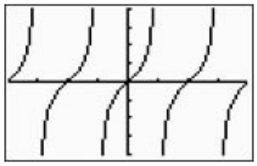
FUNC (function): plots functions with Y being a function of X.

PAR (parametric): plots relations with X and Y being functions of T.

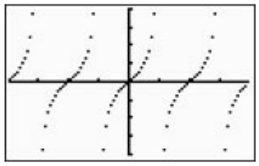
POL (polar): plots functions with r being a function of theta.

SEQ (sequence): plots sequences

CONNECTED: draws a line connecting each point calculated of the function.



DOT: plots only the calculated points of the funciton.



SEQUENTIAL: this choice evaluates and plots ONE function completely before the next function is evaluated and plotted.

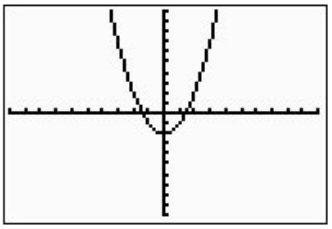
SIMUL (simultaneous): this choice evaluates and plots ALL functions for a single value of X and then evaluates and plots them ALL for the next value of X.

REAL: complex numbers are not displayed unless entered as input.

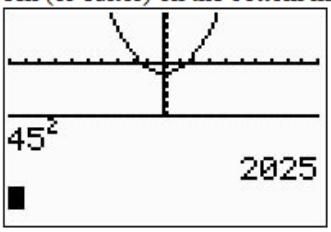
a + bi: (rectangular complex mode) complex numbers are displayed in *a + bi* form.

re^Øi: (polar complex mode) complex numbers are displayed in $re^{Øi}$ form.

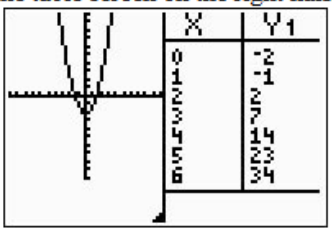
FULL: use the entire screen to display a graph



HORIZ: (horizontal) displays graph on top half of screen, with Home screen (or editor) on the bottom half.

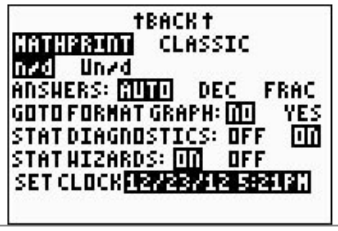


G-T: (graph-table) displays graph on the left side of the screen, with the table screen on the right half.



SET CLOCK: The clock allows you to set the time, date, and clock display.

MathPrint mode adds a second MODE page:



n/d U n/d: fractions displayed as an improper fraction OR as a mixed number.

ANSWER:

AUTO: Integer input yields integer output; integer and fraction (no decimal) input yields fraction output; decimal input yields decimal output.

DEC: output is always a decimal (either exact or approximate depending upon the input).

FRAC: output will try to be a fraction when possible.

GO TO FORMAT GRAPH: Selecting YES will automatically go to the FORMAT screen. The YES will not lock.

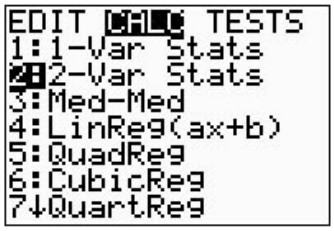


FORMAT

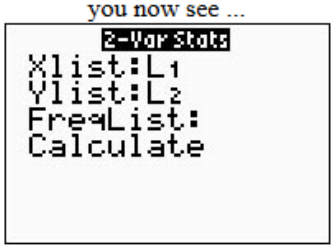
STAT DIAGNOSTICS: Diagnostics ON turns on the display of r and r^2 when calculating statistical regressions. This option is just a shortcut to turning the Diagnostic ON through the CATALOG.

STAT WIZARD:

From **STAT CALC**:



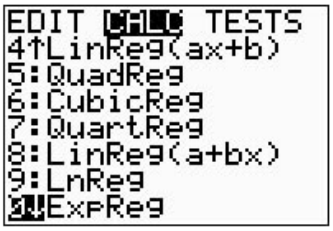
you now see ...



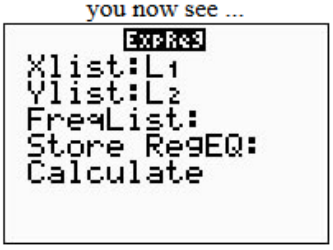
Note: For most situations, leave FreqList blank.
FreqList accepts **list names only**!
When left blank, the frequency is assumed to be 1.

After choosing Calculate (ENTER), you see ...

From **STAT CALC**:

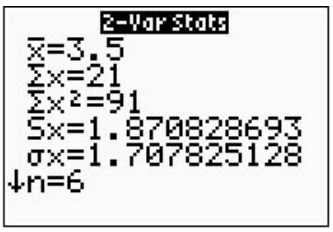


you now see ...



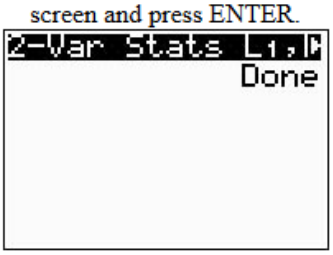
Note: For most situations, leave FreqList blank.
FreqList accepts **list names only**!
When left blank, the frequency is assumed to be 1.

Include Y1 (or other Y location) as the Store RegEQ.
Shortcut for Y1 is ALPHA, F4, ENTER.



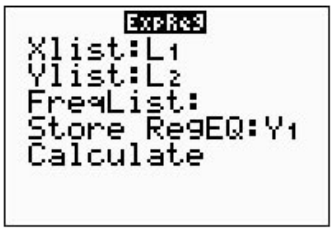
This is a temporary results screen.

Pressing CLEAR will return to a blank HOME screen. If you need to see the results again, arrow up on the HOME screen and press ENTER.

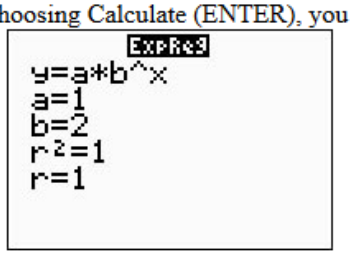


Note that lines in MathPrint mode scroll left and right. Lines in Classic mode wrap on the screen.





After choosing Calculate (ENTER), you see ...



This is a temporary results screen.

Press GRAPH to see the plot and the regression equation.

Basic Graphing

Both Classic Mode and MathPrint Mode

1. Enter equations into "Y =" area. (Be sure the PLOTS at the top of window are turned off.) When the = sign is highlighted, the graph will be displayed.

Typing Options:

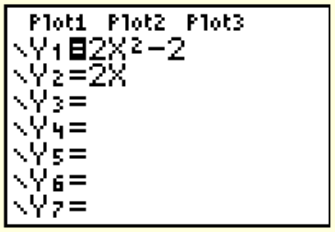
Y1 = -0.5x + 3

Y1 = -1/2x + 3

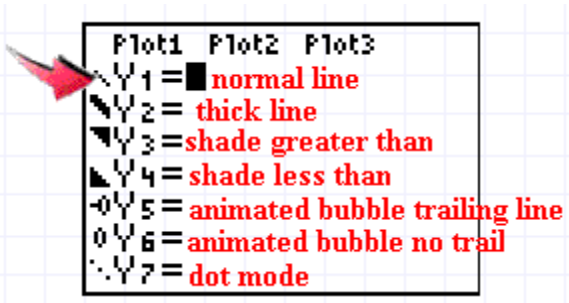
Y1 = (-1/2)x +3

The symbol to the far left of the equation indicates the style of graph you will see. For example, the -0 symbol will be an animated "bubble" with a trailing graph. To change these options, move to the far left and hit ENTER. With each hit, another choice will appear. If you go past an option, keep hitting ENTER and it will reappear.

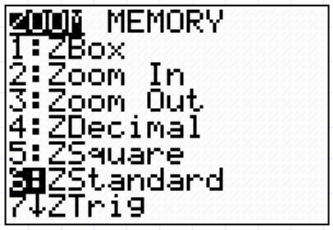
If the Plots at the top are highlighted, move to them and hit ENTER to deselect.

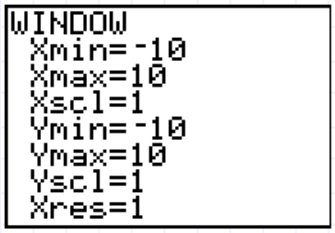


Since the equal sign in the second graph, $y = 2x$, is not highlighted, the line will not be graphed. To highlight the = sign, move to it and press ENTER.



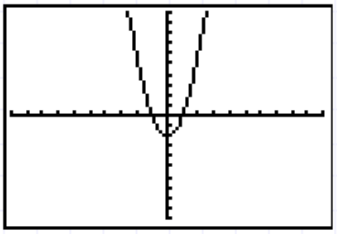
2. Choose the **viewing window**. The ZOOM key will let you chose various pre-set windows. The most common choice is #6 ZStandard which gives a [-10,10] by [-10,10] window.
Note: This window may be simply referred to as a 10 x 10 window, implying 10 units in all directions.





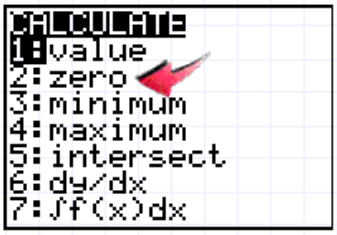
If you prefer, you can control your own viewing window by choosing WINDOW. The window shown above was the result of choosing #6 ZStandard.

3. Hit **GRAPH** to see your graphs. Notice that the second graph, $y = 2x$, did not graph. To see the graph, the = sign must be highlighted.

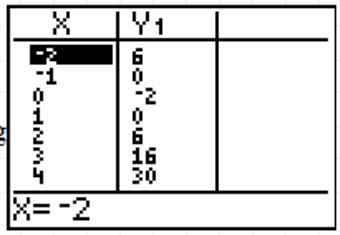


4. If you need to find the **roots of the equation** (**the *x*-intercepts**) you may choose #2 ZERO from the CALCULATE menu under CALC (above the TRACE key). The calculator will prompt you for responses.

If you need to find **points of intersection** between two graphs, you may choose #5 INTERSECT from this same menu.



5. If you need to see values relating to the graph, go to the TABLE (2nd GRAPH). You can control what you see in the table by going to TBLSET (2nd WINDOW).



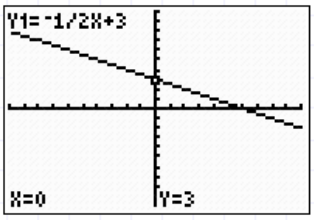


Graphing Tidbits

Both **Classic Mode** and **MathPrint Mode**

1. To find the *y*-intercept:

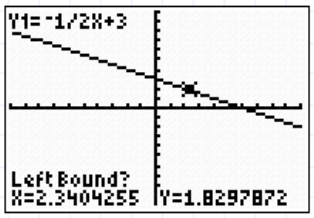
Hit the **TRACE** button. The spider will always start on the *y*-axis, showing the *y*-intercept.

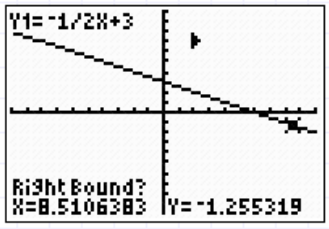


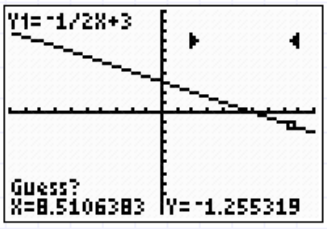
2. To find the *x*-intercept:

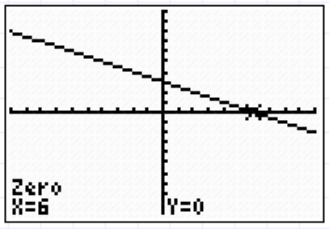
Hit **2nd TRACE** button, **#2 ZERO** (called the "Root Key") allows you to find the *x*-intercept(s).

When asked for the *Left Bound*, move the cursor to the left of the intercept. Hitting **ENTER** will place a marker on the screen. For the *Right Bound*, move to the right of the intercept. Hit **ENTER** twice.





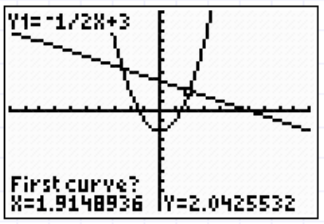


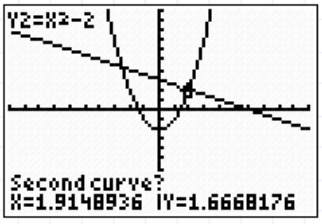


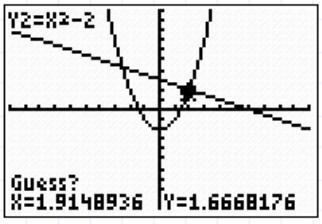
3. To find where the graphs intersect:

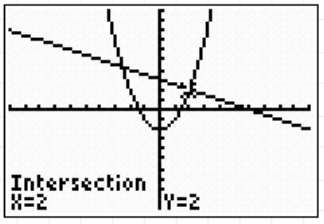
Hit **2nd TRACE** button, **#5 INTERSECT**.

When asked for the First Curve, move the cursor near the point of intersection you wish to find. Simply hit **ENTER** three times.



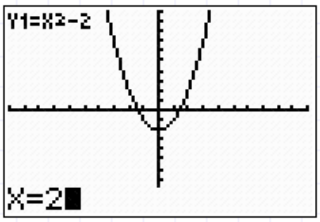






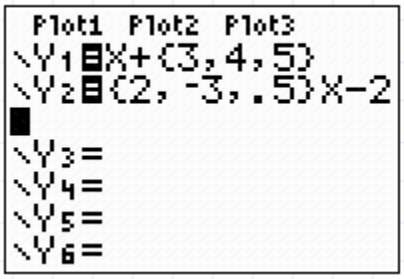
4. To find a specific value while tracing:

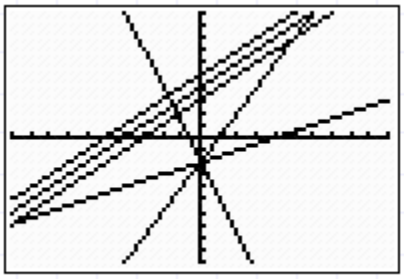
When tracing a graph, you may not be able to find the specific value that you desire by moving the arrow keys. To find your specific value while tracing, simply type the x-value that you are looking for and hit **ENTER.** The cursor will automatically move to that point and yield the y-value, if the x-value is within the viewing window. If your needed x-value is outside of the viewing window, adjust the window as to contain the needed x-value.



5. To enter multiple graphs quickly:

Lists can be used to enter several equations with similar coefficients or constants. Be sure to use French curly braces and not the parentheses. For example, Y1 = x + {3, 4, 5} will plot $y = x + 3$, $y = x + 4$, and $y = x + 5$.





Graphing a Vertical Line

(without using DRAW)

Both Classic Mode and MathPrint Mode

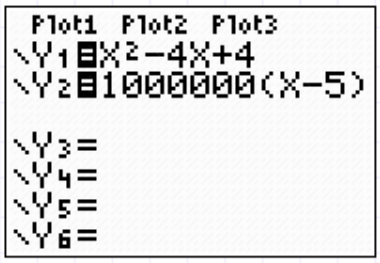
It is possible to draw vertical lines using the DRAW commands. Unfortunately, lines drawn with these commands cannot utilize any of the graphing features such as intersect, etc.

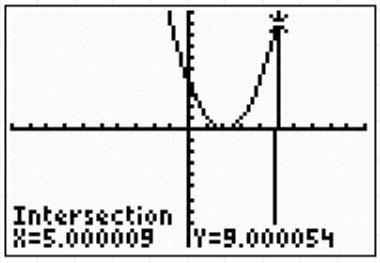
There is a way to "fake" the calculator into producing a somewhat "workable" vertical line.

To graph the vertical line $x = 5$:

Y1 = A Big Number (x - 5)

where "A Big Number" is around 1,000,000.





The intersection option can be engaged.

Rationalize:

$$x = 5$$

$$x = 5 + 0 \cdot y$$

$$x - 5 = 0 \cdot y$$

$$x - 5 = \lim_{n \to 0} n \cdot y$$

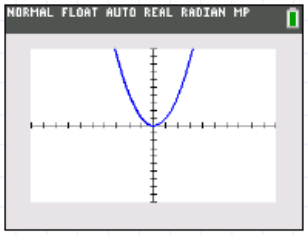
$$y = \frac{x - 5}{\lim_{n \to 0} n}$$

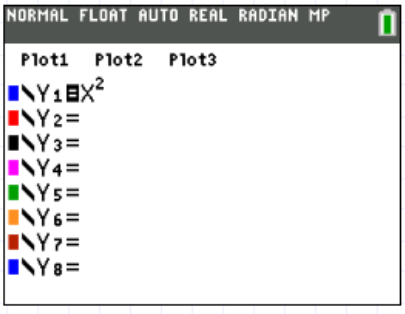
$$y \approx \infty \cdot (x - 5)$$

Investigating Graph End Behavior

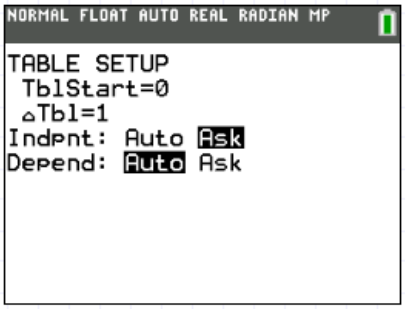
Use the **TABLE** feature to investigate the end behavior of your graphs.

1. Enter your function in "y =".



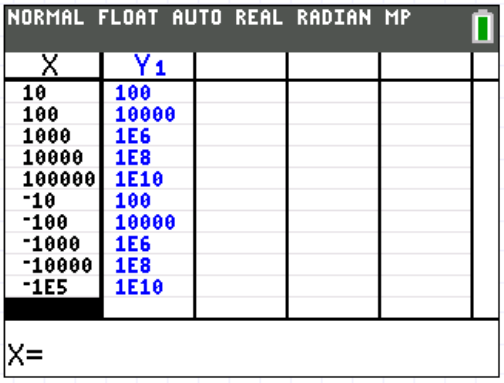


2. Set up the table to examine LARGE/SMALL values:
Hit **2nd TBLSET** and change **Indpnt:** to **ASK.**
Enter gradually increasing large values.
Enter gradually decreasing small values.
Examine what is happening to the function.



3. As the x values increased in size, the y values also increased in size.
$f(x) \to +\infty$, as $x \to +\infty$

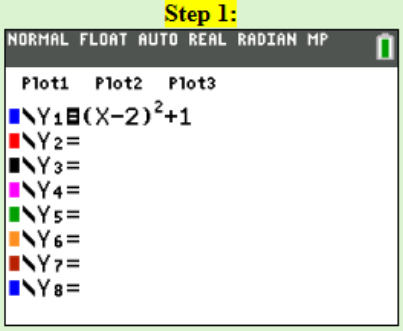
As the x values decreased in size, the y values increased in size.
$f(x) \to +\infty$, as $x \to -\infty$



Finding Maxima and Minima for Simple Functions

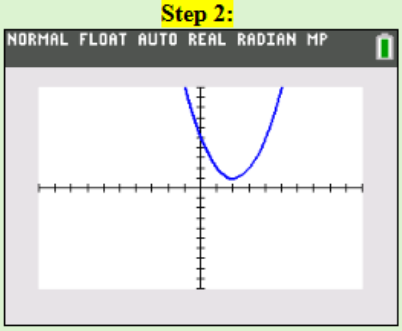
METHOD 1: Investigate relative minima/maxima for $y = (x - 2)^2 + 1$.

Step 1:



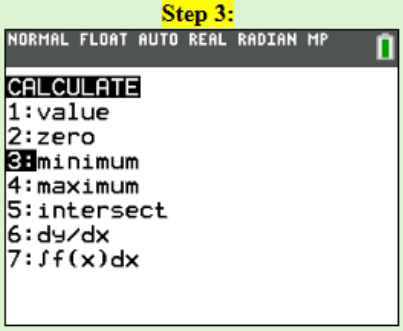
Enter the equation into **Y=**

Step 2:



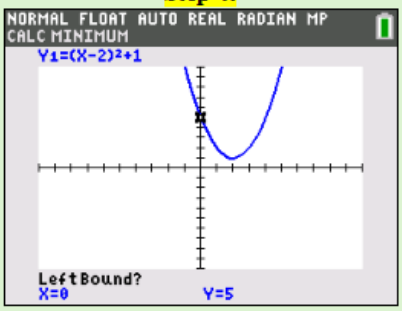
Hit **GRAPH.**
It may be necessary to adjust the **WINDOW** so you can see the "hills" and "valleys".

Step 3:



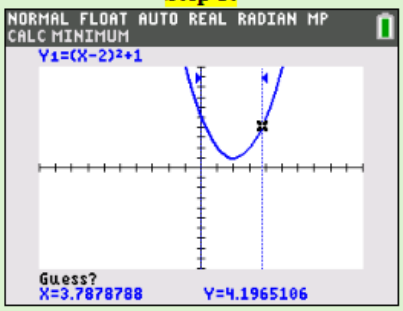
This graph has a "valley", so look for the minimum. Under the **CALC** (**2nd TRACE**) menu, choose **#3 minimum.**
Hit **ENTER.**

Step 4:



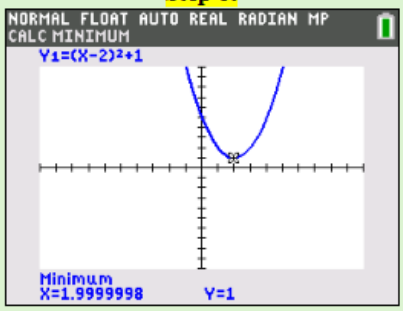
When asked for **Left Bound**, move the cursor (use arrow keys) to the left of the observed minimum location. Hit **ENTER.** You will see a ▶ mark indicating that you have "locked" this position.

Step 5:



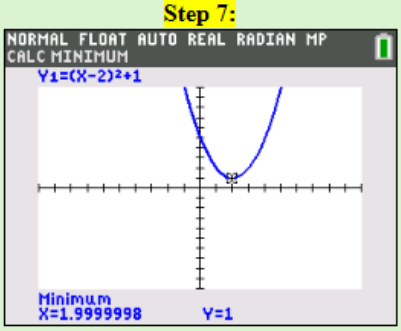
When asked for **Right Bound**, move the cursor (use arrow keys) to the right of the observed minimum location. Hit **ENTER.** You will see a ◀ mark indicating that you have "locked" this position.

Step 6:



When the last screen asked for **Guess**, simply hit **ENTER.**

Step 7:



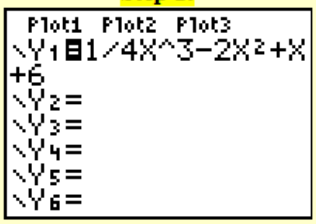
The coordinates of the miminum value (within your marked boundaries) is:
Min (2, 1)

Note:
If you are investigating more than one location on a graph, you must start this process from the beginning for each point being investigated. (Return to Step 3.)

This same process is used to examine relative maxima.
In step 3, **choose #4: maximum.**

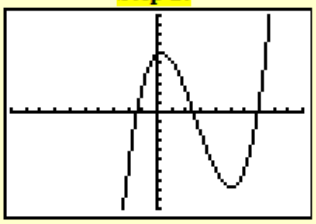
METHOD 2: Investigate relative minima/maxima for $= \frac{1}{4}x^3 - 2x^2 + x + 6$.

Step 1:



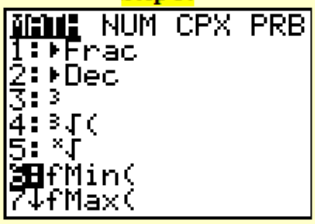
Enter the equation into **Y=**

Step 2:



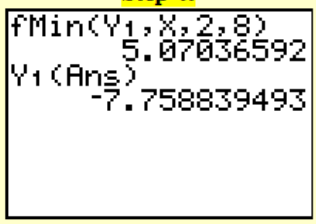
Hit **GRAPH.** Be sure the graph is viewable in the graphing window. Adjust the **WINDOW** if needed.

Step 3:



From the **HOME** screen, hit the **MATH** key. Choose either **#6 fMin** or **#7 fMax.** Hit **ENTER.**

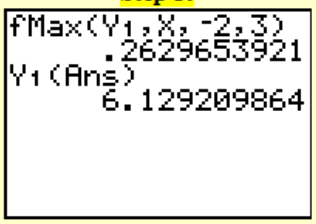
Step 4:



The parameters for **fMin** and **fMax** the same:
fMin(*expression, variable, left bound, right bound*)

Be careful: The answer from **fMin** is the ***X*-coordinate where the minimum occurs.** It is not the actual *y*-value minimum. You must then calculate the *y*-value.

Step 5:



Again, remember that the answer from **fMax** is the ***X*-coordinate** where the maximum occurs. You must then calculate the *y*-value.

HINT: To get **Y_1 (Ans)**
Y_1: VARS → Y-VARS #1Function
Ans: 2nd (-) key

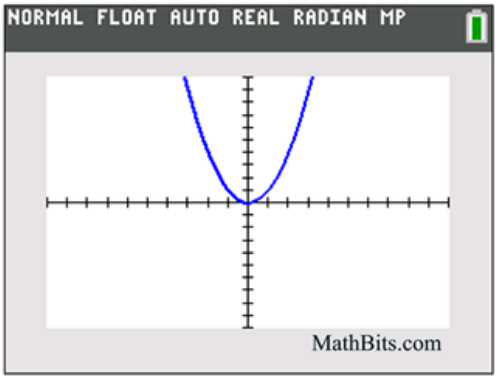
ANSWERS:
rounded to nearest thousandths

Min(5.070, -7.759)

Max(0.263, 6.129)

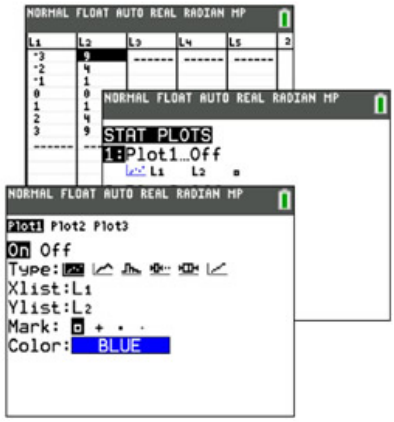
Graphing Continuous versus Discrete Functions

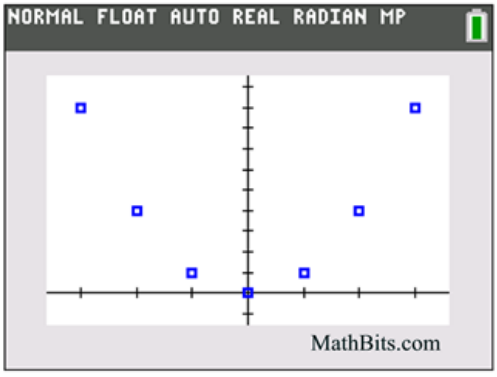
A **continuous function** allows the *x*-values to be **ANY points** in the interval, including fractions, decimals, and irrational values.	A **discrete function** allows the *x*-values to be **only certain points** in the interval, usually only integers or whole numbers.:
Graph the continuous function: $y = x^2$ for all Reals.	**Graph the discrete function: $y = x^2$ for all Integers.**
• Enter the function into "y=". • Choose the window. ZOOM #6 (standard) • Graph	• We cannot limit the point choices in "y=". We need a different approach, the scatter plot. • If your viewing window is the standard 10x10 window, you will have to determine which points will be visible in this window: (x = -3, -2, -1, 0, 1, 2, 3) • STAT - Edit - enter x (L_1) and y (L_2) values.



Notice that all of the visable points are connected with a continual drawing.

- Stat-Plot - Plot 1 - turn Plot "On" - Choose "Type" scatter plot (first icon) - Xlist: L1 - Ylist: L2 - choose any Mark.
- Choose ZOOM #9 (statistics)
- Graph





Notice that only the integer values are plotted. The points are NOT connected.

Exponents (Powers)

Things to keep in mind when using your graphing calculator with exponents (powers):

To SQUARE a value: (or raise to a power of 2)



The caret symbol (^) can be used to raise a value to any power.

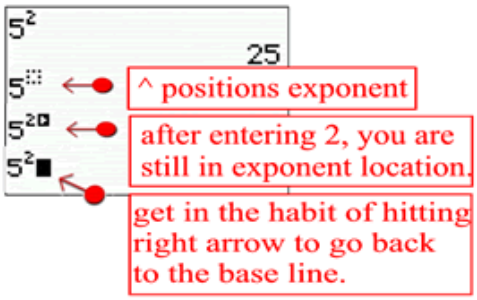
Classic Mode:



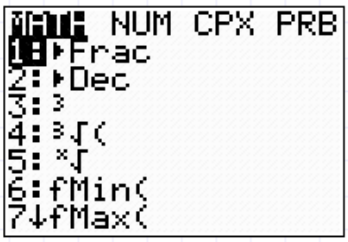
Squaring can be accomplished by using the "squared" key or by raising the value to the power of 2 with the use of the ^ (caret) symbol.

MathPrint mode: the ^ (caret) will move you up to the exponent position. Type the 2. Now, you will REMAIN up in the exponent position until you hit the right arrow to return to the base line.

Simply hitting ENTER after the 2 will engage the squaring process, BUT... get in the habit of using the right arrow after entering exponents, as you will need this skill on future problems.



To CUBE a value: (or raise to a power of 3)



Classic Mode and MathPrint Mode:

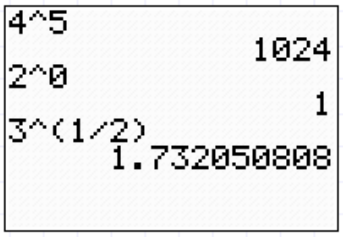
To raise to a power of three, you can utilize a special "cubing" function under the MATH key #3, or you can use the ^ (caret) symbol.



Remember, when using MathPrint Mode, the ^ (caret) will place the exponent in the raised position. Use your right arrow to leave this position and return to base line entry.

To RAISE to ANY value:

Classic Mode

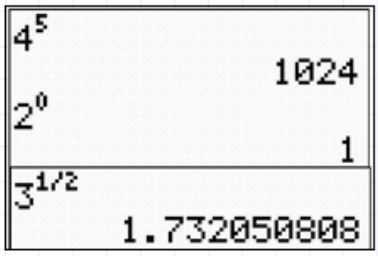


** Notice the need for parentheses when raising to a fractional power.

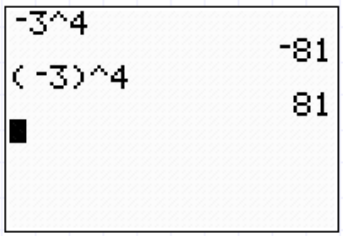
To raise to any power, utilize the ^ (caret) symbol.

Notice how raising to the zero power created an answer of 1.

MathPrint Mode



** Remember to use the right arrow to "leave" the raised exponent area.



In **BOTH modes:** Notice the need of parentheses when raising a negative number to a power. Remember, -3^4 is not the same as $(-3)^4$.

REMEMBER...

You can raise any value to any power by using the ^ (caret) symbol.

You do not have to remember the special keys for squaring and cubing.

Working with Fractions

Things to keep in mind when using your graphing calculator with fractions:

1. Yikes! There is NO fraction key!!

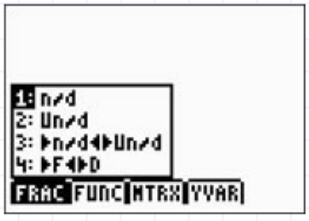
Classic Mode: Don't panic! Just remember that the fraction bar means "divide".

Enter $\frac{3}{5}$ as " 3 ÷ 5 ". The calculator will display .6

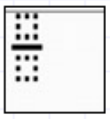
MathPrint Mode:

The MathPrint Mode has templates to help you enter fractions quickly. Press ALPHA and F1. Make a choice and use arrow keys to navigate.

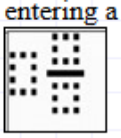




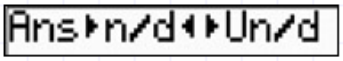
Choosing #1 will place a template on the screen for entering a fraction.



Choosing #2 will place a template on the screen for entering a mixed number.



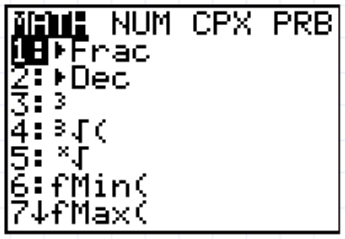
Choosing #3 will convert an entry or a previous answer to a mixed number or an improper fraction.

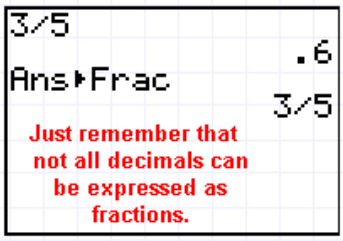


2. Convert Decimals to Fractions.

If you need to convert your decimal answer to a fraction, choose the MATH key and #1▶Frac. The screen will show Ans▶Frac. Hit Enter.
(**Note:** The fraction answer will always appear in fraction or improper form. It will not be converted to a mixed number.)

Classic Mode





MathPrint Mode



In MathPrint, you may also enter fractions as shown in #1. Choosing #4 from the F1 menu, you can also change between fractions and decimals.

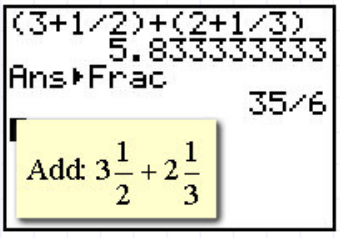


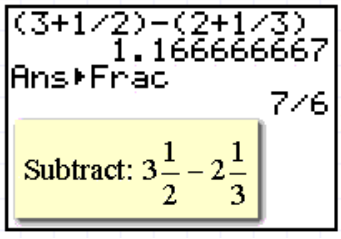
3. Mixed numbers may be entered directly.

Classic Mode: A mixed number can be entered as it is read ---
" three **AND** (+) one half "

$3\frac{1}{2}$ Enter 3 + 1 / 2

Check out the two problems below. Look carefully at the subtraction problem.

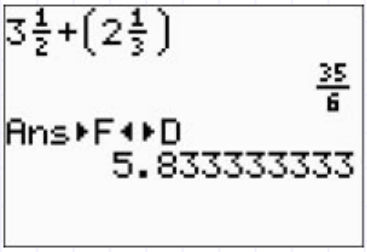


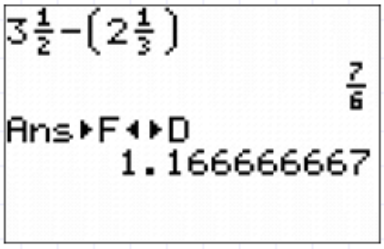


Beware!!
In the examples on the left, the **use of parentheses** was not needed when adding, but **WAS** needed when subtracting.

If the parentheses was not used in the subtraction problem, only the 2 would have been subtracted (by order of operations) and the wrong answer would have resulted.

MathPrint Mode: Mixed numbers can be entered directly using the F1 templates.

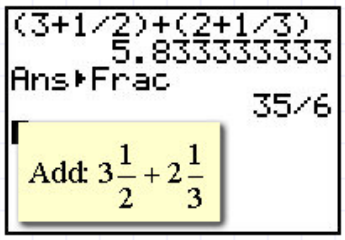




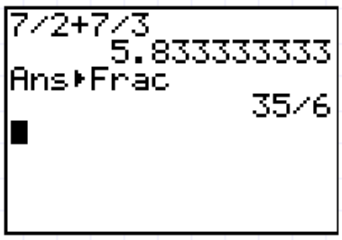
The parentheses are entered automatically by the calculator AFTER you have entered the mixed numbers. Notice that it leaves the answers in fraction form.

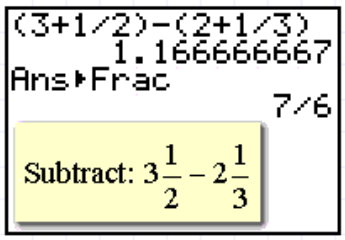
4. Mixed numbers entered as improper fractions.

Classic Mode: If you have a tendency to "forget" your parentheses when dealing with mixed numbers, you should consider changing all mixed numbers to improper fractions. This will avoid the need to worry about the parentheses.

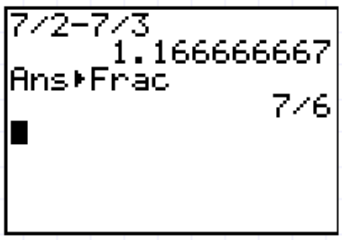


could also be ....





could also be ....

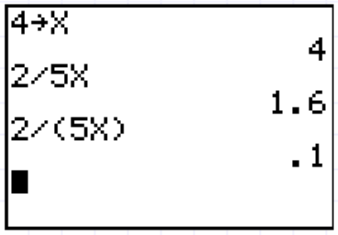


5. Entering Algebraic Fractions

Be careful when entering algebraic fractions.

Consider this problem: $\frac{2}{5x}$

Classic Mode



For testing purposes, the value 4 was stored in x using the STO▶ key.

Notice the different answers!!! The calculator only recognizes division between the character immediately before and after the division symbol.

Parentheses were needed for this problem (the second answer (.1) is correct.)

MathPrint Mode



The same problem as seen above will occur with MathPrint if you enter 2/5*x*. To avoid such problems, use the F1 template to enter the fraction. When using the template, the denominator will stay in place and avoid the problem.

When in doubt, use parentheses!

Largest Number on the Calculator

We have such faith in our calculators. It is hard to believe that they do not have the answers to ALL calculations, even in approximate form.

But, unfortunately, there is a point beyond which our calculator cannot compute. Most calculators, including the TI-83+/84+, cannot handle numbers larger than $9.99999999 \times 10^{99}$. Numbers beyond this value create an overflow error.

As an example, the largest factorial that can be computed on the TI-83+/84+ is the factorial 69!. The factorial 70! produces an overflow error.

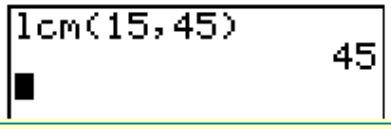
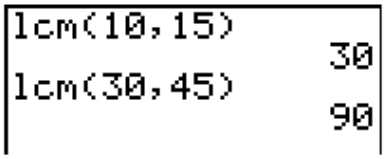
It is possible to continue investigating some larger factorials by multiplying the decimal portion of 69!.

$$69! = 1.711224524 \times 10^{98}$$
$$70! = (70 \times 1.711224524) \times 10^{98}$$
$$70! = 119.7857167 \times 10^{98} = 1.197857167 \times 10^{100}$$

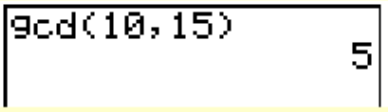
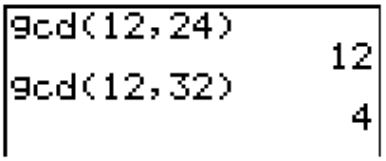
Sometimes, YOU are more powerful than your calculator!!!

LCM and GCF

Finding the **Least Common Multiple (LCM)**

LCM is found under **MATH** → **NUM**, arrow down to **#8: lcm(**	From the home screen, choose **lcm(** followed by two numbers separated by a comma.	
NOTE: **lcm(** accepts non-negative integers only and works with only two entries at a time.		
Example: Find the LCM for 10, 15, and 45.	Since LCM can only accept two entries at a time, find the LCM of the first two values, and then find the LCM of your answer and the third value.	

Finding the **Greatest Common Factor (GCF)**

NOTE: The calculator refers to the GCF as the GCD, greatest common divisor.		
GCD is found under **MATH** → **NUM**, arrow down to **#9: gcd(**	From the home screen, choose **gcd(** followed by two numbers separated by a comma.	
NOTE: **gcd(** accepts non-negative integers only and works with only two entries at a time.		
Example: Find the GCF (or GCD) for 12, 24, and 32.	Since GCD can only accept two entries at a time, find the GCD of the first two values, and then find the GCD of your answer and the third value.	

Matching Numerical Answers

The graphing calculator can be very useful for "checking" your work and for determining if your multiple choice answers are correct.
This page will address numerical style problems.

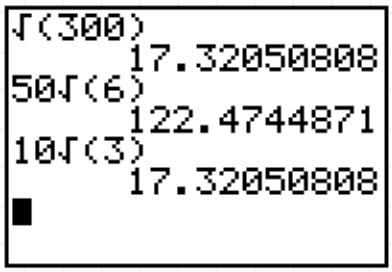
1. Determine numerical equivalents.

The calculator can be easily used to solve multiple choice style problems where numerical equivalents are needed. Find a decimal answer to the problem and then check each answer to see which answer matches your decimal.

The expression $\sqrt{300}$ is equivalent to

(1) $50\sqrt{6}$ (2) $12\sqrt{5}$

(3) $3\sqrt{10}$ **(4)** $10\sqrt{3}$



Hint: *Get in the habit of CLOSING parentheses. This habit will be most beneficial when working with longer problems.*

2. Be careful when comparing inequalities.

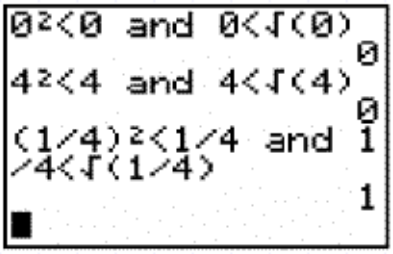
Calculators (and computers) do not evaluate "sandwich" expressions written in the form $3 < x < 10$ as you might expect. They will only evaluate the first condition ($3 < x$). To evaluate the entire expression, you will need to "tell" the calculator to also use the second expression by using the word "and". Find "and" in the CATALOG or under the TEST menu → LOGIC.

($3 < x < 10$ must be entered as $3 < x$ and $x < 10$)

If the relationship $t^2 < t < \sqrt{t}$ is true, then t could be

(1) $-\frac{1}{4}$ (2) 0

(3) $\frac{1}{4}$ (4) 4



The numerical choices have been substituted into the inequality expression.
If the answer is 0, the expression is FALSE.
If the answer is 1, the expression is TRUE.
This is called a Boolean test.

Resetting the Calculator

If you are using a school calculator only in class, you may need to reset the calculator before each use in order to erase any settings from the previous user. Ask your teacher if you should simply run the "Defaults" or completely reset "ALL Ram".

If you own your own calculator (and have been playing games), you should run the "Defaults" before math class. Games often change the calculator settings (such as turning off the axes). Remember that a "complete" reset (ALL Ram) will clear memory and delete programs and/or applications.

What are the options??

- Start by choosing 2nd MEM
- Select #7 Reset
- Choose Defaults if you simply want to return the calculator to the factory settings, such as Radian mode, Normal setting, Full screen, etc. Default will not erase programs or applications.



- Choose All Ram if you want to reset the factory settings AND erase any programs.



- Arrow to the right to ARCHIVE to erase applications and clear archived memory.



- Arrow to the right to **ALL** to clear *everything* from the calculator.

  (Teachers: This is the option that is necessary to clear calculators prior to NYS Regents Examinations.)



Certain examinations, such as the SAT Test, require that calculators' memories be completely cleared (including programs and APPS) prior to the test. If you do not want to lose your programs, be sure to back up your calculator (to another calculator or a computer) before the test.

Working with Scientific Notation

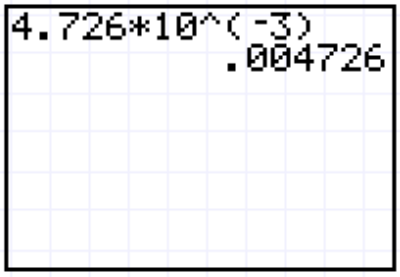
In Normal Mode:

(If a problem is already expressed IN scientific notation, it may be possible to simply enter the problem on the home screen in Normal mode to arrive at the answer.)

Expressed in decimal notation, 4.726×10^{-3} is

(1) 0.004726 (2) 0.04726

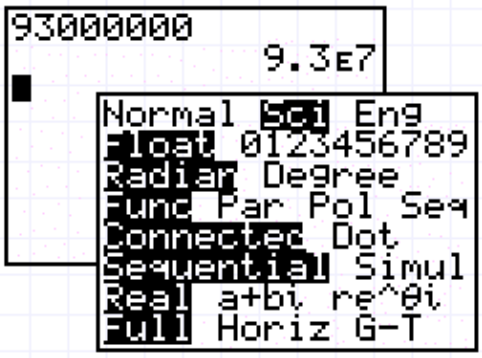
(3) 472.6 (4) 4,726



Note: In Normal mode, if the answer cannot be displayed in 10 digits or the absolute value is less than .001, the answer will appear in scientific notation.

In Scientific Mode:

(If a problem asks for the answer to be in expressed in scientific notation, change the Mode to Sci (scientific mode).



The distance from the Earth to the Sun is approximately 93 million miles. A scientist would write that number as

(1) 9.3×10^6 **(2)** 9.3×10^7

(3) 93×10^7 (4) 93×10^{10}

In scientific mode (Sci), all answers will be expressed in scientific notation.

Note: The calculator is using the notation 9.3 E 7 to mean 9.3×10^7 .

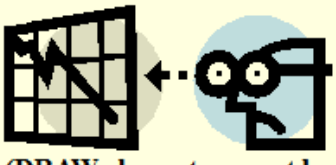
To insert the E into a calculation, press 2nd *(comma)* ,
The face of the calculator reads EE above the comma key, but only one E will appear on the screen for your scientific notation. Can be used in either Normal or Sci mode.

(*Point of interest:* The double EE is used in Engineering Mode (Eng) where the powers of 10 are multiples of 3. The form is $a = b \times 10^n$; where b is a number from 1 to less than 1000, and n is a multiple of 3.)

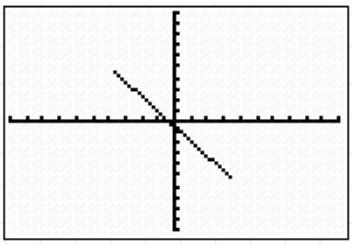
Using the DRAW Menu

Draw a Line Segment - #2 Line(

1. On the graph screen: This command draws a line segment when graph is displayed. Press DRAW, #2 Line(. Using the arrow keys, place cursor where you wish the line segment to start on the graph screen and hit ENTER. Move the cursor to the end of the line segment and press ENTER. As you can see, the segment may not be perfectly "smooth" due to the size of the screen pixels.

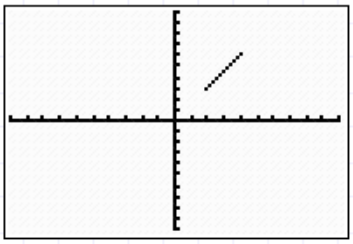


(DRAW elements cannot be traced!)



2. From the HOME screen: From the home screen, you may designate specific coordinates for the line segment. Line(X_1, Y_1, X_2, Y_2) and hit ENTER. To erase a line segment type **0** as the fifth parameter:

Line (2, 3, 4, 6, **0**) and hit ENTER.

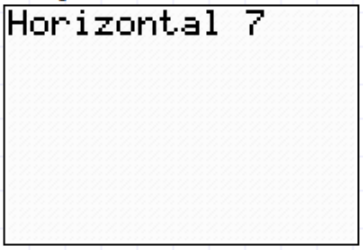


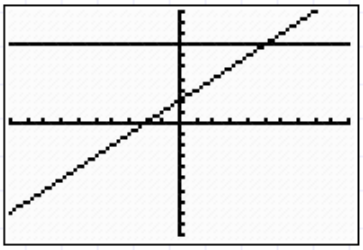
******* To erase any DRAW elements, choose #1 ClrDraw. DEFAULTS also clears the DRAW commands.***

Draw a horizontal or vertical line - #3 Horizontal, #4 Vertical

1. To Draw a moveable line on your graph: Hit GRAPH. Choose DRAW, #4 Vertical. Now move your cursor left and right to create a moving vertical line. (Nice for demonstrating the Vertical Line Test for functions.) The same applies to #3 Horizontal. (Nice for demonstrating Horizontal Line Test for existence of inverse functions.) To secure any of these lines, press ENTER. To cancel, press CLEAR.

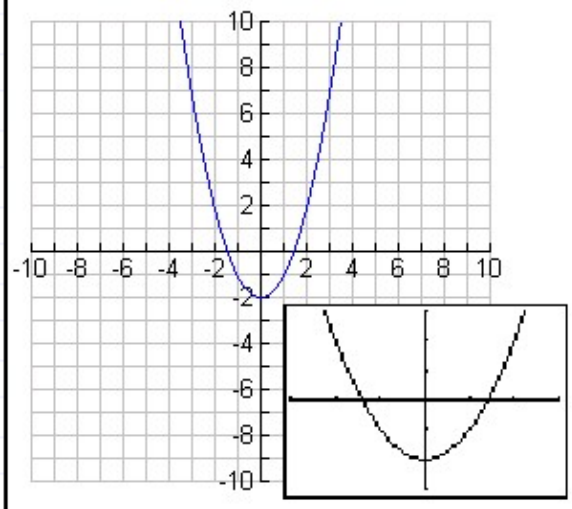
2. To Draw from the home screen: From the home screen, press 2ND, DRAW, #3 Horizontal, Horizontal 7 and press ENTER.





Window Settings

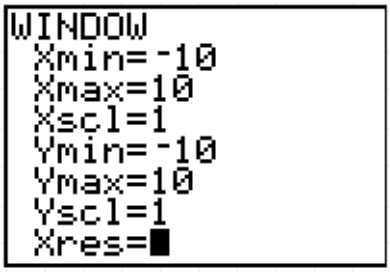
The viewing window is the part of the coordinate plane that will be visible on your graphing calculator screen.



Remember that you are only seeing a "portion" of an entire graph in your viewing window. Think of your calculator view screen as a small magnifying glass that lets you see a PART of the entire graph.

Notice that the calculator viewing window at the left shows only a small 3x3 grid section of the entire parabola. The graph paper view shows a larger portion of the parabola graph. Also notice that the calculator view shows a somewhat "wider" looking parabola. The window's aspect ratio (affecting the actual x and y scales) is 3:2 (not 1:1). The normal window is a rectangle, not a square.

You need to understand how the viewing window is set up and how to control its size.



The WINDOW key allows you to control the viewing window.

The default window (also obtained with ZOOM #6 ZStandard) is the 10 x 10 window shown at the left.

Xmin = minimum value displayed on the *x*-axis
Xmax = maximum value displayed on the *x*-axis
Xscl = distance between the tick marks on the *x*-axis
Ymin = minimum value displayed on the *y*-axis
Ymax = maximum value displayed on the *y*-axis
Yscl = distance between the tick marks on the *y*-axis
Xres = pixel resolution. Xres = 1 means that functions are evaluated and graphed at every pixel on the *x*-axis.

Xmin must be **less than** Xmax

Ymin must be **less than** Ymax

Setting
set Xscl =0 and Yscl = 0
will turn the tick marks off.

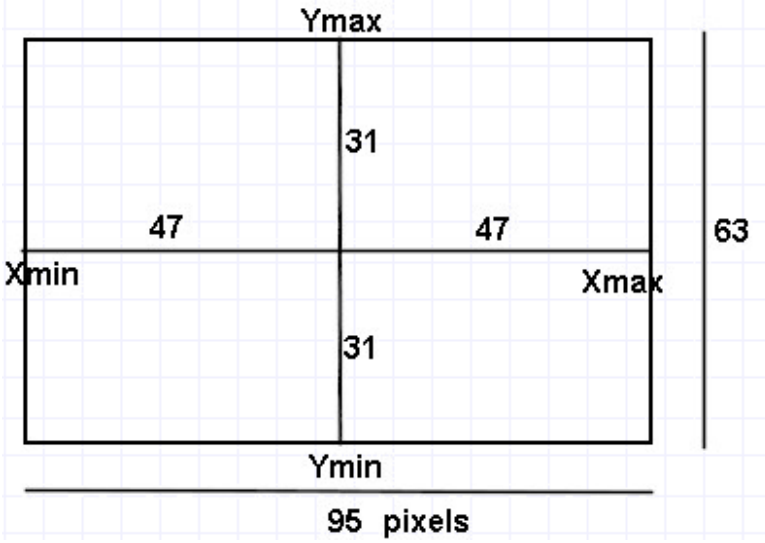
To change window settings on the WINDOW screen:
-- use your up or down arrows to move to the item to be changed
-- enter the new value (which clears the original value)
-- arrow down or hit ENTER.

Format Settings (2nd ZOOM)



... sets cursor coordinates to rectangular or polar coordinates
... sets coordinates display on or off
... sets grid on or off (GridOn shows dotted background screen)
... sets axes on or off
... sets axes label on or off (labels axes, but not ON the axes)
... sets expression display on or off

What is the physical layout of the TI-83+/84+ view screen in terms of pixels?

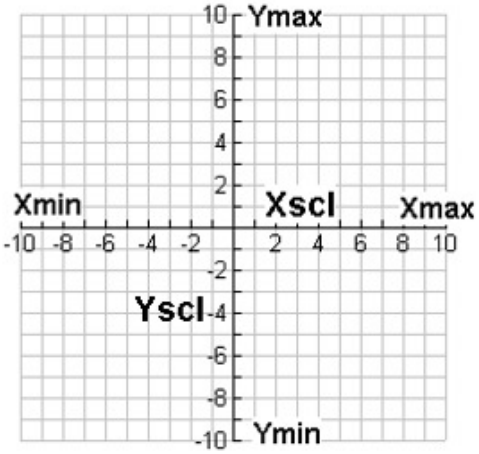


Pixels are the small cells that light up when you graph. If you turn your screen very dark (hit 2nd up arrow repeatedly) you will see the actual pixels in the screen display.

The default viewing window is set up in a 3:2 aspect ratio. The window is NOT a square.

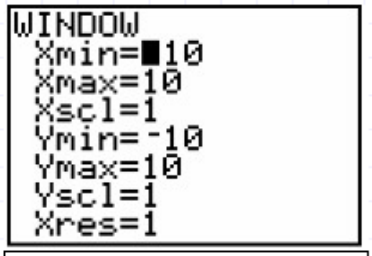
Because of these set pixels, the "nicest" viewing windows have Xmin and Xmax values that differ by a multiple of 47.

Zoom Settings



The size of the coordinate plane is defined by **Xmin, Xmax, Ymin**, and **Ymax**, as seen at the left.

- **Xmin** must be less than **Xmax**
- **Ymin** must be less than **Ymax**
- **Xscl** and **Yscl** define the distance between tick marks on the *x*-axis and y-axis
- To turn off tick marks, let **Xscl = 0** and **Yscl = 0**
- 3/2 aspect ratio in window



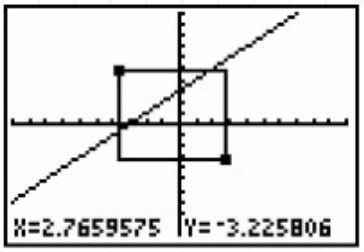
Xres sets pixel resolution 1 to 8. The default is 1. At **Xres** = 1, functions are evaluated and graphed at each pixel on the *x*-axis. At **Xres** = 8, at every eighth pixel along the *x*-axis.

Zoom - Zoom - Zoom





#1 ZBox - draws a box to define the new smaller viewing window. Move cursor to upper left corner of where your box will be placed. Hit **ENTER** to anchor that corner. Move cursor to lower right corner of the desired box, press **ENTER.** The box will fill the screen.



#2 Zoom In - magnifies the part of the graph surrounding the cursor location. Move cursor to the location you wish to magnify and hit **ENTER.** To magnify a second time, hit **ENTER** again. You can move the cursor between zooms and continue magnifying. To return to previous graph, chooose **#3 Zoom Out** and press **ENTER.** To quickly return to the 10 x 10 screen, press **#6 Zstandard.**

#4 ZDecimal - plots the function with values Xmin = -4.7, Xmax = 4.7, Ymin = - 3.1, Ymax = 3.1. The $\triangle$X and $\triangle$Y are 0.1. In plain English, you have a smaller screen which *traces nicely* in increments of one decimal place.

#5 ZSquare - $\triangle$X = $\triangle$Y. The aspect ratio is adjusted so that circles will look like circles. The domain in the window is -15 to 15 and the range is -10 to 10.

#6 Zstandard - 10 x 10 window. This window is usually large enough to show a representative portion of a graph. Remember, however, that the graph is slightly distorted because the x and y units are not equal.
#7 Ztrig - window appropriate for trig graphs. In Radian Mode, the domain is -2π to 2π, the range is -4 to 4, and Xscl = $\pi/2$. In Degree Mode, the domain is -352.5° to 352.5°, the range is -4 to 4, and Xscl = 90°.
#8 ZInteger - $\triangle$X = $\triangle$Y = 1. Xscl, Yscl = 10. Each pixel represents one unit along the horizontal and vertical axes. Move cursor to desired center, ENTER.
#9 ZStat - redefines window to appropriate size for statistical plotting. Automatically adjusts the window to show all data points in the active plot in STATPLOT.
#10 ZFit - recalculates Ymin and Ymax using current *x*-values to show best graph. The range is adjusted so that the function will enter the window at a corner on the left side. If the function is odd, it will leave the window at the other corner of the diagonal. If the function is even, the range will cover the extreme *y*-value in the other direction of the entry point.

2. Beginning Algebra

The Absolute Value Function

Where are they hiding absolute value on this calculator?

There are two ways to find the absolute value function on the calculator.

The absolute value function will look like **abs(**

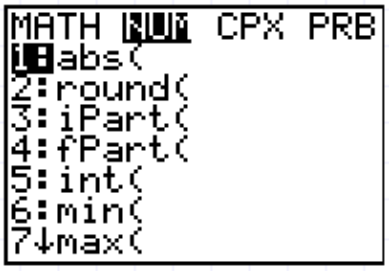
Method 1:

You can find the absolute value function by accessing the **Math** key. Arrow to the right to find the **NUM** menu.
On this screen you will find:

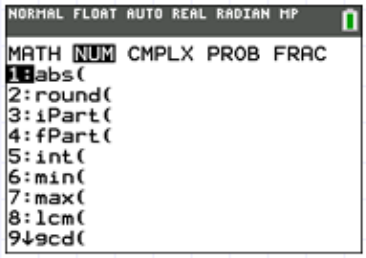
#1 abs(

the absolute value function.

Classic Mode:



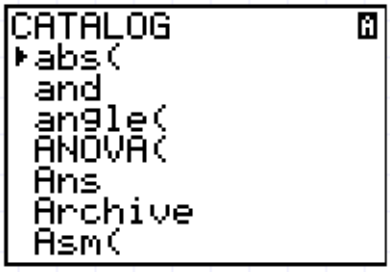
MathPrint Mode:



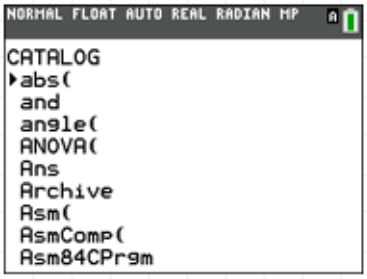
Method 2:

You can also find the absolute value function in the **CATALOG (2nd 0)**. The Catalog is an alphabetical listing of functions, and since the absolute value function starts with the letter "a", you can find the function at the beginning of the Catalog.

Classic Mode:



MathPrint Mode:

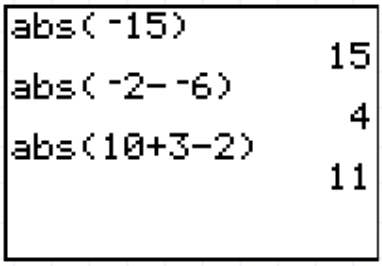


Application:

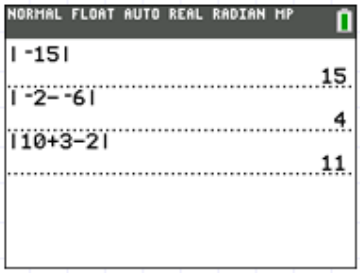
In Algebra 1, finding the absolute value of a number can be easily done without the use of a calculator. If you want, however, to check your work, you may want to use the calculator. Also, becoming familiar with the workings of your calculator will prove valuable in upcoming courses.

Example: Find |-15|, |-2-(-6)|, |10 + 3 - 2|

Classic Mode:



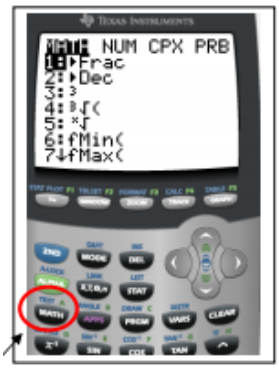
MathPrint Mode:



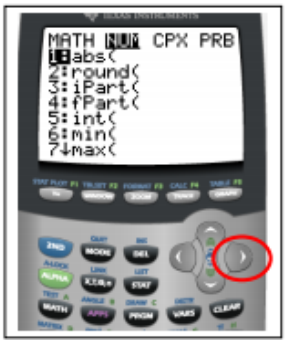
Graphing the Absolute Value Function



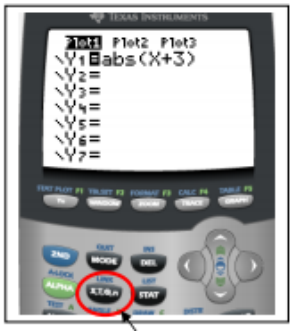
1. Press the y= button in the upper left corner.



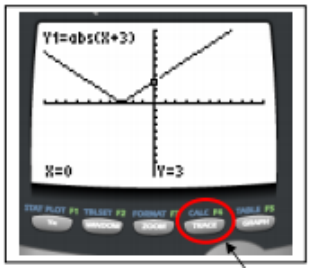
2. Press the Math button. Your screen should look like this.



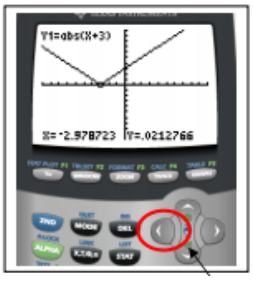
3. Press the arrow key once to move over to the NUM menu. Then press enter.



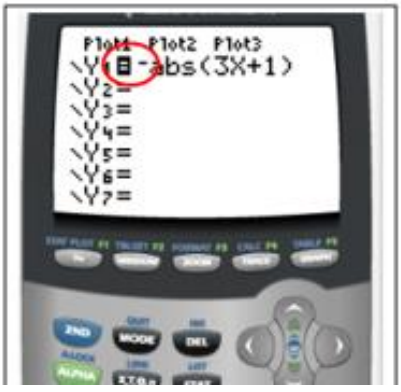
4. Let's graph the equation $y = |x + 3|$. The variable x is found under the MODE key. Type in "x + 3". Close the parenthesis when you are finished (a good habit to get into). Then press enter.



5. Your calculator will now graph the absolute value function. Press the TRACE button and the calculator will also give you the equation of the graph and the y intercept.



6. By pressing the arrow keys you can move the cursor to the approximate vertex of the graph. Note the x and y coordinates are given, but these are only approximations. To find the exam coordinates of the vertex, use the –b/a + c formula; in this case it would be -3/1 = > (-3,0).



7. Tip: when you are graphing a negative function such as $y = -|x + 3|$ be careful NOT to use the subtraction key – you will get an error message. Instead use the negative key at the bottom right of the calculator.



Numerical Checks for Multiple Choice Questions

The graphing calculator allows for several methods of checking or comparing multiple choice answers. Let's look at the checking of algebraic problems using a numerical format.

Process:

- Choose your "favorite" positive one-digit (for ease) integer value and store the value in x (do not pick 0 or 1). To store a 7: **7 STO (key) x**
- Hit **ENTER**.
- Enter the algebraic expression from the problem on the home screen using x as your variable.
- Hit **ENTER** to get your numerical value.
- Enter each of the possible multiple choice answers on the home screen.
- Hit **ENTER** to get their numerical values.
- Find the answer that matches the answer from the problem.

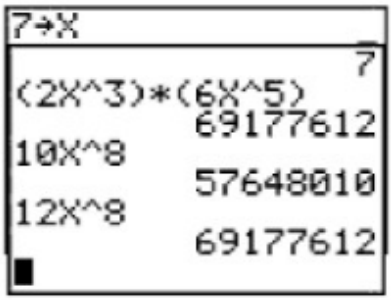
1. The product of $2x^3$ and $6x^5$ is

(1) $10x^8$

(2) $12x^8$

(3) $10x^{15}$

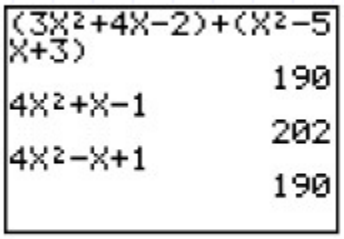
(4) $12x^{15}$



The number 7 was chosen as the "favorite" number. Remember "product" means to multiply. The expression was evaluated and the answer was 69177612. The multiple choice answers were then checked to see which yields the same answer.

2. The sum of $3x^2 + 4x - 2$ and $x^2 - 5x + 3$ is

(1) $4x^2 + x - 1$
(2) $4x^2 - x + 1$
(3) $4x^2 + x + 1$
(4) $4x^2 - x - 1$



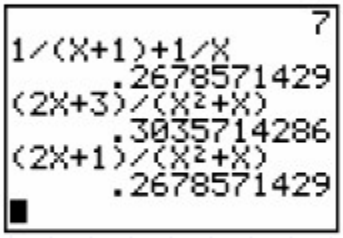
The number 7 was chosen. The sum was entered. **ENTER** was hit. The multiple choice answers were entered until the matching answer was found.

3. Express as a single fraction:

$$\frac{1}{x+1} + \frac{1}{x}, \quad x \neq 0, -1$$

(1) $\frac{2x+3}{x^2+x}$ (2) $\frac{2x+1}{x^2+x}$

(3) $\frac{2}{2x+1}$ (4) $\frac{3}{x^2}$



The number 7 was stored in x. The expression was carefully entered being sure to use parentheses for the first denominator. **ENTER** was hit. The multiple choice answers were entered, again being careful to use parentheses for the fraction's numerators and denominators.

Equation Checks for Multiple Choice Questions

The graphing calculator allows for several methods of checking or comparing multiple choice answers. Let's look at the checking of algebraic problems by forming equations.

Process

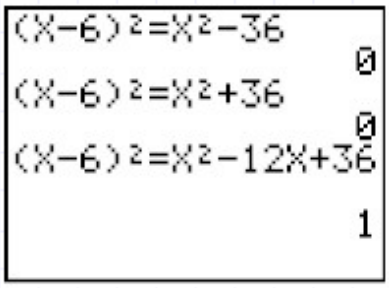
- Choose your "favorite" positive one-digit (for ease) integer value and store the value in x (do not pick 0 or 1). To store a 7: **7 STO (key) x**
- Hit **ENTER**.

- Enter the algebraic expression from the problem on the home screen using x as your variable.
- Set this expression "=" to the each of the multiple choice answers. The "=" sign is under **2nd TEST (MATH) #1 =.**
- Hit **ENTER**.
- If a **0** appears, this is NOT the correct answer. If a **1** appears, this IS the correct answer.
- The "recall entry" feature of the calculator can be used to speed up data entry. (**2nd ENTER**)

1. The expression $(x - 6)^2$ is equivalent to

(1) $x^2 - 36$
(2) $x^2 + 36$
(3) $x^2 - 12x + 36$
(4) $x^2 + 12x + 36$

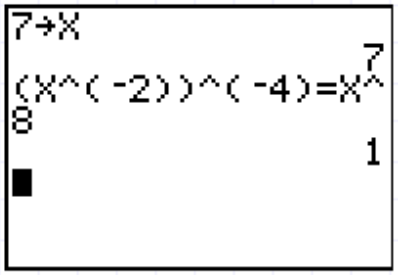
The "favorite" number used here was 7. When you hit ENTER and a "1" appears, you have found the answer (the equation is "true").



2. Which of the following statements is true?

(1) $(x^{-2})^{-4} = x^8$
(2) $-x^2 = (-x)^2$
(3) $(5x^2)^0 = 5$
(4) $3x^{-1} = \frac{1}{3x}$

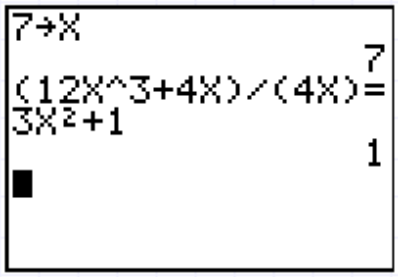
When in doubt, use parentheses when entering information.



3. When $12x^3 + 4x$ is divided by $4x$, the answer is

(1) $3x^2 + 1$
(2) $3x + 1$
(3) $3x^2 + x$
(4) $3x^2$

Parentheses are needed for both the numerator and the denominator in the division of this problem.



Writing Equation of a Line

Write the equation of the line passing through the points A(2,-5) and B(4,1).

Solution:

Normal Solution: Of course you can simply use your calculator to help you find the slope (using the slope formula) and substitute your values into the point-slope form for the equation of a straight line.

Slope Formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-(-5)}{4-2} = \frac{6}{2} = 3$$

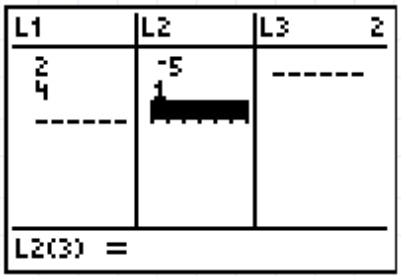
Point-Slope Form for Line:

$$y - y_1 = m(x - x_1)$$
$$y - 1 = 3(x - 4)$$
$$y = 3x - 12 + 1$$
$$y = 3x - 11$$

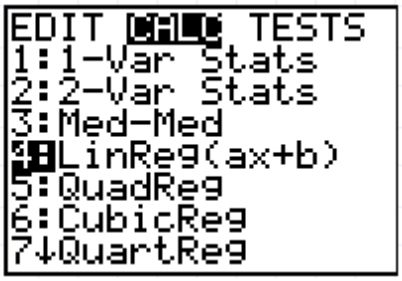
Calculator Solution: The Statistics features of the graphing calculator can be used to write the equations of straight lines given points.

1. Enter the points into lists **L1** and **L2**.
(See Basic Commands for entering data.)

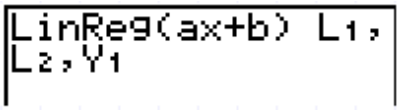
Be careful to keep the x and corresponding y coordinates lined up horizontally.



2. Choose STAT. Arrow to the right to **CALC** at the top of the screen. Choose **#4:LinReg(ax+b).**



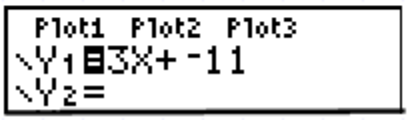
3. On the home screen you will need to tell the calculator which lists contain your points. If you also wish to quickly graph your new line, include **Y1**.
(**L1** and **L2** are above the numbers **1** and **2** on the calculator.)
(To get **Y1:** Choose **VARS → Y-VARS, #1Function.** Choose **Y1**)

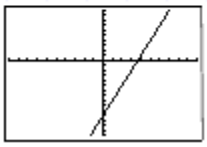


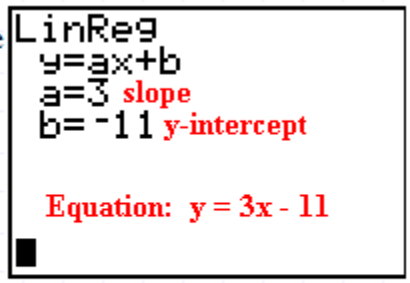
The Y1 is only needed if you wish to also see the graph of the line.

4. Hitting **ENTER** will show the screen at the right. "a" is the slope, and "b" is the y-intercept.

If you hit y = , you will see the equation for the line. Hitting **GRAPH** will show the graph.







Another way for the solution of the linear equations (downloadable software)

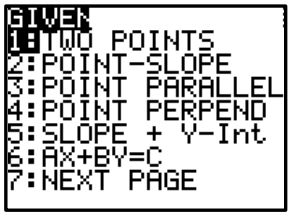
Step 1. After downloading, press the key **PRGM**. Select **EXEC** and **LINE**.



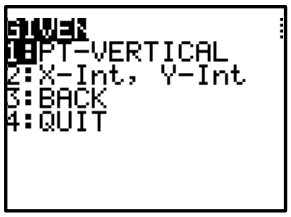


Step 2. Press **ENTER** two times.





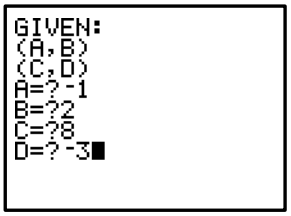
Depends from the given values choose the option from the previous screen. If you press **7: NEXT PAGE**, you will see the new screen. If you press **3: BACK**, you will open the previous screen, if you choose **4: QUIT**, you close the program.

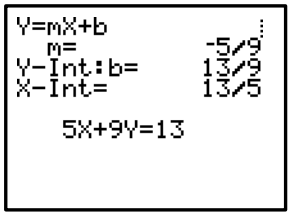


Example:

Create the equation of the line and it characteristics by the points (-1, 2) and (8, -3).

Choice **1: TWO POINTS** and hit **ENTER**. Type given x and y parameters of two points and hit **ENTER**. You will receive: the slope of the line $\boldsymbol{m = -5/9}$; intercepts **y-int** $\boldsymbol{= 13/9}$ and $\boldsymbol{x}$**-int** $\boldsymbol{= 13/5}$, and the equation of the line $\boldsymbol{5x + 9y = 13}$.





Linear Regression: The Line of the Best Fit

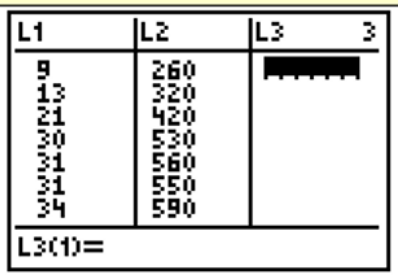
We discovered that finding the line-of-best-fit may lead to many "different" equations depending upon the points chosen to construct the line.

Let's examine our fast food problem with the use of the graphing calculator

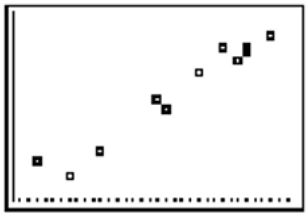
Sandwich	Total Fat (g)	Total Calories
Hamburger	9	260
Cheeseburger	13	320
Quarter Pounder	21	420
Quarter Pounder with Cheese	30	530
Big Mac	31	560
Arch Sandwich Special	31	550
Arch Special with Bacon	34	590
Crispy Chicken	25	500
Fish Fillet	28	560
Grilled Chicken	20	440
Grilled Chicken Light	5	300

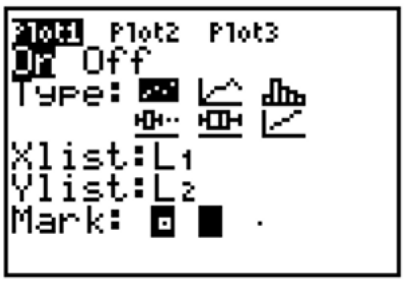
Can we predict the number of total calories based upon the total fat grams?

1. Enter the data in the calculator lists. Place the data in L_1 and L_2.
STAT, #1Edit, type values into the lists



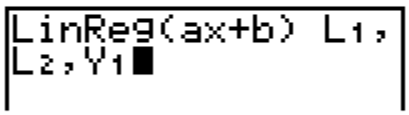
2. Prepare a scatter plot of the data.
Set up for the scatterplot.
2nd StatPlot - choices shown at right.
Choose ZOOM #9 ZoomStat. Graph shown below.



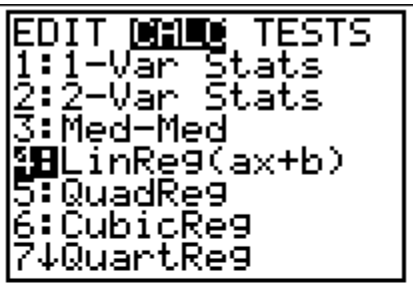


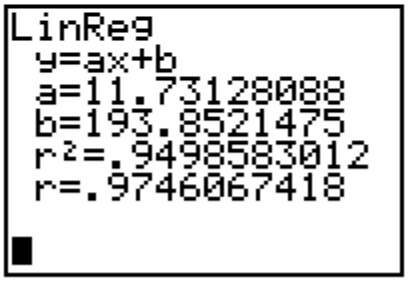
3. Have the calculator determine the line of best fit.
STAT → CALC #4 LinReg(ax+b)

Include the parameters L_1, L_2, Y_1.
(Y_1 comes from VARS → YVARS, #Function, Y_1)



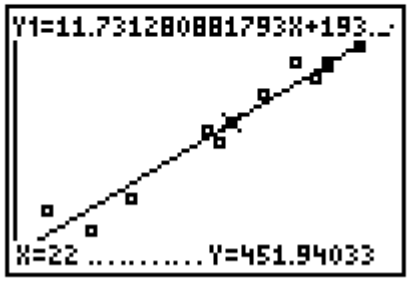
You now have the values of a and b needed to write the equation of the line of best fit. See values at the right.
$y = 11.73128088x + 193.8521475$

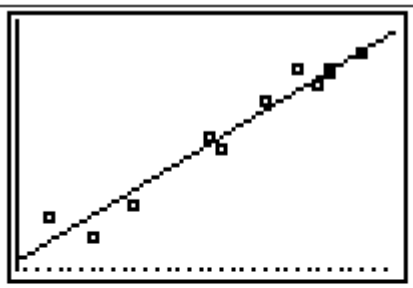




4. Graph the line of best fit.
Simply hit GRAPH.

To get a predicted value **within the window**, hit TRACE, up arrow, and type the desired value.





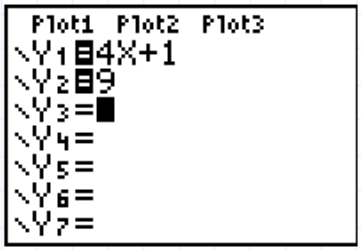
Question: Predict the total calories based upon 22 grams of fat.
ANS: 451.940 calories

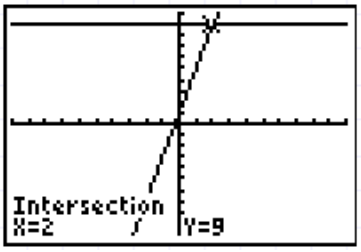
Solving Equations

The graphing calculator can be easily used to solve and/or check algebraic equations.

Example 1: Solve for x: $4x + 1 = 9$

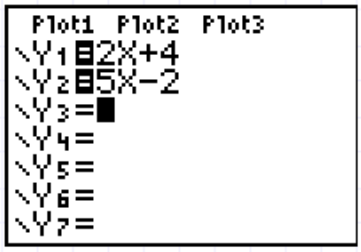
- Enter the left side of the equation into Y_1.
- Enter the right side of the equation into Y_2.
- **Graph** (you may need to adjust your window to see where the two graphs intersect)
- Find the point of intersection (the answer).
  (2nd CALC (above TRACE), #5 Intersect)
- **Answer:** $x = 2$

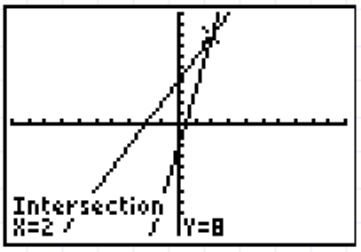




Example 2: Solve for x: $2x + 4 = 5x - 2$

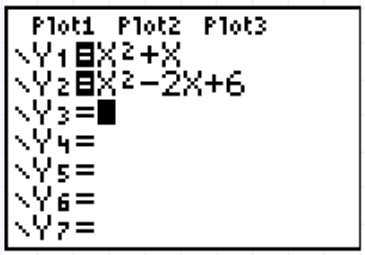
- **Answer:** $x = 2$

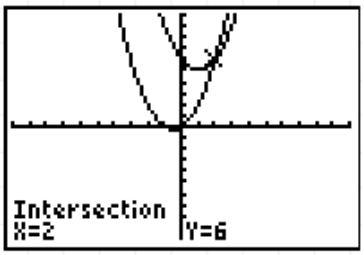




Example 3: Solve for x: $x^2 + x = x^2 - 2x + 6$

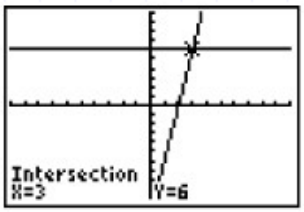
- **Answer:** $x = 2$





Example 4: **Solve for x:** $15x - 3(3x + 4) = 6$. **Choose the correct value for x.**

1) 1 (2) $-\frac{1}{2}$ (3) 3 (4) $-\frac{1}{3}$



Evaluating Algebraic Expressions, Relations and Functions

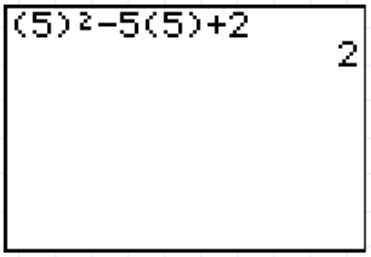
When working with algebraic expressions, relations and functions, the graphing calculator can be used in a variety of ways to help evaluate the expression or determine the *y*-values for given domain elements. The following methods may be used with expressions, relations and/or functions. Relations and functions must be entered in the form of "*y* = ".

Consider the following methods for evaluating:

Substitute Directly:

Evaluate: $x^2 - 5x + 2$ when $x = 5$

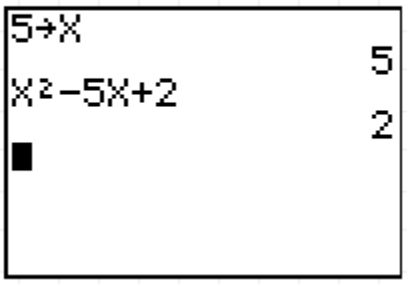
- On the Home Screen, type the expression replacing x with the desired value of 5.
- Place your desired value in parentheses to avoid any possible computational errors.
- Press **ENTER**.
- Your answer of 2 will appear on the Home Screen.



Store Domain Values:

Evaluate: $x^2 - 5x + 2$ when $x = 5$

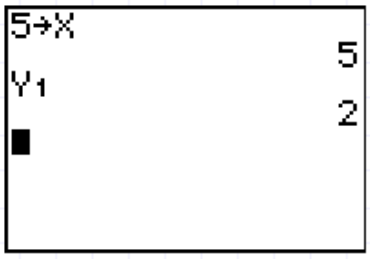
- On the Home Screen, store the value of 5 into the variable x using the **STO▶** button. Type **5 STO▶ x** and press **ENTER**.
- Type the expression $x^2 - 5x + 2$ on the Home Screen and press **ENTER**.
- Your answer of 2 will appear on the Home Screen.



Store Domain Values and Utilize Y= :

Evaluate: $y = x^2 - 5x + 2$ when $x = 5$

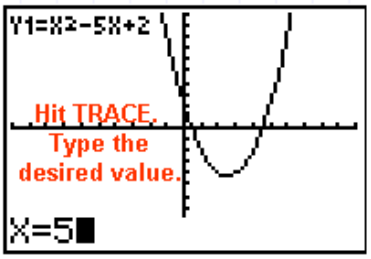
- Enter the function into **Y=** .
- On the Home Screen, store the value of 5 into the variable x using the STO▶ button. Type **5 STO▶ x** and press **ENTER**.
- Type **Y1** on the Home Screen. (**VARS**, arrow right to **Y-VARS**, **#1 Function, Y1**).
- Press **ENTER**.
- Your answer will appear on the Home Screen.

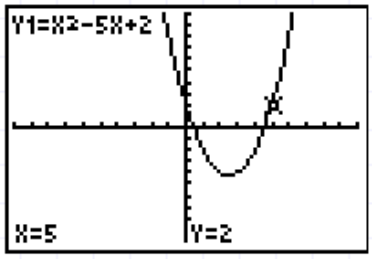


Use TRACE with Appropriate Window:

Evaluate: $y = x^2 - 5x + 2$ when $x = 5$

- Enter the function into **Y=** .
- ➡ Set the **WINDOW** such that the desired value of 5 will appear on the x-axis. (**Zoom #6** will set a 10x10 viewing window.)
- Press **GRAPH.**
- Press **TRACE**.
- Now, just type the desired value on the keypad and hit **ENTER.**
- Your answer will appear at the bottom of the screen after y = , and the spider will mark the location on the graph.



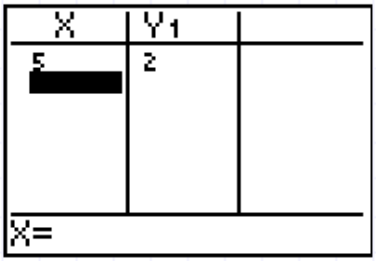


Use the TABLE:

Evaluate: $y = x^2 - 5x + 2$ when $x = 5$



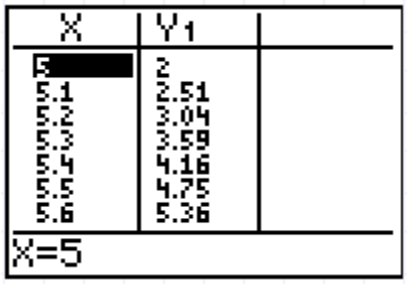
- Enter the function into Y= .
- ➧ Set the **TBLSET** (above **WINDOW**) such that the **Indpnt: ASK** is highlighted. The **ΔTbl** can be set to any increment of your choosing and the starting value is of no importance.
- Press **TABLE** (above **GRAPH**). The table will appear empty and the **X** = entry will appear at the bottom of the screen. Enter your value of x = 5 and the answer will appear in the table. If you need another value entered, simply type it in at the bottom of the screen.
- Your answer(s) will appear as a listing in the table.



or

- Enter the function into **Y=** .
- ➧ Set the **TBLSET** (above **WINDOW**) such that the **TblStart** is set equal to your desired value of 5. The **ΔTbl** can be set to any increment of your choosing. Choose **AUTO, AUTO**.
- Press **TABLE** (above **GRAPH**).
- Your answer will appear at the top of the table as the first value under the **Y1** column.





Factoring

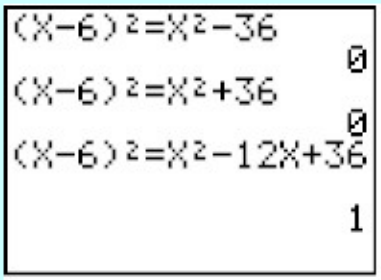
The graphing calculator can be a tremendous help when attempting to multiply or factor algebraic expressions. There are several approaches you may use depending upon the known information.

Answers can be "checked" to see if the correct factors have been found:
(particularly helpful when dealing with multiple choice questions)

Check Method 1: Check factors/answer on the home screen in equation form.

Factor $x^2 - 12x + 36$. OR Find $(x - 6)^2$.

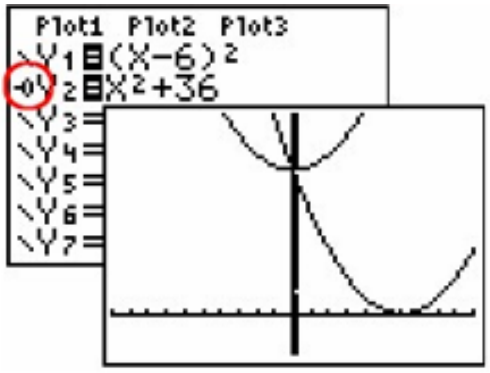
- Choose your "favorite" positive one-digit (for ease) integer value and store the value in x (do not pick 0 or 1). For example, to store a 7: **7 STO► x**
- Hit **ENTER.**
- Enter the problem to be factored (or multiplied) and set "=" to one of the possible answers (or the answer you want to check). The "=" sign is under **2nd MATH (TEST) #1 =.**
- Hit **ENTER.**
- If a **0** appears, this is NOT the correct answer.
  If a **1** appears, this IS the correct answer.



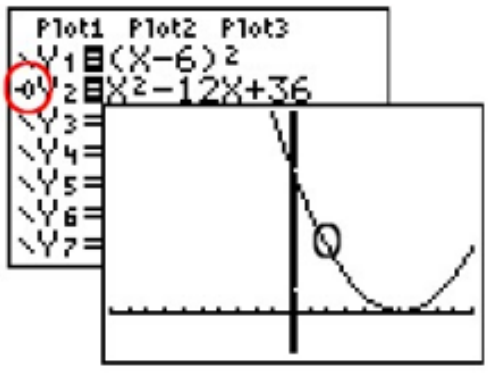
Check Method 2: Check factors/answer by graphing.

Factor $x^2 - 12x + 36$. OR Find $(x - 6)^2$.

- Enter the expression into the **Y1=** on the calculator.
- Enter the possible answer response into **Y2=**.
- Move to the left of the **Y2=** and choose the open circle. This setting will allow you to clearly see the animated circle crossing the screen.
- When the answer is correct, the animated circle will be "riding" on top of the original graph.
- Keep checking until the correct answer is found.
- The only difficult feature of this method is the occasional need to adjust the viewing window to see the graph.



The animated bubble does not run along the original curve. It forms its own graph. This is **not** the correct solution.



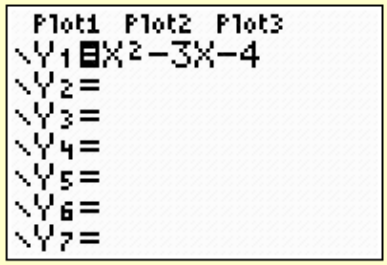
The animated bubble runs along the original graph. This is the answer.

Answers can be "found" by graphing and finding the zeros of an equation.

Graph Method: Factors can be found by finding the zeros of an equation.

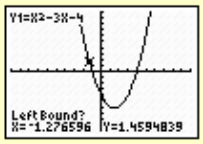
Find the factors of $x^2 - 3x - 4$.

- Enter the expression to be factored into **Y=.**
- **Graph.** The window may need to be adjusted to see where the graph crosses the x-axis (the x-intercepts or zeros).
- Use the **ZERO (2nd CALC #2:zero)** function to find the x-intercepts or zeros (also called the *roots*). See steps below:

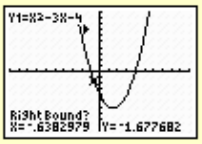




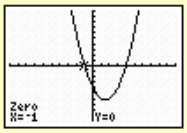
2nd CALC to locate **ZERO**



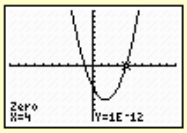
Move spider to left of x-intercept. **ENTER.**



Move spider to right of x-intercept. **ENTER.**

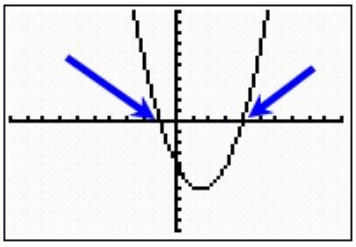


When asked for Guess, hit **ENTER.** Write down the $x = -1$



Repeat the process to find the other x-intercept. Write down $x = 4$.

We know have the *zeros*,
$x = -1$
and
$x = 4$.



The roots (zeros) of this equation (where it crosses the x-intercept) will help us find the factors.

- Using the zeros (roots), we can find the **factors.**

$x = -1$ $\quad$ $x = 4$

$x + 1 = 0$ $\quad$ $x - 4 = 0$

$(x + 1)(x - 4) = 0$

$x^2 - 3x - 4 = 0$

Check to see if the factors are correct. Multiply the factors to see if they give you the original expression.

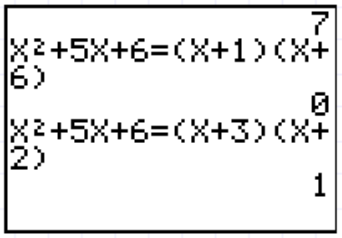
ANSWER:
Factors of $x^2 - 3x - 4 = 0$ are $(x + 1)(x - 4)$

Example 1: *(Multiple choice question)*

The factors of $x^2 + 5x + 6$ are

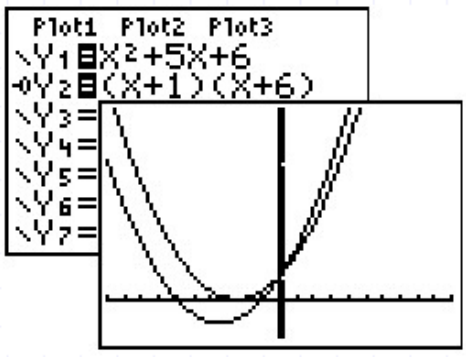
(1) $(x + 1)(x + 6)$
(2) $(x + 3)(x + 2)$
(3) $(x + 5)(x + 1)$
(4) $(x + 6)(x - 1)$

Method 1:

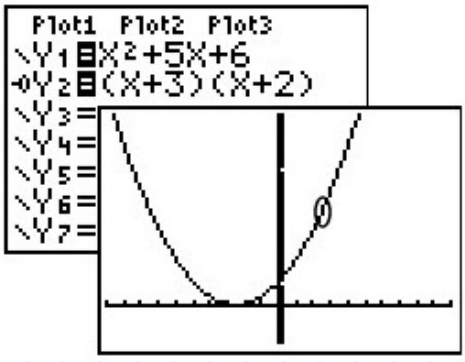


The "favorite" number stored was 7.
When you hit **ENTER** and a "1" appears, you have found the answer.

Method 2:



The two graphs do not overlap.
This is **not** the answer.



The animated bubble overlaps the original graph. This is the answer.

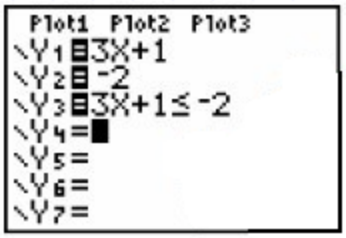
Linear Inequalities

(single variable)

Example 1: **Solve the inequality:**

$$3x + 1 \leq -2$$

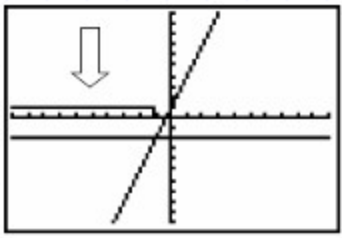
- Enter each side of the inequality into Y_1 and Y_2



- Enter the inequality statement into Y_3:

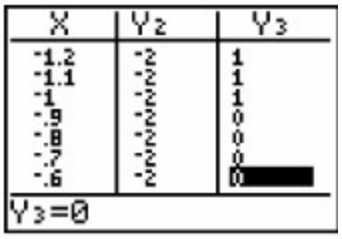
  $Y_3 = 3x + 1 \leq -2$

  (The inequality symbols are under the **TEST** Menu - **2nd MATH.**)
  This step could also be entered as $Y_1 \leq Y_2$.



- **Graph.** Use the intersect option (**2nd Trace #5 Intersect**) to find the exact point where the inequality answer begins.

- **Do a Boolean Check:** Where the inequality is true, the y-values on the graph will be a 1.



- If you look at the table, 0's will be listed where the inequality is false and 1's will be listed where the inequality is true. (*arrow to the right to see additional tables*)

- **Answer:** $x \leq -1$

- Remember that the calculator cannot draw an open or closed circle on the intervals. You will have to determine which circle is needed based upon whether the inequality includes "equal to".

If you turn off the axes (**FORMAT - 2nd ZOOM**), you will be able to see the graphing of the 0's and 1's more clearly. Notice that the small vertical segment connecting the 0's to the 1's is simply the calculator being set in "connected" mode. Change to "dot" mode to eliminate this segment.

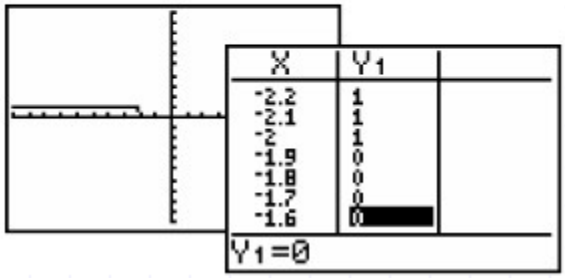
Example 2: Solve the inequality:

$$-2x - 1 \geq 3$$

- Enter the inequality into Y_1

$$Y_1 = -2x - 1 \geq 3$$

- **Graph**
- If you enter the inequality directly into Y_1, you will see only the Boolean graph. It may be necessary to look carefully in your table to find the exact point where the inequality answer begins.
- Such answers can be found easily if the answers are integer values. Otherwise, you will want to use the intersect option mentioned above to find an exact answer.
- **Answer:** $x \leq -2$
- In this problem, the calculator automatically took care of reversing the direction of the inequality when solving.



Example 3: Graph the solution set for $2x - 4 \leq 8$ and $x + 5 \geq 7$

Algebraic solution:

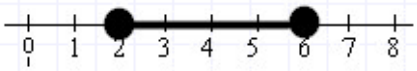
Solve each equation separately. The answer will be points that make BOTH sets true.

$2x - 4 \leq 8$ $\qquad$ $x + 5 \geq 7$

$2x \leq 12$ $\qquad$ $x \geq 2$

$x \leq 6$

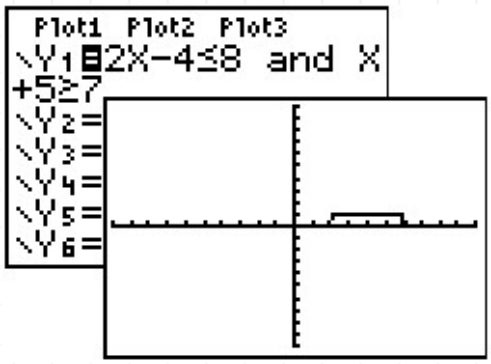
Answer: $2 \leq x \leq 6$



Graphical solution:

Both inequalities are entered into Y_1 with the use of "**and**".

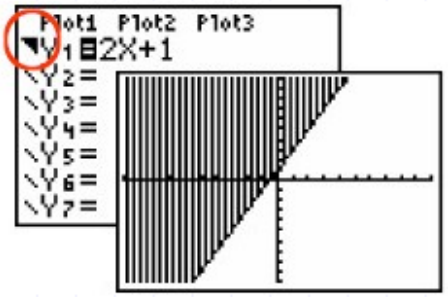
Find "**and**" by using the **CATALOG (2nd 0)** or **TEST (2nd MATH)→ LOGIC, #1 and.**



Graphing Linear Inequalities

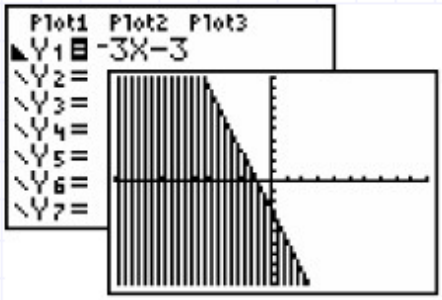
Graphing Linear Inequalities

Example 1: Graph $y \geq 2x + 1$



- Enter $2x + 1$ into **Y_1**
- Arrow to the far left side of **Y_1**
- Hit **ENTER** until the "shade above" symbol is displayed.
- Hit **ZOOM #6 ZStandard** (for a 10x10 window)
- **Graph**
- **NOTE:** You will have to determine whether to draw a solid line or a dotted line for $y = 2x + 1$. This problem uses a solid line because of the "less than or equal to" sign. The calculator will display a solid line at all times.
  (The Application program called "Inequalities" will display solid and dotted lines on the graphs.)

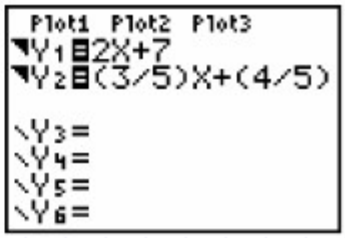
Example 2: Graph $y < -3x - 3$



- Notice the "shade below" symbol to the far left of the **Y_1.**
- Again, you must determine whether to draw a solid or dotted line. This problem uses a dotted line because it is strictly "less than" (no "equal to").

Example 3: Solve the system:

$$y > 2x + 7$$
$$3x - 5y < -4$$

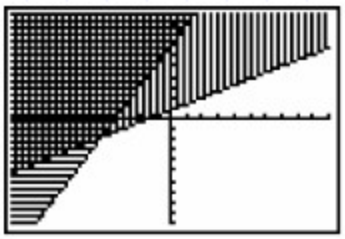


- You will need to isolate the y variable in the second equation so that it can be entered into the calculator with the "shade above" indicated.

Solving algebraically,

$3x - 5y < -4$ becomes $5y > 3x + 4$ and $y > \frac{3}{5}x + \frac{4}{5}$

(be careful of the direction of the inequality symbol in this problem.)

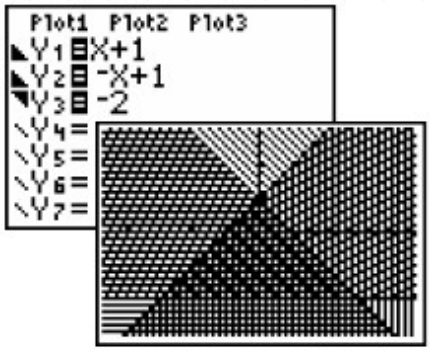


- The answer is the double shaded region on the graph.

Example 4:

Solve $y \le x + 1$ and $y \le -x + 1$ and $y \ge -2$

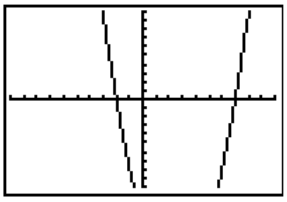
- Enter all three equations with appropriate "shade above" or "shade below" symbol.
- Answer is the darkest shading.



Zoom Decimal view.

Solving Quadratic Equations

Find the solutions (roots) of a quadratic equation using the graphing calculator.
Consider the following examples:

1.	**Using the ZERO Command**	
	Solve: $x^2 - 5x - 14 = 0$ Since this equation is set equal to zero, the roots will be the locations where the graph crosses the x-axis (if the roots are real numbers). (Remember that the x-axis is really just $y = 0$.) **1.** Set $Y_1 = x^2 - 5x - 14$ **2.** Use the ZERO command to find the roots -- 2nd TRACE (CALC), #2 zero	Standard 10 x 10 window 

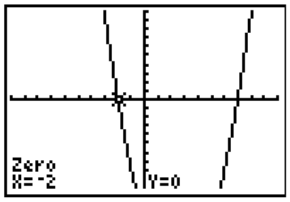
3. Left bound? Move the spider as close to the root (where the graph crosses the x-axis) as possible. Hit the left arrow to move to the "left" of the root. Hit ENTER. A "marker" ▶ will be set to the left of the root.

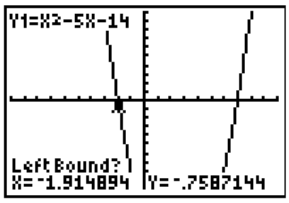
4. Right bound? Move the spider as close to the root (where the graph crosses the x-axis) as possible. Hit the right arrow to move to the "right" of the root. Hit ENTER. A "marker" ◀ will be set to the right of the root.

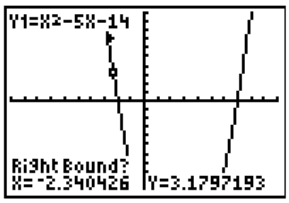
5. Guess? Just hit ENTER.

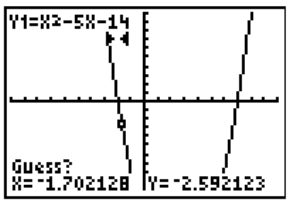
6. Repeat the entire process to find the second root (which in this case happens to be $x = 7$).

Answer: one of the roots is $x = -2$









2. Using the INTERSECT Command

Solve: $2x^2 + 2x = 7x - 2$

Since this equation is NOT set equal to zero, the ZERO command cannot be used to look for roots(unless you re-write the equation so that it IS set equal to zero).

If you do not want to re-write the equation, solve using the intersect command to find the points where the two expressions intersect (if the roots are real numbers):

1. Set $Y_1 = 2x^2 + 2x$, $Y_2 = 7x - 2$

2. Use the intersect command to find the roots, 2nd TRACE (CALC), #5 intersect.

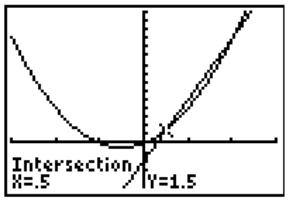
3. First Curve? Move the spider close the point of intersection that you want to find. Hit ENTER.

4. Second Curve? Just hit ENTER.

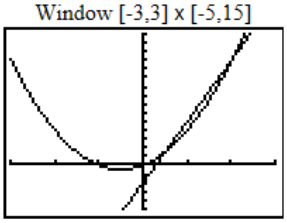
5. Guess? Just hit ENTER.

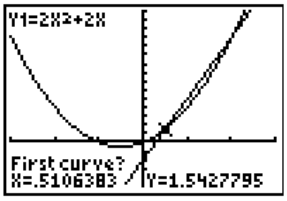
6. Repeat the entire process to find the second root (which in this case happens to be $x = 2$).

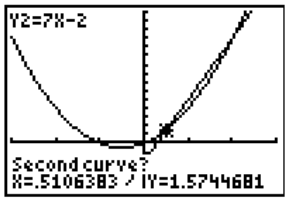
Answer: one of the roots is $x = 0.5$

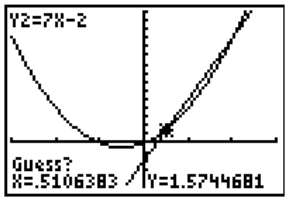


Window [-3,3] x [-5,15]





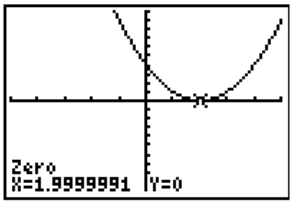




3. Only One Root?

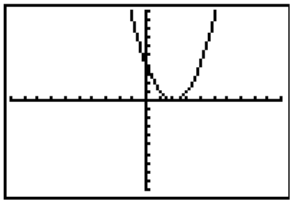
Solve: $x^2 - 4x + 4 = 0$

When graphed, this equation only intersects the x-axis in one location. This tells you that this root repeats itself.



If a reading appears such as 1.9999991, the actual answer is most likely the number 2. Check your assumption that the answer is 2 by substituting 2 into the original equation to see if it satisfies the equation.

Standard 10 x 10 window



$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x = 2; \quad x = 2$

4. What if the graph does not intersect the x-axis???

(or the "intersect" option shows no intersections)

Solve: $x^2 - 3x + 9 = 0$

When graphed, this equation does NOT intersect the x-axis. This tells you that the roots of this equation are complex (imaginary) values. Dig out the quadratic formula to find the roots.

$$x^2 - 3x + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

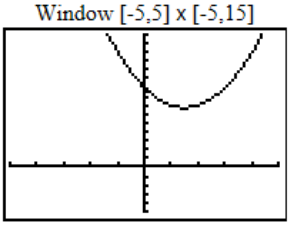
$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 - 36}}{2}$$

$$x = \frac{3 \pm \sqrt{-27}}{2}$$

$$x = \frac{3 \pm 3i\sqrt{3}}{2} = \frac{3}{2} \pm \frac{3i\sqrt{3}}{2}$$

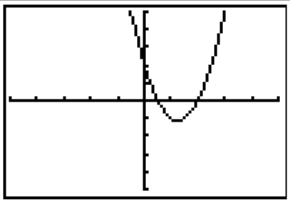
Window [-5,5] x [-5,15]



5. Need to WRITE a quadratic equation from a graph?

If you are given the visual graph of a quadratic equation and you are given (or can identify) at least 3 points, you can use the Quadratic Regression process to create the equation of the graph.

Write the equation of the parabola shown at the right, given that the points (0,2), (-1,9) and (3,5) are on the graph.



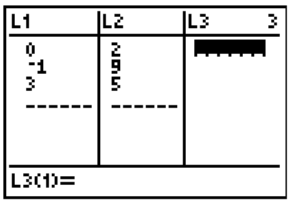
1. Enter the points into L_1 and L_2 (x-coordinate in L_1 and matching y-coordinate in L_2)
(See Basic Commands for Stats if you need more assistance.)

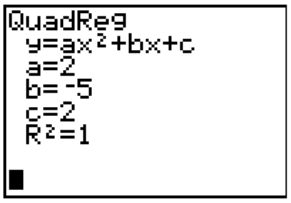
2. Press STAT. Arrow over to CALC. Choose #5 QuadReg.

3. On the home screen: QuadReg L_1, L_2

4. Press ENTER. You will see the screen at the right showing you the coefficients and constant of the quadratic equation.
A possible equation is

$$y = 2x^2 - 5x + 2$$



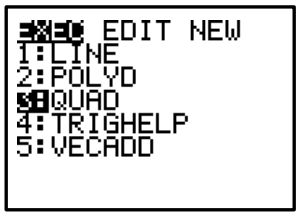


Solving Quadratic Equations (downloadable software)

The program solve the quadratic equation with the real roots by the given coefficients A, B and C at the given equation at the form of $Ax^2 + Bx + C = 0$.

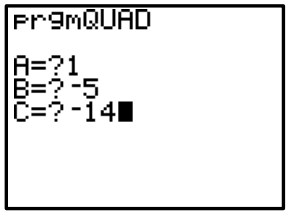
Press **PRGM** and choice **3: QUAD**

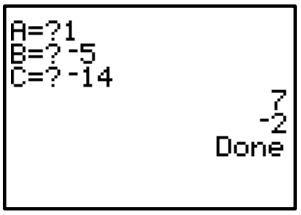




Press **ENTER** twice. Type the values of the coefficients ***A***, ***B*** and ***C***. After typing of the each value, hit **ENTER**.
Let solve the equation $x^2 - 5x - 14 = 0$. Type the coefficients: $A = 1, B = -5$ and $C = -14$. Press **ENTER** and read the answer.







If, even one of the roots from the quadratic equation will be irrational, the screen shows for you:



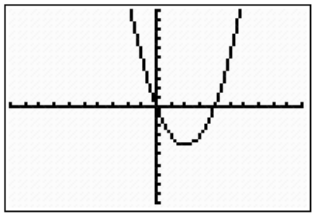
Working with Parabolas

Solve the following problem involving a parabola:

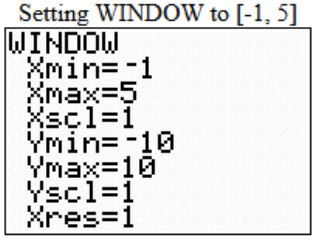
a. Graph the parabola $y = x^2 - 4x$ on the interval [-1, 5].
b. Find the turning point (the vertex).
c. Find the axis of symmetry.
d. Find the roots.

a. To graph the parabola, enter the equation into $y =$. The graph can be seen by using a standard 10 x 10 window (Zoom 6), but this question specifies the interval [-1, 5].

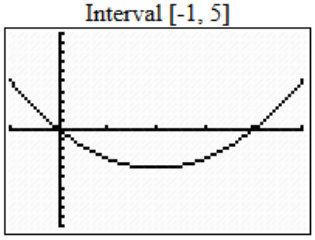
Standard 10 x10 window



Setting WINDOW to [-1, 5]



Interval [-1, 5]

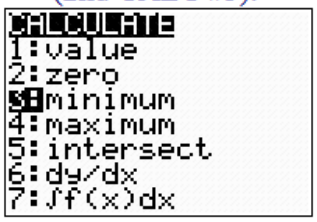


b. To find the turning point, you are looking for the minimum (or maximum) value of the parabola. For this problem, we will need the **minimum** option.

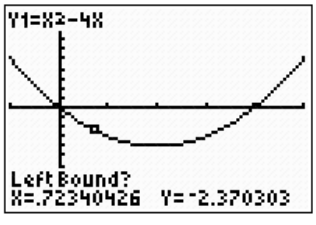
Graph the parabola.

You may be tempted to use the TRACE option to move to the turning point, but be careful. Used in this manner, the TRACE option may only place you "close" to the turning point and not at its "exact" location. To be sure you have the "exact" location, use the minimum and maximum options.

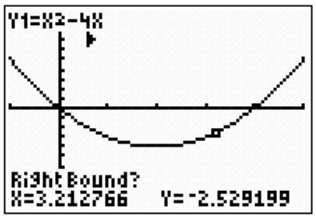
Locate **minimum** (2nd CALC #3).

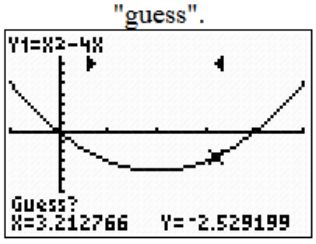
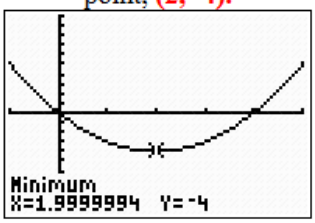


For the "left bound", choose any location to the left of the turning point. Press ENTER.

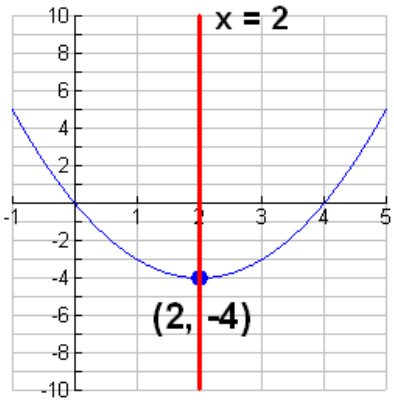


An anchor for the left position will appear. For the "right bound", choose any location to the right of the turning point. Press ENTER.

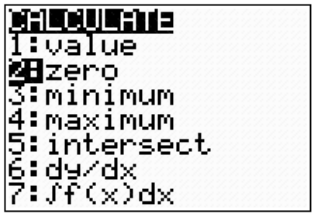
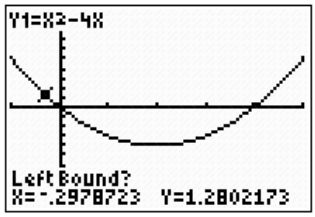
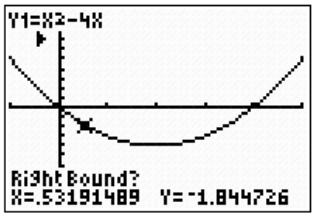


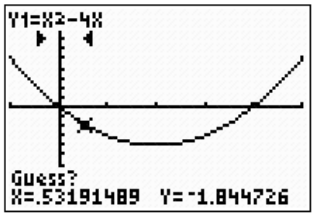
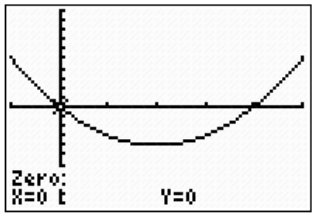
Press ENTER to by-pass "guess". 	The minimum coordinates will appear. These are the coordinates of the turning point, **(2, -4).** 	Remember, when you see an answer such as 1.9999994, the actual answer is most likely 2. To verify, check the value $x = 2$ into the equation and the result will be $y = -4$.

c. Once you know the turning point, you know where the line which is the axis of symmetry is located. The axis of symmetry passes through the turning point. For this example, the axis of symmetry is $x = 2$. There is no need to graph the axis of symmetry on the calculator.



d. To find the roots, you need to find the points at which the graph crosses the x-axis. Again, do not be tempted to use the TRACE option as it may not give the "exact" locations. Use the ZERO command to find the roots.

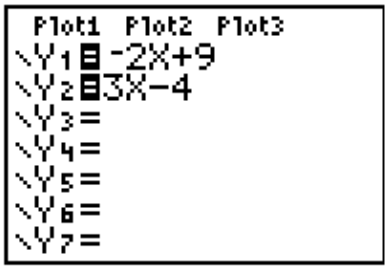
Locate the **zero command**. 2nd TRACE (CALC), #2 zero 	Left bound? Move the spider as close to the root (where the graph crosses the x-axis) as possible. Now, hit the left arrow again to move to the "left" of the root. Hit ENTER. 	A "marker" (anchor) ▶ will be set to the left of the root. Right bound? Move the spider as close to the root (where the graph crosses the x-axis) as possible. Now move to the "right" of the root. Hit ENTER. 

A "marker" (anchor) ◀ will be set to the right of the root. Guess? Just hit ENTER. 	The coordinates of the zero (the root) will appear. **Root at $x = 0$.** 	Repeat the entire process to find the second root (which in this case happens to be $x = 4$).

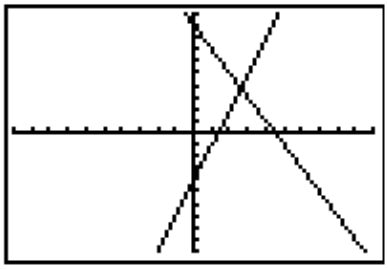
Solving Systems of Equations

1. Solve the system: $y = -2x + 9$ and $y = 3x - 4$

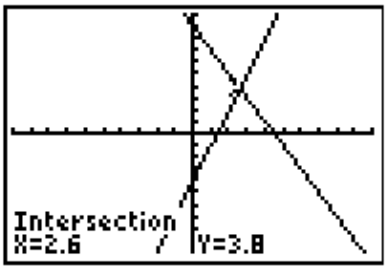
1. Enter the first equation into $\mathbf{Y_1}$.
2. Enter the second equation into $\mathbf{Y_2}$.
3. Hit **GRAPH.**



4. Use the **INTERSECT** option to find where the two graphs intersect (the answer).
   2nd TRACE (CALC) #5 intersect
   Move spider close to the intersection.
   Hit **ENTER** 3 times.



5. Answer: $x = 2.6$ and $y = 3.8$



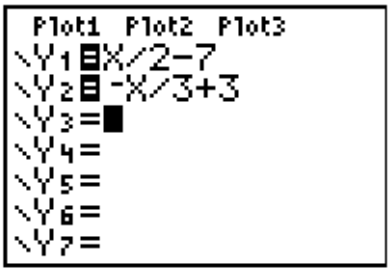
2. Solve the system: $x - 2y = 14$ and $x + 3y = 9$

The graphing calculator will only accept entries that start with $y =$, so we need to solve these equations for $y =$.

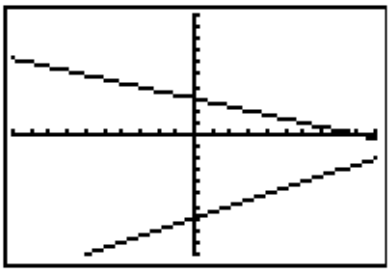
$$y = \frac{x}{2} - 7$$

$$y = -\frac{x}{3} + 3$$

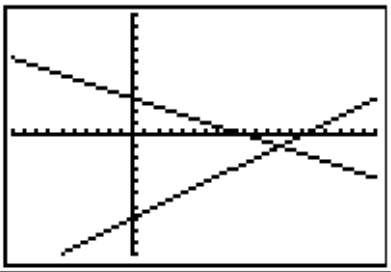
1. Enter the first equation into **Y_1.**
2. Enter the second equation into **Y_2.**
3. Hit **GRAPH**. The graphs appear to intersect OFF the window. We need MORE x-values to the right hand side of the graph. Go to **WINDOW**. Increase the size of **Xmax**. Hit **GRAPH.**
4. Use the **INTERSECT** option to find where the two graphs intersect (the answer).
   2nd TRACE (CALC) #5 intersect
   Move spider close to the intersection.
   Hit **ENTER** 3 times.
5. Answer: $x = 12$ and $y = -1$

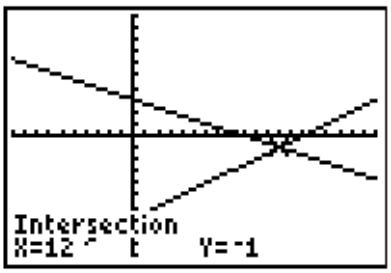


Oops!! They don't cross in the window.



Better! Xmax was increased to 20.



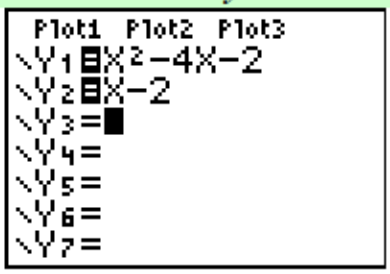


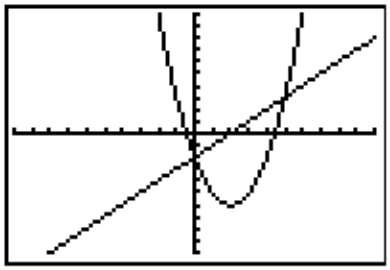
3. Solve linear quadratic system: $y = x^2 - 4x - 2$ and $y = x - 2$

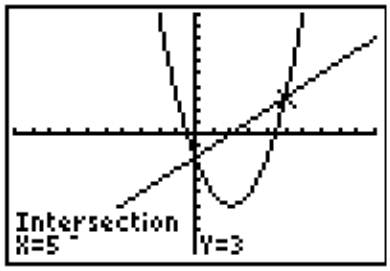
1. Enter the first equation into $\mathbf{Y_1}$.
2. Enter the second equation into $\mathbf{Y_2}$.
3. Hit **GRAPH.**

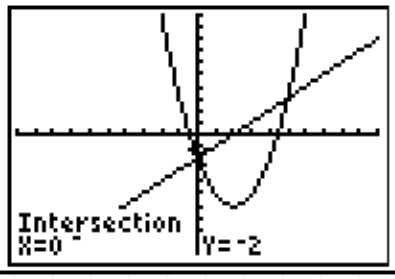
4. Use the **INTERSECT** option twice to find the two locations where the graphs intersect (the answers).
   2nd TRACE (CALC) #5 intersect
   Move spider close to the intersection.
   Hit **ENTER** 3 times.

5. Answer: (5,3) and (0,-2)







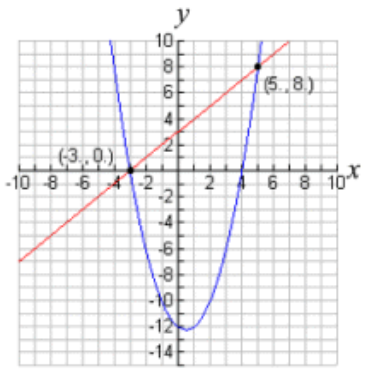
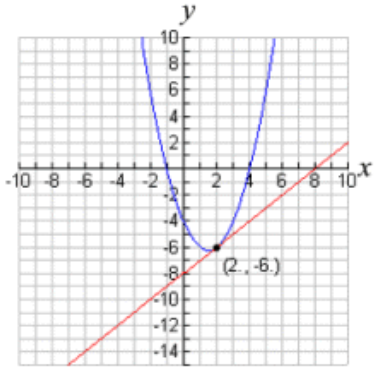
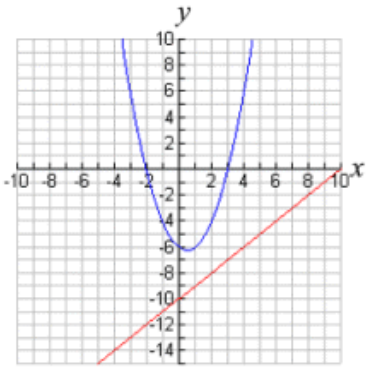


Solving a Linear - Quadratic System

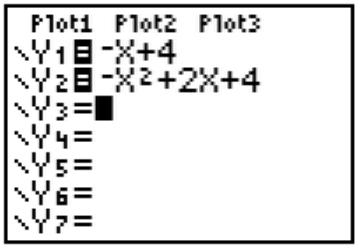
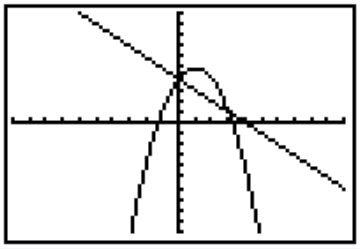
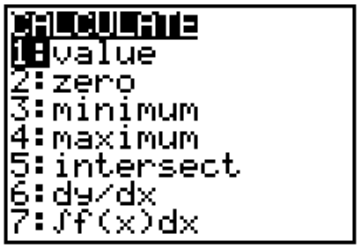
A quadratic equation is defined as an equation in which one or more of the terms is squared but raised to no higher power. The general form is $ax^2 + bx + c = 0$, where a, b and c are constants.

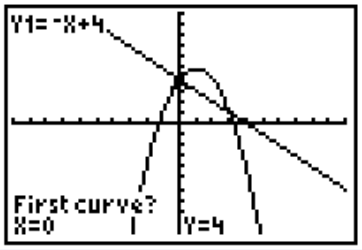
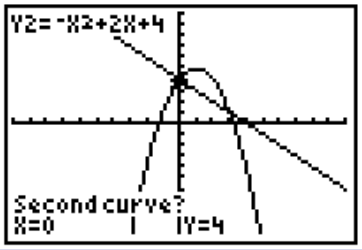
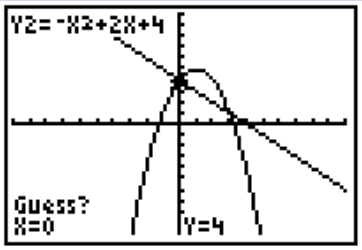
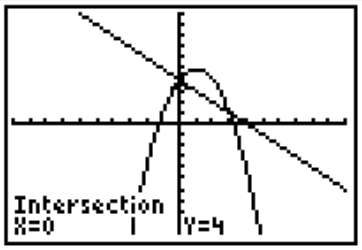
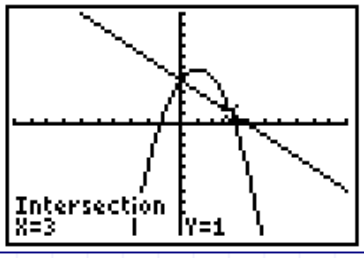
Linear - quadratic system:
(where the quadratic is in one variable - only one variable is squared)

In a linear- quadratic system where only one variable in the quadratic is squared, the graphs will be a parabola and a straight line. When graphing a parabola and a straight line on the same set of axes, three situations are possible.

		
The equations will intersect in two locations. Two real solutions.	The equations will intersect in one location. One real solution.	The equations will not intersect. No real solutions.

Solve graphically: $y = -x^2 + 2x + 4$ (*quadratic - parabola*)
$x + y = 4$ (*linear*)

1.	Change the linear equation to "y=" form.	$y = -x + 4$
2.	Enter the equations as "y_1=" and "y_2=". (Be sure to use the negative key, not the subtraction key, for entering negative values.)	
3.	Hit GRAPH to see if and where the graphs intersect. (Using **ZOOM #6: ZStandard** creates a 10 x 10 viewing window. You may need to adjust the **WINDOW** to see a clear picture of the intersection locations for the two graphs.)	
4.	Under **CALC** (2nd Trace) choose **#5 intersect** to find the points where the graphs intersect.	

5.	When prompted for the "First curve?", move the spider on, or near, a point of intersection. Hit **Enter.**	
6.	When prompted for the "Second curve?", just hit **Enter.**	
7.	Ignore the prompt for "Guess?", and hit **Enter.**	
8.	Read the answers as to the coordinates of the point of intersection. These coordinates appear at the bottom of the screen. Point of intersection (left side): **(0,4)**	
9.	If your graphs have a second point of intersection, repeat this process to find the second point. Choose the **#5 intersect** choice and repeat the steps for finding the intersection. Point of intersection (right side): **(3,1)**	

Linear - quadratic system:

(where the quadratic is in two variables - both variables are squared)

Solve graphically: $x^2 + y^2 = 25$ (*quadratic - circle*)

$x - y = 5$ (*linear*)

1. Change the linear equation to "y=" form.

$y = x - 5$

2. Change the quadratic equation to "y=" form.

$x^2 + y^2 = 25$

$y^2 = 25 - x^2$

$y = \pm\sqrt{25 - x^2}$

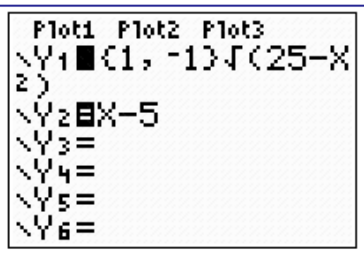
$y = \pm\sqrt{25 - x^2}$ can be entered as two separate entries:

$y_1 = \sqrt{(25 - x^2)}$

$y_2 = -\sqrt{(25 - x^2)}$

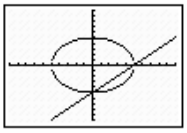
or a "list" notation may be used:

$y_1 = \{1, -1\}\sqrt{(25 - x^2)}$

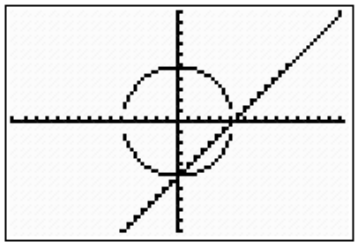


Enter the equations.
The circle entry shown here uses the "list" notation.

3. If you hit GRAPH at this point using a standard window (**ZOOM #6: ZStandard**) your circle will not resemble a circle due to the 3 to 2 aspect ratio of this screen.

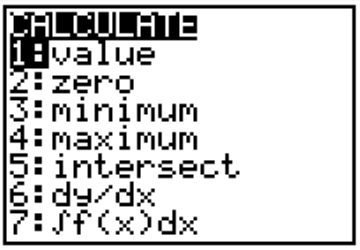


Choose **ZOOM #5: ZSquare** to get a true looking circle.

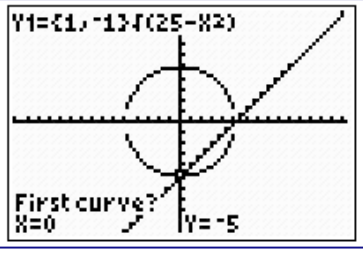


Notice the missing pixels near the x-axis.

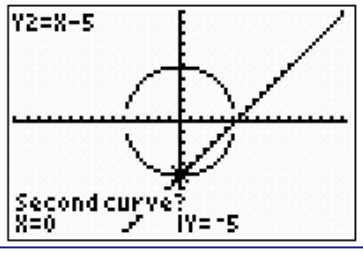
4. Under **CALC** (2nd Trace) choose **#5 intersect** to find the points where the graphs intersect.

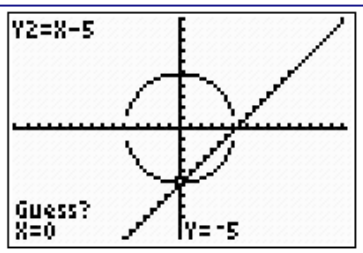
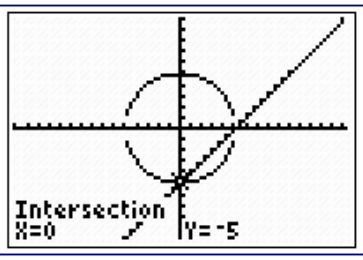
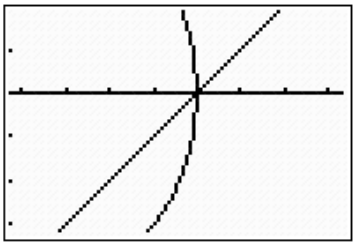
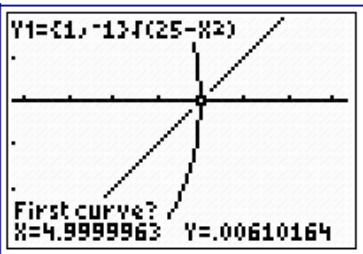
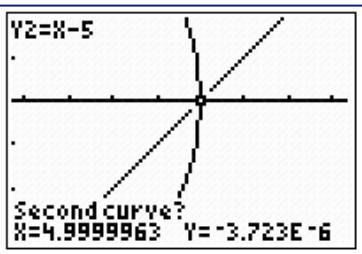
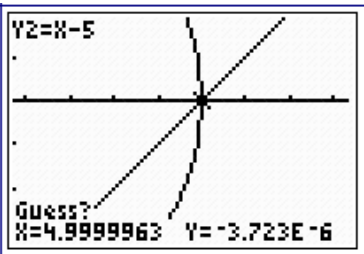
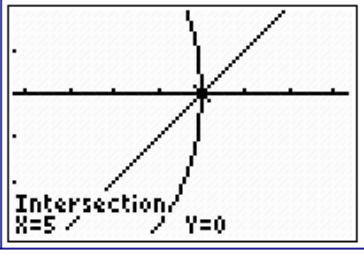


5. We will find the left point of intersection first. When prompted for the "First curve?", use the **down arrow** key to move to the bottom portion of the circle. Hit **Enter.**



6. When prompted for the "Second curve?", just hit **Enter.**



7.	Ignore the prompt for "Guess?", and hit **Enter.**	
8.	Read the answers as to the coordinates of the point of intersection. These coordinates appear at the bottom of the screen. Point of intersection (left side): **(0,-5)**	
9.	We will find the right point of intersection next. This point is occurring where the missing pixels are located. We need to **zoom in** at this location to place more pixels on the screen. After zooming, go to **CALC** (2nd Trace), choose **#5 intersect,** and find the points where the graphs intersect.	
10.	  Notice the graph is set to **Y1**. Use up and down arrows to get **Y2.**	 No need to guess. Hit **ENTER.**
11.	Point of intersection (right side): **(5,0)**	
When working with a circle, always check the equation statement in the upper left corner to see where you are. The up and down arrows will move you between the equations.		

3. Geometry

Transformations

Using the graphing calculator to examine transformations:

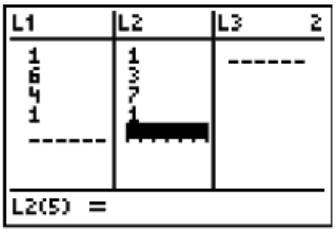
Figure being used will be a triangle: A(1,1), B(6,3), C(4,7).

1. Clear entries in the Y= positions (or turn them off).

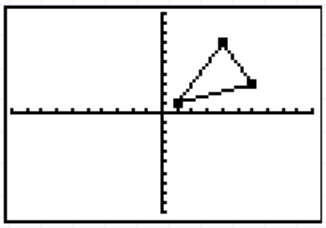
2. Turn on the **Connected Graph** icon under StatPlots.
Go to **StatPlot - #1 Plot** - highlight **On** - highlight second icon.
Xlist is the name of the list where the *x*-coordinates will be found.
Ylist is the name of the list where the *y*-coordinates will be found.
Mark: choose the heavier mark to represent the original figure.



3. Enter values into **L1** and **L2**. Enter the *x*-coordinate in **L1** and its corresponding *y*-coordinate in **L2**. **Enter the first coordinate again at the end to complete the connected drawing.**
(See Basic Commands for help entering data in lists.)



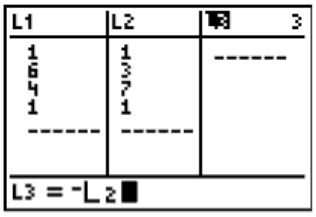
The original triangle is now residing in **Plot1.** When graphing do NOT choose **ZoomStat** for the window. We want to have a coordinate axes for examining our transformations. Use a standard 10X10 window (**ZStandard**) for this problem.



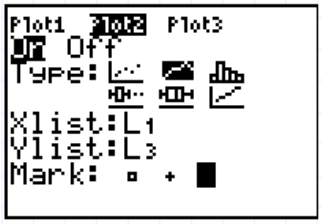
4. We will be placing our transformation coordinates into **L3** and our transformation figure will reside in **Plot2**.

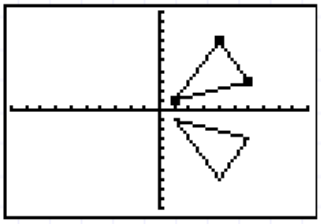
Reflection over the *x*-axis: (negate *y*-values)

Store the negated *y*-values into list **L3**. You can type them in yourself or let the calculator create the values. Arrow up ONTO **L3** and enter - **L2** (to negate the *y*-values). Press **ENTER.**



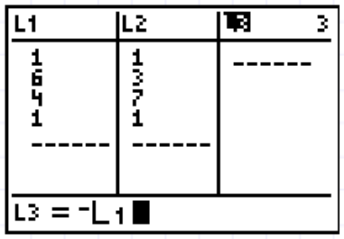
Set up **Plot2: Xlist: L1** and **Ylist: L3**
(I like to choose a different "Mark" so that the transformation can be easily distinguished from the original drawing.)

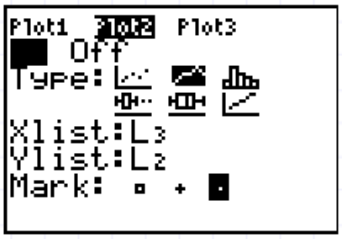


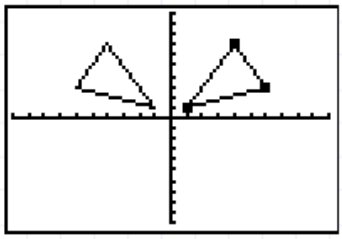


Reflection over the *y*-axis: (negate *x*-values)

Let **L3** be **-L1.** Under **Plot 2**, assign **Xlist: L3** and **Ylist: L2. Graph.**

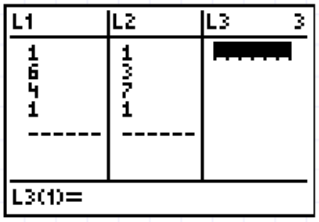


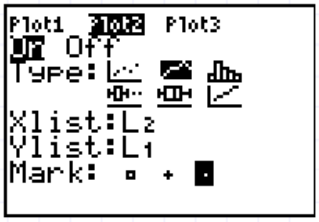


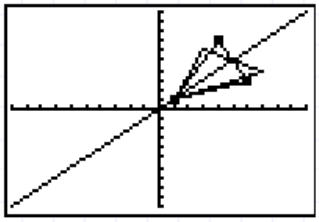


Reflection over the line *y* = *x*: (switch coordinates)

Under Plot 2, assign **Xlist: L2** and **Ylist: Y1. Graph.** Your might also wish to graph $Y_1 = x$ to show the line of reflection.

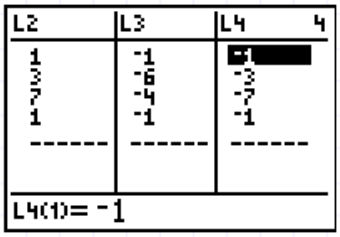


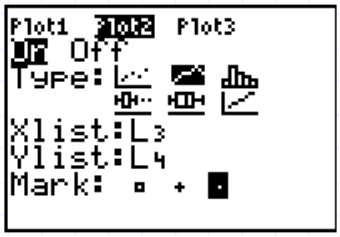


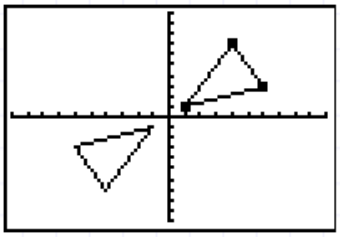


Reflection in origin: (negate both coordinates)

Let **L3 = -L1.** Let **L4 = -L2**. Under **Plot 2**, assign **Xlist: L3** and **Ylist: Y4. Graph.**

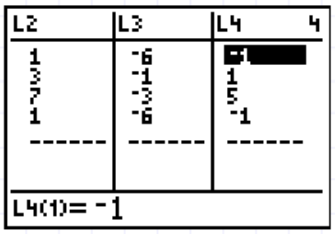


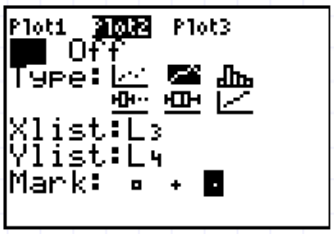


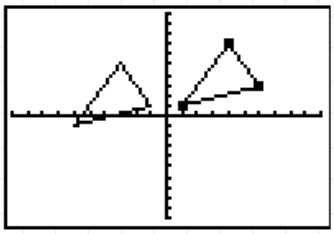


Translation $(x,y) \rightarrow (x - 7, y - 2)$:

Let **L3 = L1 - 7**. Let **L4 = L2 - 2.** Under **Plot 2**, assign **Xlist: L3** and **Ylist: Y4. Graph.**

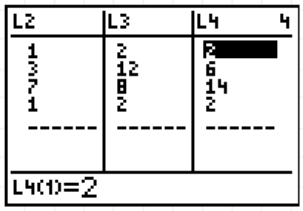


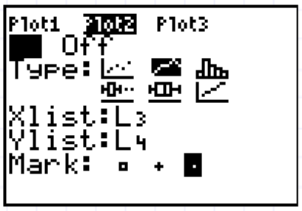


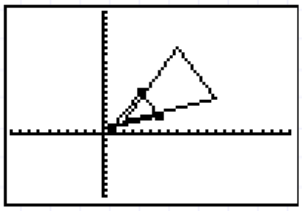


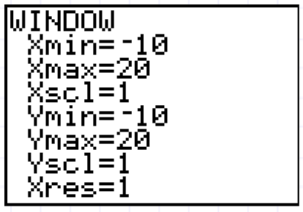
Dilation of scalar factor 2: (in the origin)

Let **L3 = L1 * 2**. Let **L4 = L2 * 2**. Under **Plot 2**, assign **Xlist: L3** and **Ylist: Y4. Graph.** Adjust the window to see the dilation.









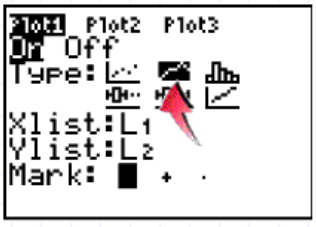
Transformations – Rotations

Using the graphing calculator to examine transformations:

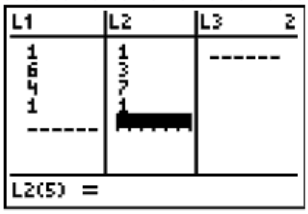
Figure being used will be a triangle: A(1,1), B(6,3), C(4,7).

1. Clear entries in the Y= positions (or turn them off).

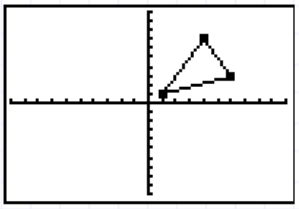
2. Turn on the **Connected Graph** icon under StatPlots.
Go to **StatPlot - #1 Plot -** highlight **On -** highlight second icon.
Xlist is the name of the list where the *x*-coordinates will be found.
Ylist is the name of the list where the *y*-coordinates will be found.
Mark: choose the heavier mark to represent the original figure.



3. Enter values into **L1** and **L2**. Enter the *x*-coordinate in **L1** and its corresponding *y*-coordinate in **L2**. **Enter the first coordinate again at the end to complete the connected drawing.**
(See Basic Commands for help entering data in lists.)



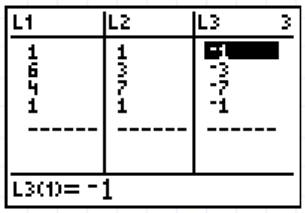
The original triangle is now residing in **Plot1.** When graphing do NOT choose **ZoomStat** for the window. We want to have a coordinate axes for examining our transformations. Use a standard 10X10 window (**ZStandard**) for this problem.

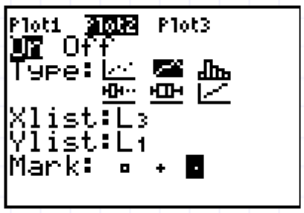


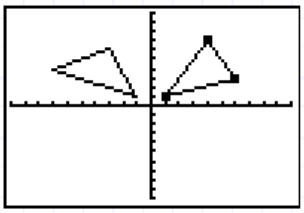
4. We will be placing our transformation coordinates into **L3** and our transformation figure will reside in **Plot2**.

Rotation of 90° counterclockwise: $(x, y) \rightarrow (-y, x)$

Store the negated *y*-values into list **L3**. You can type them in yourself or let the calculator create the values. Arrow up ONTO **L3** and enter **-L2** (to negate the *y*-values). Press **ENTER.**

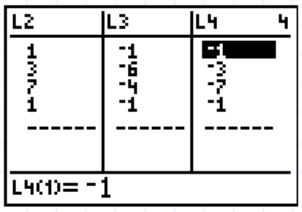


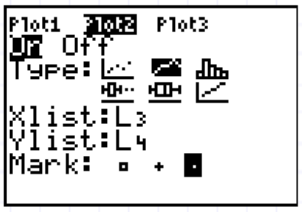


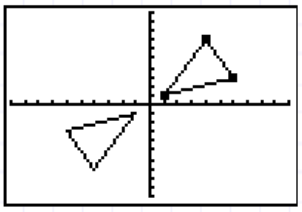


Rotation of 180° counterclockwise: $(x, y) \rightarrow (-x, -y)$

Store the negated *x*-values into list **L3** and the negated *y*-values in **L4**. Under **Plot 2**, assign **Xlist: L3** and **Ylist: Y4. Graph.**

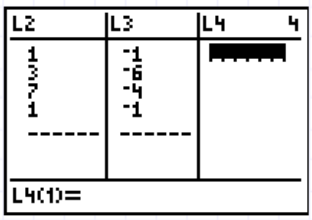


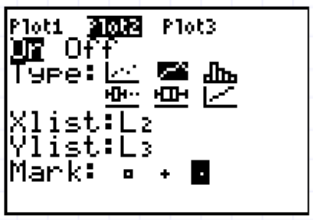


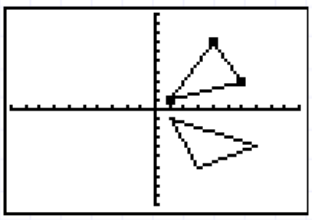


Rotation of 270° counterclockwise: $(x, y) \rightarrow (y, -x)$

Store the negated *x*-values into list **L3**. Under **Plot 2,** assign **Xlist: L2** and **Ylist: Y3. Graph.**





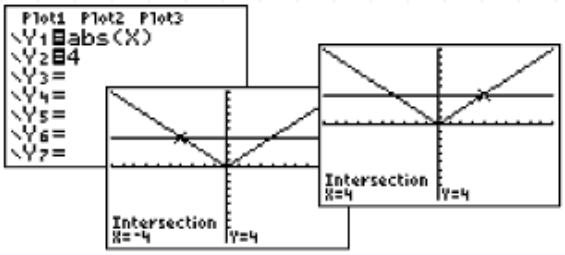


4. Advanced Algebra

Equations and Inequalities with the Absolute Value

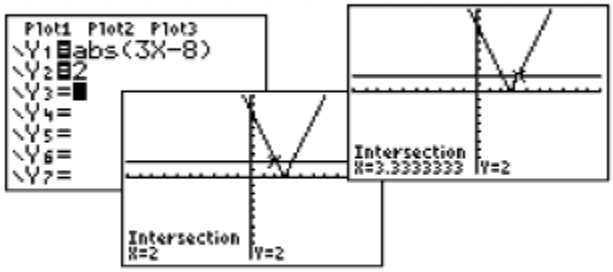
EQUATIONS:

Example 1: **Solve:** $|x| = 4$



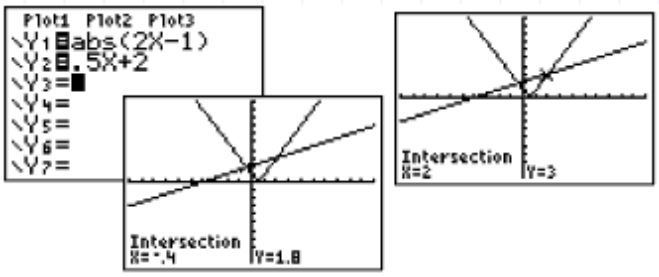
- Enter left side in Y1. You can find **abs( )** quickly under the CATALOG (above 0) (or MATH → NUM, #1 abs( )
- Enter right side in Y2
- Use the Intersect Option (2nd CALC #5) to find where the graphs intersect. Move the spider near the point of intersection, press ENTER. Simply hit ENTER twice more. You must repeat this process to find the second point of intersection.
- **Answer:** $x = 4;\ x = -4$

Example 2: **Solve:** $|3x - 8| = 2$



- **Answer:** $x = 2;\ x = 3.3333333$
  The x value is stored in the calculator's memory. If you wish to change 3.333333333 to a fraction, simply return to the home screen, hit x, hit Enter. Now, change to fraction. (MATH, #1▶Frac)
  Answer: $x = 2;\ x = 10/3$

Example 3:
Solve: $|2x - 1| = 0.5x + 2$



- **Answer:** $x = -.4;\ x = 2$

INEQUALITIES:

Example 4: **Solve:** $|2-x| > 3$

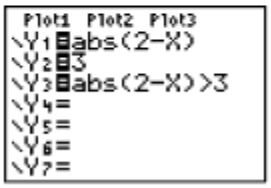
Boolean Check:

$Y_1 = abs(2-x)$

$Y_2 = 3$

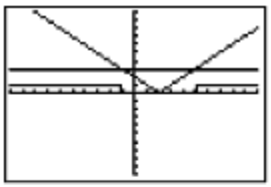
$Y_3 = abs(2-x) > 3$

(The inequality symbols are under the TEST Menu - 2nd MATH.)



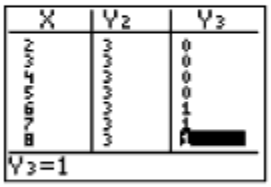
Answer: $x < -1;\ x > 5$

You could also enter $Y_3 = Y_1 > Y_2$ (using Y_1 and Y_2 found under Vars →Y-Vars, Function)



Where the inequality is true, y-values on the graph will be a 1.

If you look at the table, 0's will be listed where the inequality is false and 1's will be listed where the inequality is true.

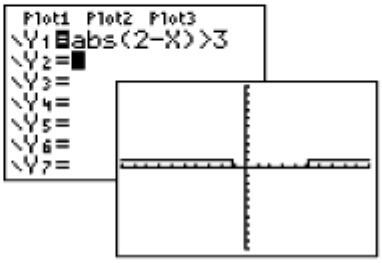


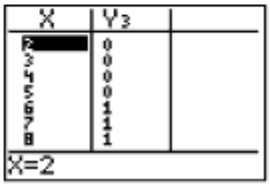
Determine exact cut off points by using the intersection option.

Remember that the calculator cannot draw an open or closed circle on the intervals. You will have to determine which circle is needed based upon whether the inequality includes "equal to". Find the endpoints by using the intersect option (2nd TRACE #5 intersect).

If you turn off the axes (FORMAT - 2nd ZOOM), you will be able to see the graphing of the 0's and 1's more clearly. Notice that the small vertical segment connecting the 0's to the 1's is simply the calculator being set in "connected" mode. Change to "dot" mode to remove this segment.

You "can" enter the inequality directly. Since you will not have the two graphs from which to determine the intersection points, you will need to examine the table to determine the exact value of the open/closed endpoints.





Example 5: **Solve:** $2 < |x+1| < 6$

Algebraic solution:

$2 < |x+1|$ $\qquad$ $|x+1| < 6$

$2 < x+1$ **or** $-2 > x+1$ $\qquad$ $x+1 < 6$ **and** $x+1 > -6$

$x > 1$ **or** $x < -3$ $\qquad$ $x < 5$ **and** $x > -7$

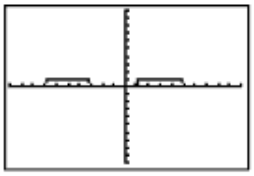
Answer: The values that make the compound inequality true are

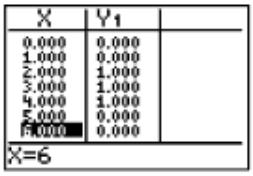
$-7 < x < -3$ **and also** $1 < x < 5$.

Calculator solution: (inequality expression is entered using "and")

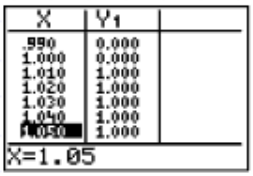
[**and** can be found quickly under the CATALOG (above 0) -
or TEST (2nd MATH)→LOGIC, #1 and]

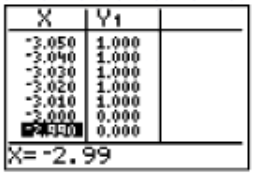






Different table settings can be used to examine the data:





Algebra of the Functions

The graphing calculator can be used to investigate algebra of functions.

Algebra of Functions:

$(f+g)(x)$; $(f-g)(x)$; $(f \cdot g)(x)$; $(f/g)(x)$

$Y_1 = f(x)$

$Y_2 = g(x)$

$Y_3 = Y_1 + Y_2$ **represents** $(f+g)(x)$

$Y_3 = Y_1 - Y_2$ **represents** $(f-g)(x)$

$Y_3 = Y_1 Y_2$ **represents** $(f \cdot g)(x)$

$Y_3 = Y_1 / Y_2$ **represents** $(f/g)(x)$

* When entering the multiplication of functions, several notation entries are possible:
$Y_3=Y_1Y_2$ OR $Y_3=Y_1*Y_2$ BUT **NOT** $Y_3=Y_1(Y_2)$ as it is composition.

* Remember when performing the division of functions, that zero denominators cause problems and will affect the look of the graph.

* Remember that you can find the Y-subscsripted variables under the **VARS** key: arrow right to **Y-VARS**, and choose **1. Function.**

Example:

Given:

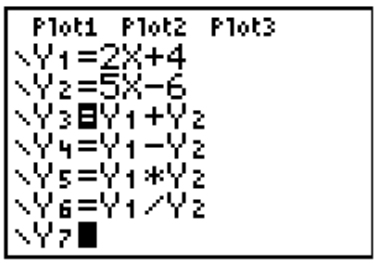
$f(x) = 2x+4$

$g(x) = 5x-6$

a) graph: $(f+g)(x)$; $(f-g)(x)$; $(f \cdot g)(x)$; $(f/g)(x)$

b) find:

$(f+g)(5)$; $(f-g)(-2)$; $(f \cdot g)(0)$; $(f/g)\left(\frac{1}{2}\right)$



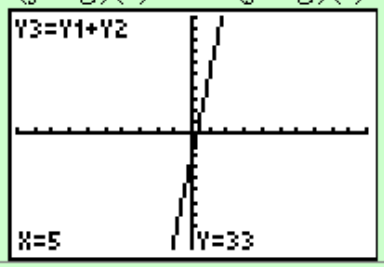
Solution:

1. Enter $f(x)$ into Y_1 and $g(x)$ into Y_2.
2. Enter the expression for the operation you will be examining into Y_3 (or into successive locations).
3. Turn off the first two entries, as well as any other entries that you do not want graphed.
4. Set an appropriate viewing window for the graph.
5. Graph. (this will be the answer to part a)
6. While on the graphing screen, hit TRACE. Simply enter the number you wish to find (if that number is within the viewing window.) Hit ENTER for the answer. (this will be the answer to part b)

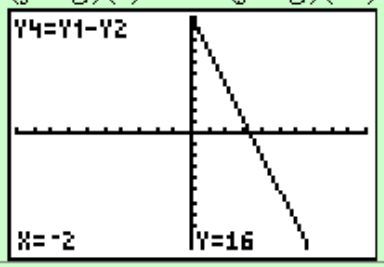
OR

From the home screen, enter $Y_3(\ 5\ )$, assuming your functions are entered in Y_1 and Y_2 with the operation expression in Y_3. Hit ENTER. (this will be the answer to part b)

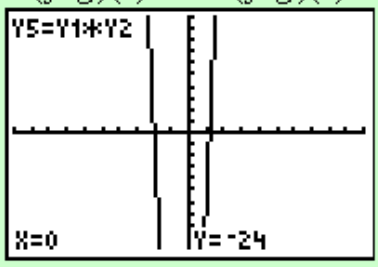
$(f+g)(x)$ and $(f+g)(5)$



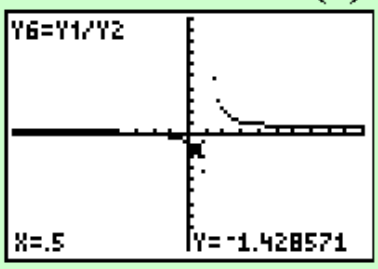
$(f-g)(x)$ and $(f-g)(-2)$



$(f \cdot g)(x)$ and $(f \cdot g)(0)$



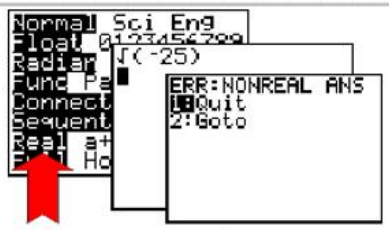
$(f/g)(x)$ and $(f/g)\left(\frac{1}{2}\right)$



Dot Mode was used here to avoid seeing the connected "heart beat" where the denominator equaled zero.

Complex Numbers

The graphing calculator can be a very useful tool for checking your work with complex numbers. Keep in mind, when working with a graphing calculator, that there may be more than one way to arrive at an answers



You get an error message when you try to enter the square root of a negative number when in Real Mode.



You need to be in $a + bi$ mode to evaluate the square root of a negative number.

Now, let's look at the arithmetic of complex numbers:

Add: (2 + 4i) + (3 - 2i)	Subtract: (6 - 3i) - (4 + 5i)	Multiply: (3 + 2i) (4 - 2i)
(2+4i)+(3−2i) 5+2i	(6−3i)−(4+5i) 2−8i	(3+2i)*(4−2i) 16+2i

Divide : (2+3i)/(4-3i) (answer appears in *a* + *bi* form) **WOW!!**

The calculator did the conjugate work for you.

For TI-83+ and for TI-84+ with OS prior to OS 2.53MP.

(2+3i)/(4−3i) -.04+.72i
Ans▸Frac -1/25+18/25i

In this display, ***i*** is NOT in the denominator! The calculator is simply listing order of operations.

TI-84+ with OS 2.53MP

(2+3i)/(4−3i) -.04+.72i
-.04 -.04
Ans▸Frac $-\frac{1}{25}$
.72 .72
Ans▸Frac $\frac{18}{25}$

Be careful to write your final answer correctly.
Note the location of the *i* in the final answer:

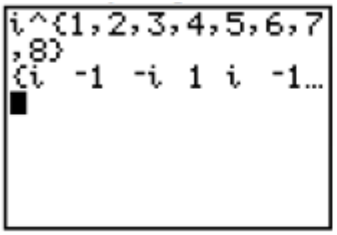
$$-\frac{1}{25}+\frac{18}{25}i$$

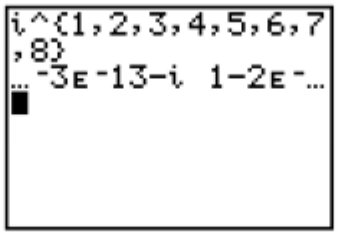
NOTE: On OS2.53MP, if the decimals you are trying to "grab" to convert to fractions are really long, use **MATH - CPX - real(** and **imag(** to "grab" the full decimal value for converting "a" and "b" to fractions.

Using the calculator to investigate powers of *i*:

Investigate the powers of *i*.		These values will appear when you are in either Real Mode or in *a* + *bi* mode.
You can look at many powers at once by using a list	... use right arrow to scroll to the right to see all of the answers	

i^2 i
i^3 -1
i^4 -i
1





What kind of number is
$-3E-13-i$???
This number is really just $-i$.
$-3E-13$ is so small, it is considered to be zero.
(E-13 is Scientific Notation meaning 10 raised to -13 power.)

There are also special functions on the graphing calculator to deal with complex numbers (but you probably won't need a calculator for many of these functions):



1. conj(returns the complex conjugate of a complex number.
   conj(2+5i) gives 2-5i
2. real(returns the "a" value in an a+bi complex number.
   real(2+5i) gives 2
3. imag(returns the "b" value in an a+bi complex number.
   imag(2+5i) gives 5

5. abs(returns the absolute value of the complex number.
abs(5+12i) gives 13

(*Note:* The **absolute value** of a complex number may also be called its magnitude. It you plot a complex number as a single point, the absolute value represents the distance from the origin to that point. If you plot a complex number as a vector, the absolute value represents the length of the vector.)

Composition of Functions

The graphing calculator can be used to investigate the composition of functions (composite functions).

Working with composite functions:

$(f \circ g)(x) = f(g(x))$

$Y_1 = f(x)$
$Y_2 = g(x)$
$Y_3 = Y_1(Y_2)$ represents $(f \circ g)(x)$

* Remember that you can find the Y-subscsripted variables under the **VARS** key: arrow right to **Y-VARS**, and choose **1. Function.**

Example:

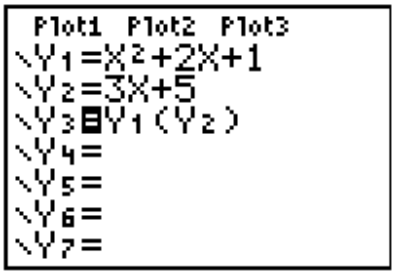
Given:

$f(x) = x^2 + 2x + 1$

$g(x) = 3x + 5$

a.) **graph** $(f \circ g)(x)$

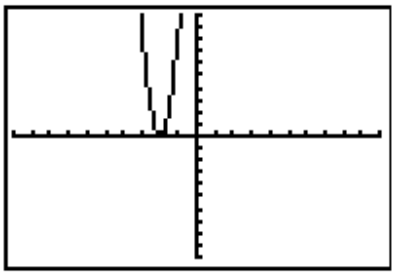
b.) **find** $(f \circ g)(3)$



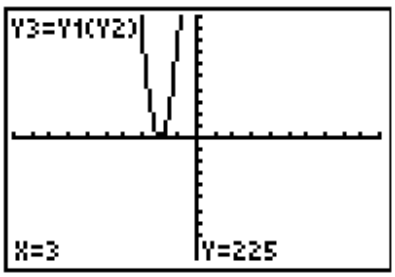
Solution:

1. Enter $f(x)$ into Y_1 and $g(x)$ into Y_2.
2. Enter the expression for composition into Y_3.

   $Y_3 = Y_1(Y_2)$
3. Turn off the first two entries so that their graphs do not appear when the composition is graphed.
4. Set an appropriate viewing window for the graph.
5. Graph. (this will be the answer to part a)

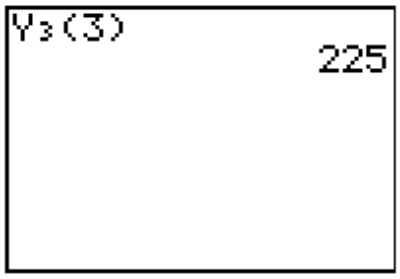


6. While on the graphing screen, hit TRACE. Simply enter the number you wish to find (if that number is within the viewing window.) Hit ENTER for the answer. (this will be the answer to part b)



OR

From the home screen, enter $Y_3(\ 3\ )$, assuming your functions are entered in Y_1 and Y_2 with the composition expression in Y_3.
Hit ENTER. (this will be the answer to part b)



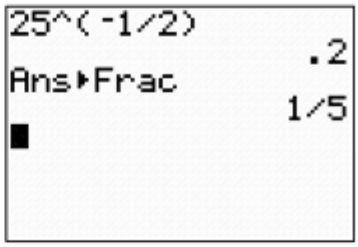
Solving Exponential Equations

While exponential equations can be solved on the graphing calculator, you will, in most cases, have to adjust your window to see the graphs.

1. $y = 25^{-\frac{1}{2}}$

Enter the right hand side of the equation on the home screen to perform the calculation. Notice the use of parentheses.

Answer: y = .2 or 1/5

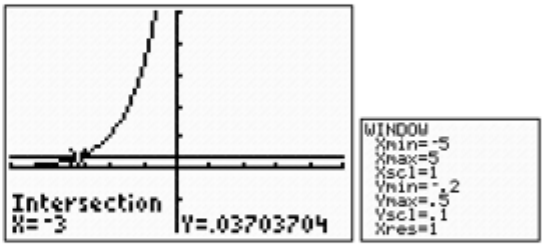


2. $3^x = \frac{1}{27}$

Enter the left side into **Y1** and the right side into **Y2**. Keep in mind that 1/27 is a very small value. Adjust your window accordingly. If necessary, start with the standard 10 x 10 window and adjust until you see the graph clearly. Then use your intersect option.

2nd CALC (above TRACE), #5 intersect

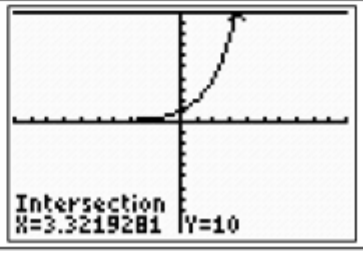
Answer: $x = -3$



3. $2^x = 10$

Proceed as in #2. A standard 10 x 10 window was used here.

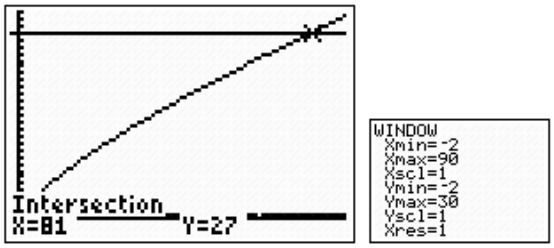
Answer: $x = 3.3219281$



4. $x^{\frac{3}{4}} = 27$

Proceed as in #2. Be sure to adjust the window to allow for a y-value of 27. Also notice the need to move to the right to find the intersection. Increase the XMax value.

Answer: $x = 81$



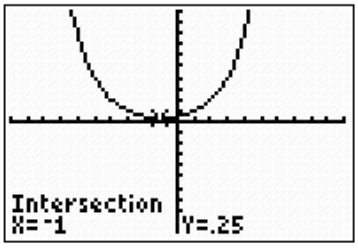
5. Solve this system:

$y = 2^{x-1}$

$y = \left(\frac{1}{2}\right)^{x+3}$

Proceed as in #2. Use the intersect option. A standard 10 x 10 window was used here.

Answer: $x = -1$, $y = 0.25$

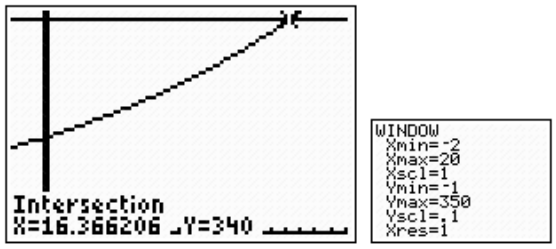


6. Solve for x, to the *nearest thousandth*:

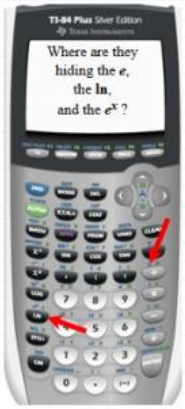
$150e^{0.05x} = 340$

Proceed as in #2. Adjust the window as to see the point of intersection of the two functions.

Answer: $x = 16.366$



Solving Exponential Expressions (ln and *e*)



Start by finding the keys you will be using:

e (to the first power) can be found above the division key.

ln has its own key on the left side of the keypad.



e^x is found above the **ln** key.

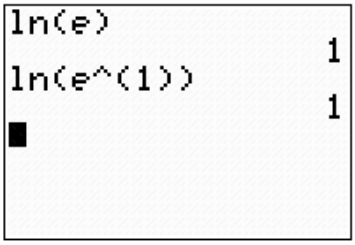
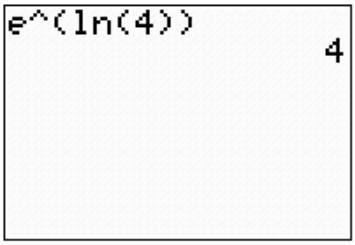
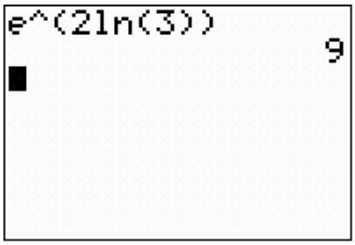
Remember: e is an irrational number, approximately 2.71828183, named after the 18th century Swiss mathematician, Leonhard Euler.

$f(x) = e^x$, is called the **natural exponential function.**

$f(x) = \ln x$, is called the **natural logarithmic function.**

These two functions are inverses of one another.

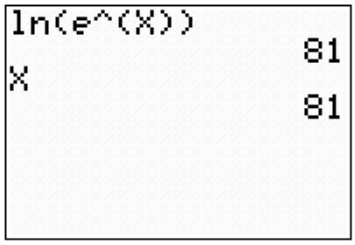
When composed, these two functions return the starting value, thus creating the identity function, $y = x$.

1. $\ln e$ Notice how the calculator automatically forces the use of the parentheses. Get in the habit of closing the parentheses. The first entry uses the ***e*** value above the division key. The second entry uses the e^x above the **ln** key. Notice how this second entry illustrates the composition of the two inverse functions **ln** and e^x, returning the starting value of 1. **Answer: 1**	
2. $e^{\ln 4}$ Simply enter the expression on the home screen. Again, notice the composition of functions at work on the two inverse functions. **Answer: 4**	
3. $e^{2\ln 3}$ Again, simply enter the expression on the home screen. Did you notice that this is NOT a composition of the two inverse functions? The "2" is in the way. Of course, the problem could be rewritten using properties of logs to utilize the composition of the inverses: $e^{\ln 3^2}$ **Answer: 9**	

4. $\ln e^x$

Why 81? What is happening here? Since the calculator does not know the numerical value for x, it is using the last value stored in its memory for x. This calculator's last stored value for x was 81. Your calculator most likely shows a different value. The thing to notice is that it returns what ever value represents x, thus telling us that $\ln e^x = x$, which we can verify from the composition of these inverse functions.

Answer: x



Inverse Functions

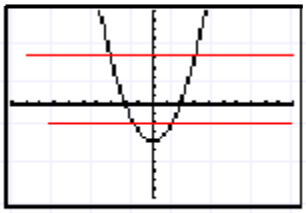
The graphing calculator can be used in several ways to check your work when you are dealing with inverse functions.

1. Use the calculator to quickly determine if the inverse of your graph is a function.

Graph the original function and observe the results of the horizontal line test.

Example: Is the inverse of $y = x^2 - 4$ a function?

Since this parabola does not pass the horizontal line test , the inverse will not be a function.

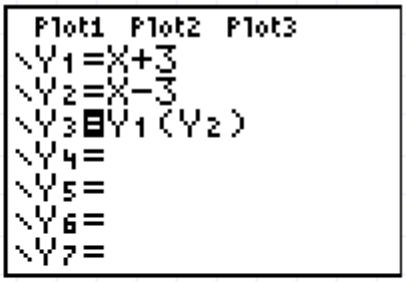


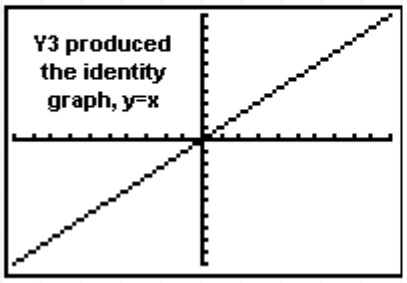
The red lines will not appear on the calculator.

2. Use the calculator to determine if your algebraic inverse answer is correct.
Method 1:

A quick way to determine if your algebraic inverse answer is correct is to:

a.) enter your starting function in Y1.
b.) enter your algebraic inverse answer in Y2.
c.) enter Y1(Y2) in Y3. (composition of functions)
d.) GRAPH
e.) if Y3 produces the graph of the identity function, y = x, your algebraic inverse answer is correct.





Use the calculator to determine if your algebraic inverse answer is correct.
Method 2:

The graphing calculator has a DrawInv command that will DRAW a function's inverse. To use this feature, place the original starting function into Y1. Press 2^{nd} PRGM (DRAW) #8 DrawInv. The command will appear on the home screen waiting for a parameter. Enter the location of the function to be investigated, Y1. The graph of the inverse will appear.



If you wish to use this feature to check an algebraic answer,

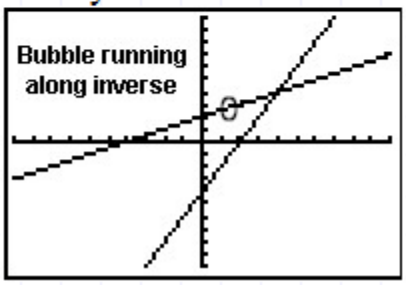
a. Enter the original function into Y1.

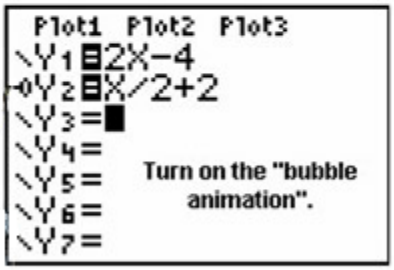
b. DrawInv Y1 (must be done **before** entering your algebraic answer)

c. Solve for the inverse algebraically on a piece of paper.

d. Enter your algebraic answer for the inverse into Y2 with the bubble animation (in front of Y2), and hit GRAPH.

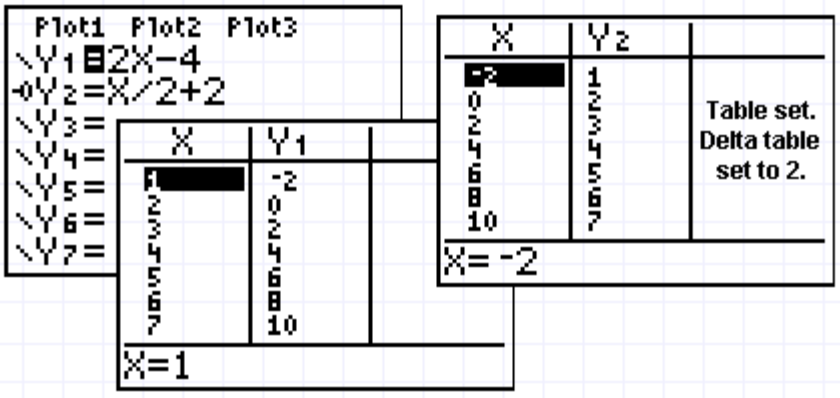
e. If your algebraic answer is correct, the bubble animation will follow the inverse graph drawn by the calculator.





3. Use the calculator to investigate the relationship between the x and y values of functions and inverses.

Use the table feature of the calculator to verify the "swapping" of x and y values between functions and their inverses.



Working with Logarithms

A **logarithm** is an exponent.

answer ↓ $\log_2 8 = 3$ ← exponent; ↑ base

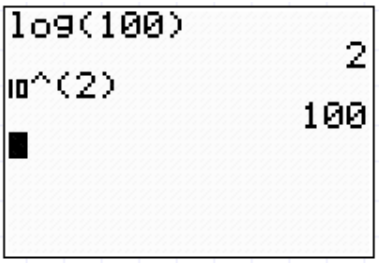
As this example shows, 3 is the exponent to which the base 2 must be raised to create the answer of 8, or $2^3 = 8$. In general terms:

$y = \log_b x$ if and only if $x = b^y$

(where $x > 0$ and b is a positive constant not equal to 1)

BASE 10:

Logarithms with base 10 are called **common logarithms.**
When the base is not indicated, base 10 is implied.
The **log** key on the graphing calculator will calculate the common (or base 10) logarithm.
2nd log will calculate the antilogarithm or 10^x

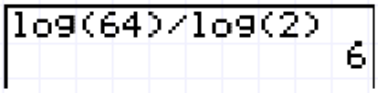


OTHER BASES:

To enter a logarithm with a different base on the graphing calculator, use the Change of Base Formula:

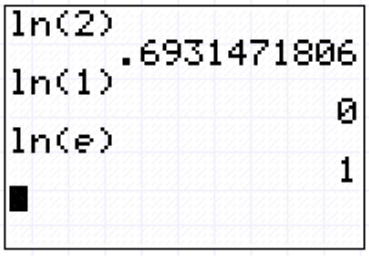
$$\log_b x = \frac{\log x}{\log b}$$

$\log_2 64$



BASE *e*:

Logarithms with base *e* are called natural logarithms.
Natural logarithms are denoted by **ln.**
On the graphing calculator, the base *e* logarithm is the **ln** key.



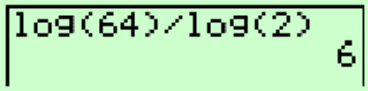
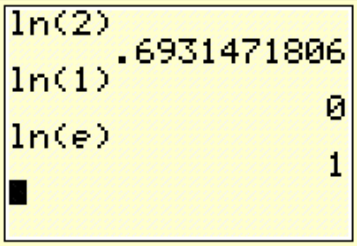
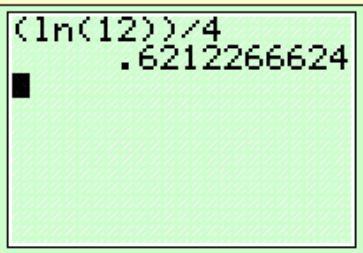
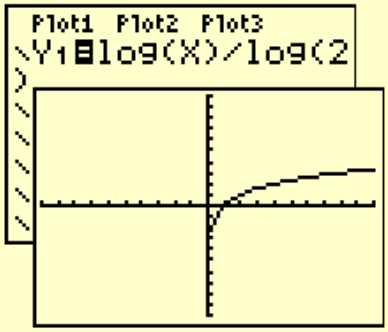
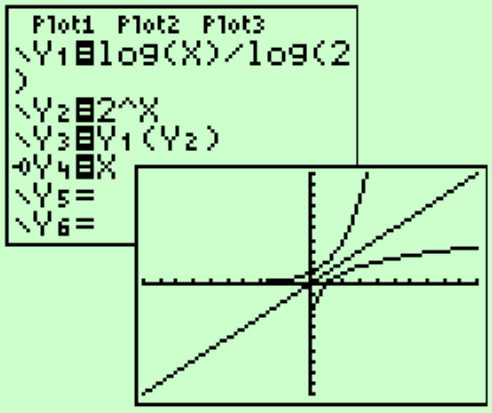
$\ln x = \log_e x$

$e \approx 2.71828183$

When working with logarithms on your graphing calculator, you must remember the "Change of Base Formula":

$\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$	Remember, the notation: log *x* is with respect to base 10 ln *x* is with respect to base *e*

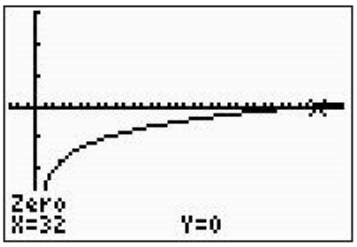
Examples:

1. Evaluate: $\log_2 64$	
2. Evaluate: $\ln 2;\ \ln 1;\ \ln e$	
3. Evaluate: $\frac{\ln 12}{4}$	
4. Sketch the graph of $f(x) = \log_2 x$	
5. Use a graph to support the conclusion that the inverse of $f(x) = 2^x$ is $f(x) = \log_2 x$. A function composed with its inverse creates the identity line $y = x$. Showing that the composition of these two functions is the identity line shows that these functions are inverses of one another. Note the bubble animation set for Y_4 to show that the linear lines in Y_3 and Y_4 are equivalent.	

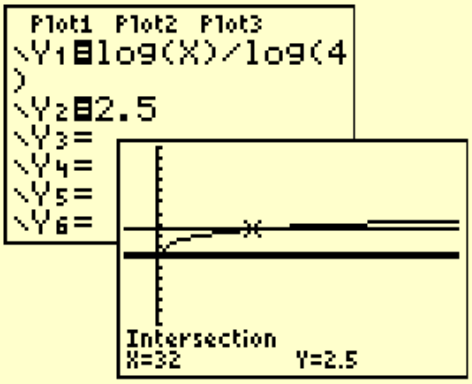
6. Solve graphically: $\log_4 x = 2.5$

Method 2: This problem can also be solved by setting the equation equal to 0 and finding the x-intercepts, or zeros (2nd CALC, #2), of the function.

$$\log_4 x - 2.5 = 0$$



Method 1:
The window must be adjusted to show a sufficient amount of the x-axis to locate the intersection point (2nd CALC, #5). The answer is $x = 32$.



7. Using your graphing calculator, determine which of the following statements are true.

$a.)\ \log(3x) = \log(3) + \log(x)$

$b.)\ \log(3 + x) = \log(3) + \log(x)$

$c.)\ \log(x^3) = (\log(x))^3$

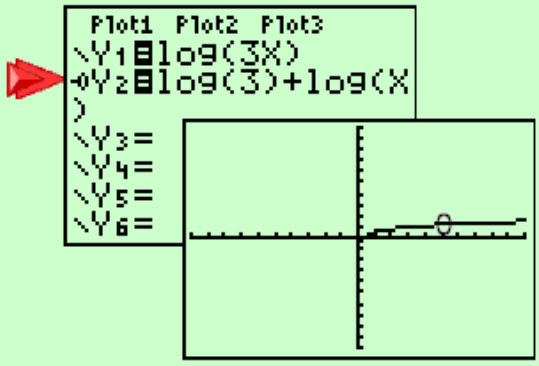
$d.)\ \log(x^3) = 3\log(x)$

$e.)\ \log\left(\frac{3}{x}\right) = \frac{\log(3)}{\log(x)}$

$f.)\ \log\left(\frac{3}{x}\right) = \log(3) - \log(x)$

a.) T b.) F c.) F d.) T e.) F f.) T

Solution for part a: **(TRUE)**
Place the left side of the equation in Y_1 and the right side in Y_2. Turn on the "bubble animation" to the left of the Y_2. This will allow you to see the bubble floating over the graph when the equation is true. Parts b - f are solved in a similar manner.



Solving Quadratic Equations

Find the solutions (roots) of a quadratic equation using the graphing calculator.
Consider the following examples:

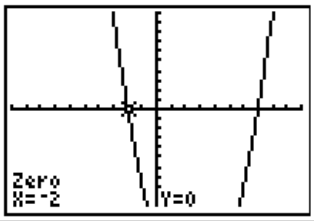
1. Using the ZERO Command

Solve: $x^2 - 5x - 14 = 0$

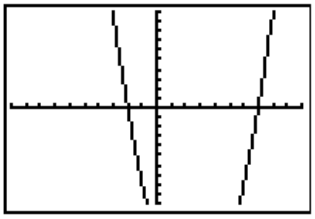
Since this equation is set equal to zero, the roots will be the locations where the graph crosses the x-axis (if the roots are real numbers).
(Remember that the x-axis is really just $y = 0$.)

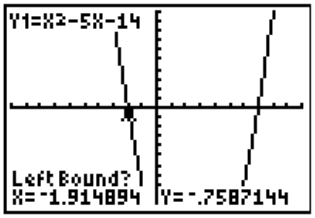
1. Set $Y_1 = x^2 - 5x - 14$
2. Use the ZERO command to find the roots -- 2nd TRACE (CALC), #2 zero
3. Left bound? Move the spider as close to the root (where the graph crosses the x-axis) as possible. Hit the left arrow to move to the "left" of the root. Hit ENTER. A "marker" ▶ will be set to the left of the root.
4. Right bound? Move the spider as close to the root (where the graph crosses the x-axis) as possible. Hit the right arrow to move to the "right" of the root. Hit ENTER. A "marker" ◀ will be set to the right of the root.
5. Guess? Just hit ENTER.
6. Repeat the entire process to find the second root (which in this case happens to be $x = 7$).

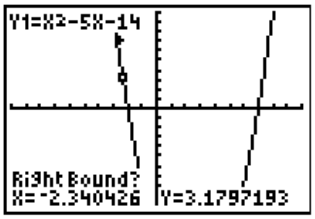
Answer: one of the roots is $x = -2$

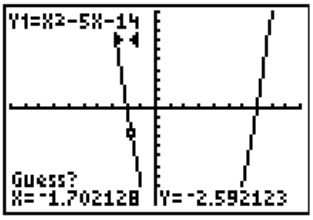


Standard 10 x 10 window







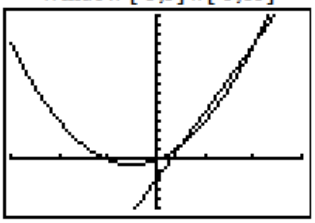


2. Using the INTERSECT Command

Solve: $2x^2 + 2x = 7x - 2$

Since this equation is NOT set equal to zero, the ZERO command cannot be used to look for roots(unless you re-write the equation so that it IS set equal to zero).

Window [-3,3] x [-5,15]



If you do not want to re-write the equation, solve using the intersect command to find the points where the two expressions intersect (if the roots are real numbers):

1. Set $Y_1 = 2x^2 + 2x, \quad Y_2 = 7x - 2$

2. Use the intersect command to find the roots, 2nd TRACE (CALC), #5 intersect.

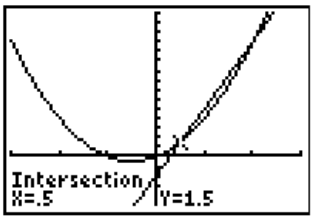
3. First Curve? Move the spider close the point of intersection that you want to find. Hit ENTER.

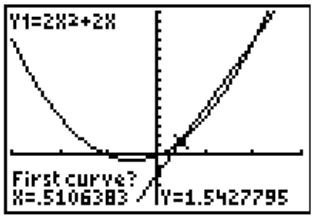
4. Second Curve? Just hit ENTER.

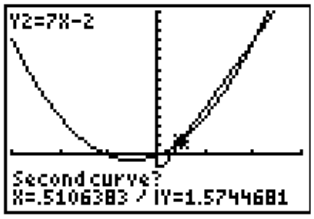
5. Guess? Just hit ENTER.

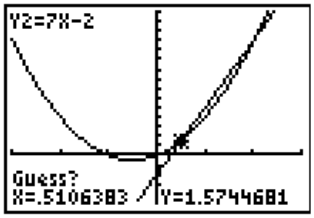
6. Repeat the entire process to find the second root (which in this case happens to be $x = 2$).

Answer: one of the roots is $x = 0.5$





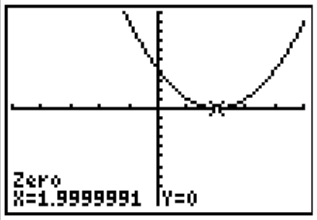




3. Only One Root?

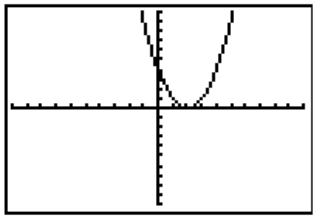
Solve: $x^2 - 4x + 4 = 0$

When graphed, this equation only intersects the x-axis in one location. This tells you that this root repeats itself.



If a reading appears such as 1.9999991, the actual answer is most likely the number 2. Check your assumption that the answer is 2 by substituting 2 into the original equation to see if it satisfies the equation.

Standard 10 x 10 window



$x^2 - 4x + 4 = 0$

$(x - 2)(x - 2) = 0$

$x = 2; \quad x = 2$

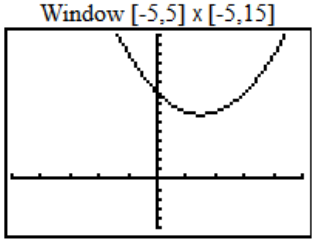
4. What if the graph does not intersect the x-axis???

(or intersect option shows no intersections)

Solve: $x^2 - 3x + 9 = 0$

When graphed, this equation does NOT intersect the x-axis. This tells you that the roots of this equation are complex (imaginary) values. Dig out the quadratic formula to find the roots.

Window [-5,5] x [-5,15]



$$x^2 - 3x + 9 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{3 \pm \sqrt{(-3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 - 36}}{2}$$

$$x = \frac{3 \pm \sqrt{-27}}{2}$$

$$x = \frac{3 \pm 3i\sqrt{3}}{2} = \frac{3}{2} \pm \frac{3i\sqrt{3}}{2}$$

Solving Quadratic Inequalities

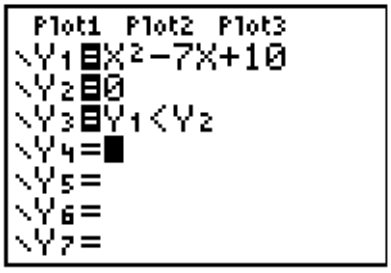
Working with quadratic inequalities in one variable is similar to working with linear inequalities in one variable. (See Linear Inequalities)

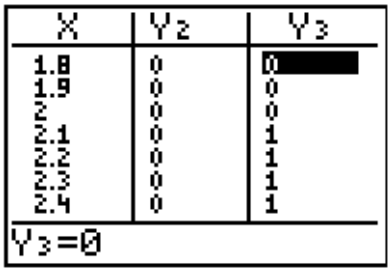
Consider the following examples:

1. Solve: $x^2 - 7x + 10 < 0$

1. Enter the left side of the inequality into Y_1.
2. Enter the right side of the inequality into Y_2.
3. Enter the inequality into Y_3 using the inequality symbols found under 2nd MATH (TEST).
   Remember the Y_1 and Y_2 are found under Vars →Y-VARS, #1Function.
4. Hit GRAPH.

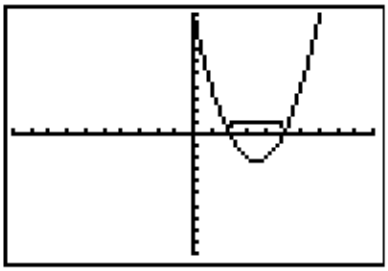
Just as we saw in Linear Inequalities, a small bar appears representing the graph of the value 1, showing where the inequality is true. In this case, the inequality is true for $2 < x < 5$. To determine the "exact" values for the endpoints, use the INTERSECT option (2nd TRACE (CALC) #5 intersect) with Y_1 and Y_2.





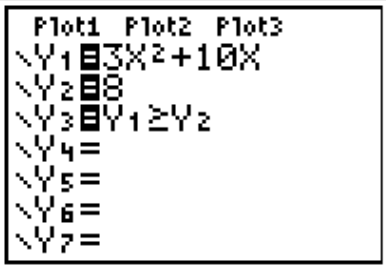
Looking at the table shows that the Y_3 values alternate between being a 0 (false) and a 1 (true). (Arrow to the right to "see" the column for Y_3.

Remember that < will not "include" the endpoints.

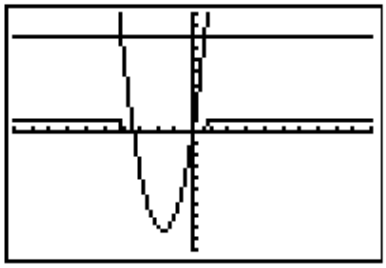


2. Solve: $3x^2 + 10x \geq 8$

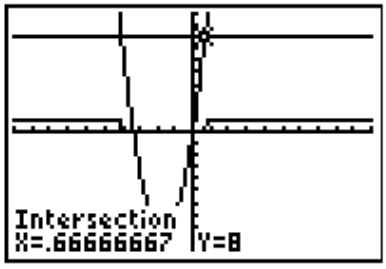
1. Enter the left side of the inequality into Y_1.
2. Enter the right side of the inequality into Y_2.
3. Enter the inequality into Y_3 using the inequality symbols found under 2nd MATH (TEST). Remember the Y_1 and Y_2 are found under Vars →Y-VARS, #1Function.
4. Hit GRAPH.



The small bars appear representing the graph of the value 1, showing where the inequality is true.

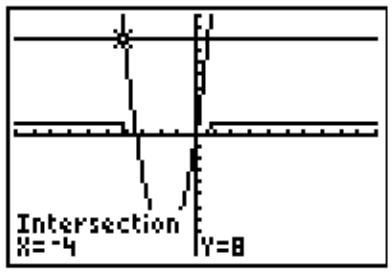


Use the INTERSECT option (2nd TRACE (CALC) #5 intersect) with Y_1 and Y_2 to determine the exact values of the endpoints of the bars.



This inequality is true for

$$x \le -4 \quad \text{or} \quad x \ge \frac{2}{3}$$



Solving Radical Equations

As you know, radical equations involving square roots are typically solved by isolating a radical and squaring both sides of the resulting equation. This process of squaring may lead to an answer that is actually NOT one of the roots of the original equation. This "*extra*" answer is called an *extraneous root.* The graphing calculator is a wonderful back-up checking tool to determine if a root is extraneous. Note the example below:

Example: Solve the following equation algebraically and check.

$$x - 3 = \sqrt{30 - 2x}$$

Algebraic Solution:

$$(x-3)^2 = \left(\sqrt{30-2x}\right)^2$$

$$x^2 - 6x + 9 = 30 - 2x$$

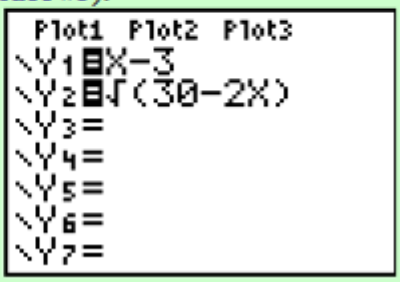
$$x^2 - 4x - 21 = 0$$

$$(x-7)(x+3) = 0$$

$$x = 7; x = -3$$

Graphical check:

Enter the left side of the equation into Y1.
Enter the right side of the equation into Y2.
Use the INTERSECT option
(2nd Trace #5).



Check:

$7 - 3 = \sqrt{30 - 14}$

$4 = \sqrt{16}$

$4 = 4$

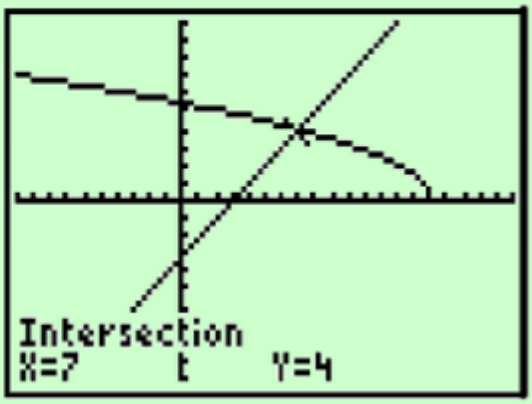
Check:

$-3 - 3 = \sqrt{30 - (-6)}$

$-6 \neq \sqrt{36}$

extraneous root

The calculator clearly shows that there is only one solution ($x = 7$), thus showing that $x = -3$ is an extraneous root.



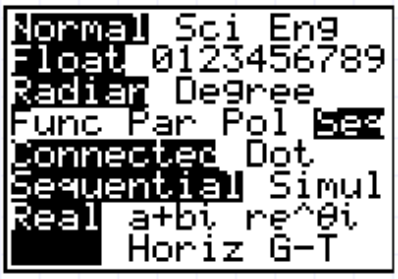
Hint: When working with radical equations, it may be difficult to "see" the intersection point if the viewing window is a small representation of the graph. You may want to enlarge the viewing window by adjusting the WINDOW settings or by using ZOOM (#2 Zoom In). Remember, when using Zoom In, hitting ENTER the first time only registers the function. You must hit ENTER a second time to activate the Zoom In option. You can quickly return to the 10 x 10 viewing window by pressing ZOOM (#6 ZStandard).

Remember: In the example above, you can see that there is only one answer to the equation since there is only one point of intersection. Should there be NO points of intersection, the answer will be the empty set.

Working with Sequences

(in both Seq and Func Modes)

In Seq MODE:

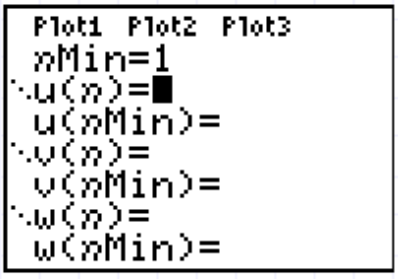


To work with sequences in the **Seq** mode:

- Press MODE key
- Choose Seq in the fourth line
- Hit ENTER to highlight Seq
- Leave other settings as default (on the left side)

Press Y= and notice the difference in this screen.

To define a sequence, you must specify:

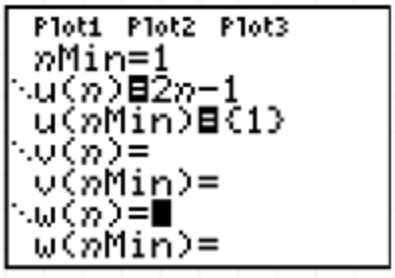


- *n*Min - where you will start counting the numbers in the sequence (most likely this value is 1)
- u(*n*) - the pattern for the sequence (use X,T,θ,*n* key to enter the pattern)
- u(*n*Min) - the first number in the sequence (When you enter the first number in your sequence, notice the list braces that appear. These braces will allow for the entry of more than one term if needed.)

Example 1: Consider the sequence defined as $a_n = 2n - 1$.

List the first 6 terms of the sequence and find the 105th term.

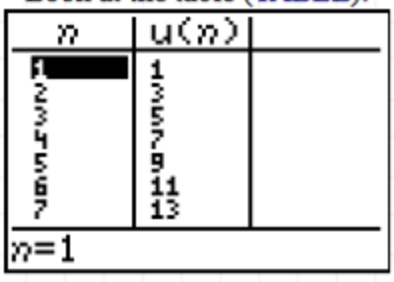
Enter the information in Y=:



Set the table (TBLSET):



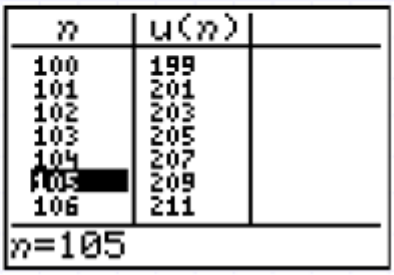
Look at the table (TABLE):



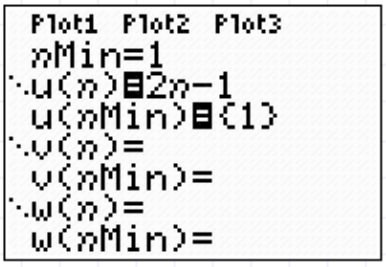
The first 6 terms are easily visible in the table.

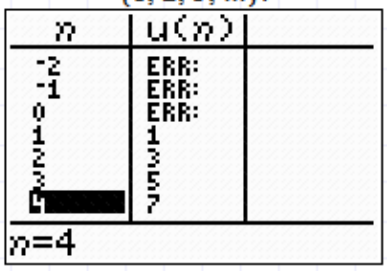
To find the 105th term, reset the TblStart to a value near (or at) the desired term. Read the answer from the table.

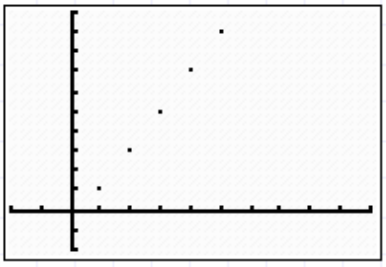




If you **GRAPH** when in **sequence** mode, even if your calculator is set in **connected mode**, you will only see plots of points. Remember that a sequence is a listing of numbers without other values in between. Notice, in the center calculator screen below, how the **sequence** mode table shows answers only for n values of 1 or larger. Remember that the domain of a sequence is the set of natural numbers $\{1, 2, 3, \ldots\}$.







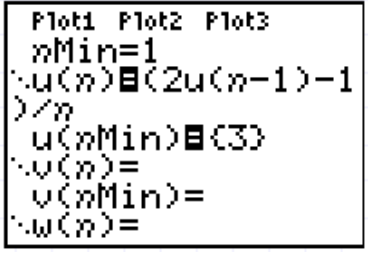
If you express this sequence as a function and return to **function** mode, you will see a connected graph (a line in this case), rather than the plotted points.

Example 2: (**Recursive sequence**) List the first 7 terms of the sequence:

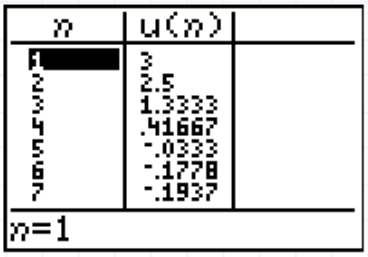
$$a_1 = 3$$

$$a_n = \frac{2a_{n-1} - 1}{n}$$

Enter the recursive formula information. Enter the **u** by using the alpha letter **u** (2nd 7). Note the starting value of 3.

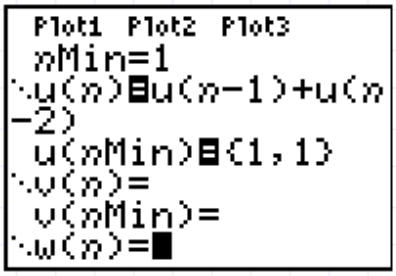


View the terms of the sequence in the table.

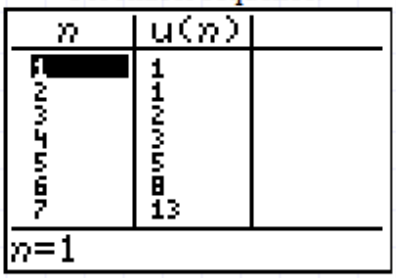


Example 3: Generate the Fibonacci Sequence (where each term, after the first two terms, is determined by adding the two previous terms).

Enter the pattern for the sequence. Notice how the u(*n*Min) displays the first two terms using the list braces.

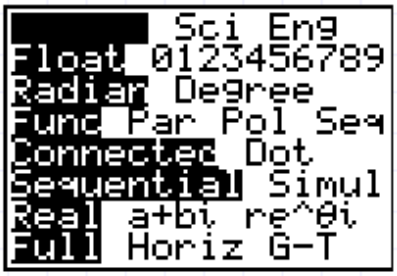


Go to the table to view the Fibonacci sequence.



In Func MODE:

Certain sequences can be examined in **Func** mode. Consider the following examples:

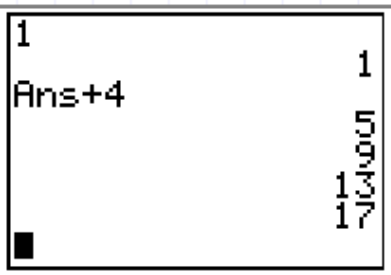


Example 4: Arithmetic and geometric sequences can be quickly generated in Func mode if the common difference or common ratio, respectively, are known, along with the starting value.

List the first five terms of an arithmetic sequence whose first term is 1 and whose common difference is 4.

A "quick - see it on the screen" method:

- Enter the first term on the home screen.
- Press ENTER to register the first term.
- Press + 4 (the expression Ans+4 will appear).
- Press ENTER.
- Continuing to press ENTER will generate successive terms in this sequence.

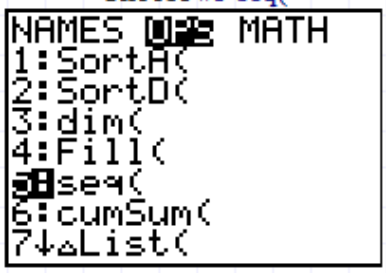


Unfortunately, finding specific terms with this method, such as the 100th, will require that you press ENTER and count very carefully - not always an easy process.

Example 5: If an expression/formula is known for the generation of a sequence, the terms can be listed in Func mode using the seq(function.

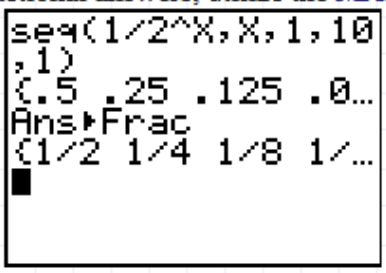
List the first 6 terms of the sequence defined by: $a_n = \frac{1}{2^n}$

2nd STAT (LIST) → OPS
Choose #5 seq(



Using seq(, type the formula, variable, starting value for n, last value of n needed, increment. (For ease, X may be used as the variable -- or type N using the alpha key.) Arrow to the right to see more of the terms of the sequence.

If you need fractional answers, utilize the MATH - #1▶Frac



This method of listing terms of a sequence will be limited to 999 terms of the sequence at any one time.

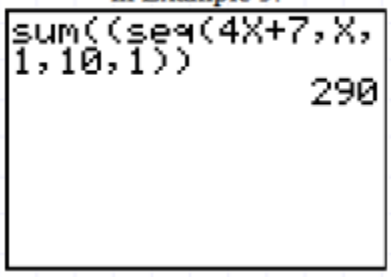
Example 6: If an expression/formula is known for the generation of a sequence, the sum of a specific number of terms can be found in Func mode.

Find the sum of the first 10 terms of the sequence defined by: $a_n = 4n + 7$

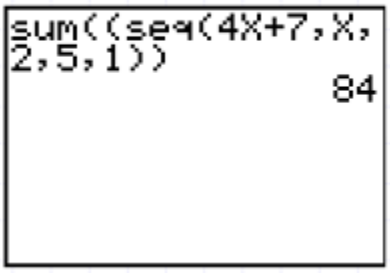
2nd STAT (LIST) → MATH
Choose #5 sum(



Notice that sum is followed by the entry of the sequence as shown in Example 5.



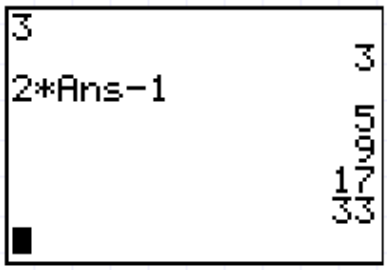
Find the sum of the second through fifth terms of the sequence: $a_n = 4n + 7$



Example 7: Sequences represented by a recursive formula can be generated in Func mode.

List the first 4 terms of the sequence defined by: $a_1 = 3$, $a_{n+1} = 2a_n - 1$

- Type the starting value and press ENTER.
- Use the ANS function to create the formula used to generate the sequence. Type 2*ANS-5 and press ENTER.
- Now, simply press ENTER to create successive terms of the sequence.



Evaluating Summation Notation

(Function Mode)

General summations:

A shorthand notation is used to represent sums with more than a few terms. This shorthand notation uses the Greek letter sigma (which denotes sum). If we have a rule such as $f(n)$ which assigns the value $f(n)$ to each integer n in the set of integers $\{j, j+1, j+2, \ldots k\}$, we can represent the sum as:

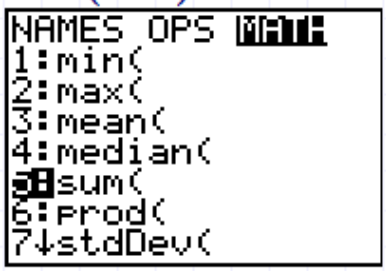
$$\sum_{n=j}^{k} f(n) = f(j)+f(j+1)+f(j+2)+\ldots+f(k)$$

On the graphing calculator:

We will be using the **sum** feature and the **seq** feature to work with a general summation. On the calculator, we will interpret a summation as follows:

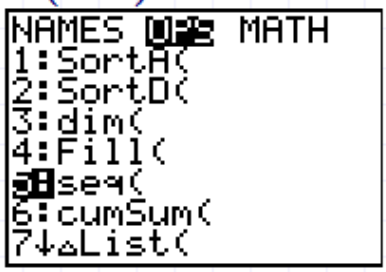
$$\sum_{n=j}^{k} f(n) = sum(seq(f(n),n,1,k,1))$$

2nd STAT (LIST) → MATH #5 sum



The format for **sum** is **sum(** *list* **)**
where *list* will be the terms of the rule
$f(j), f(j+1), f(j+2), \ldots, f(k)$

2nd STAT (LIST) → OPS Choose #5 seq(

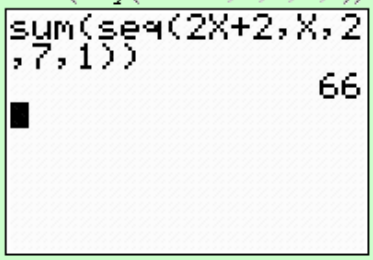


The format for **seq:** ***expression, variable, starting value, ending value, increment.***

While sequences have a domain of natural numbers, {1, 2, 3, 4, ...}, in **Func** mode, the **seq**uence command will accept integer domain values less than one. (In **Seq** mode, however, the **seq**uence command will accept only integer domain values of 1 or greater.)

Example 1: Evaluate: $\sum_{k=2}^{7}(2k+2)$

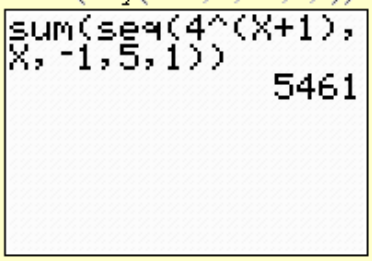
$sum(seq(2x+2,x,2,7,1))$



notice that this sum starts with $k = 2$

Example 2: Evaluate: $\sum_{r=-1}^{5}4^{r+1}$

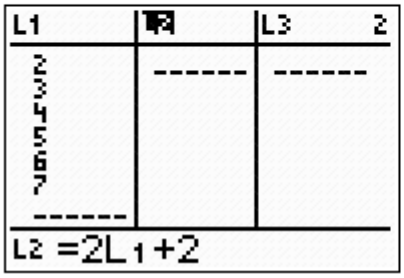
$sum(seq(4^{x+1},x,-1,5,1))$

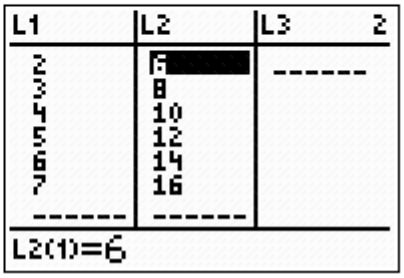


(For ease of entry, **X** was used as the variable in both examples above.
If you wish to use the designated variable, type it into the calculator using the alpha key.)

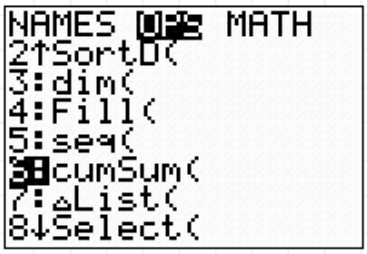
Another approach to ***Example 1****:*

Type the number of terms needed into **L1**.
Type the rule into **L2**.
Add the answers manually on the **HOME** screen.

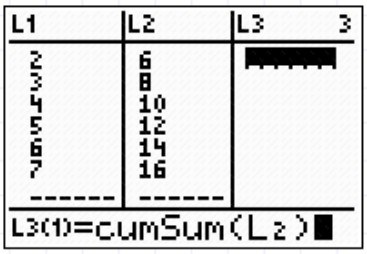




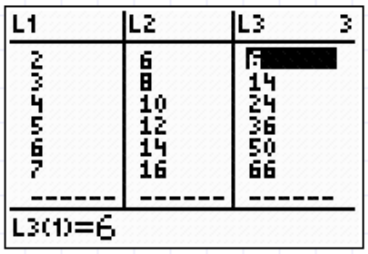
Or, have the calculator do the adding. If you do not want to add manually on the home screen, you can use the **cumulative summation feature (cumSum)** to add **L2**.
Remember that Example 1 starts with $k = 2$.



To find **cumSum(** , go to **LIST** (**2nd STAT**), arrow to **OPS** heading, and choose **#6 cumSum(**.



Enter **cumSum(L2)** as the pattern to determine **L3.**



The entry in column **L3** corresponding to the value 7 in **L1** will be the sum.

Summations and Sequences:

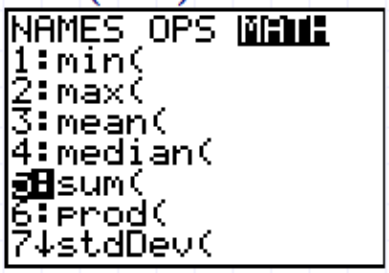
Sums, such as $\sum_{j=1}^{n} a_j$, where $j = 1$, may represent the summation of a sequence, called a series.

Remember, sequences have a domain of natural numbers, {1, 2, 3, 4, ...}.
In the sequence related problems below, the starting value in the summation will be greater than or equal to 1.

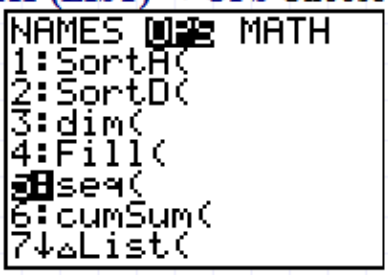
Again, we will be using the **sum** and **seq** features of the calculator to find the summations. The summation is interpreted as:

$$\sum_{j=1}^{n} a_j = \text{sum}(\text{seq}(a_j, j, 1, n, 1))$$

2nd STAT (LIST) → MATH #5 sum

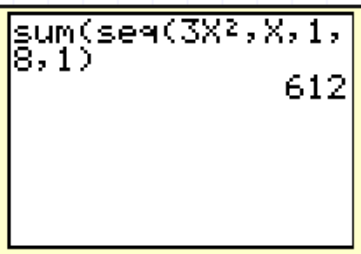


2nd STAT (LIST) → OPS Choose #5 seq(



Example 3:

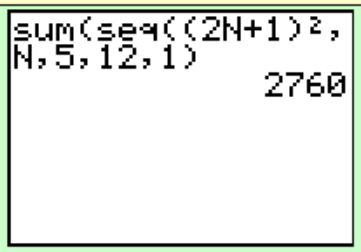
Evaluate: $\sum_{j=1}^{8} 3j^2$



This example uses x as the variable, instead of using j. The x is simply easier to enter into the calculator.

Example 4:

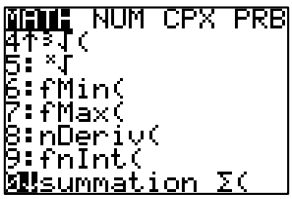
Evaluate: $\sum_{n=5}^{12} (2n+1)^2$



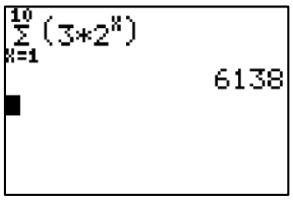
This example uses n as the variable -- obtained by engaging the alpha key.

Example 5:

Find $\sum_{r=1}^{10}\left(3\cdot 2^{r}\right)$



Press **MATH** and choose **0↓ summation ∑(** and hit **ENTER.**



As you will finish to type the sequence, press **ENTER.** The answer will be at the left side at the middle of the screen.

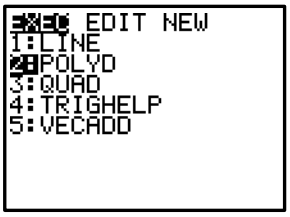
Polynomials Division

Downloadable software

This program designed for the real numbers only in the quotient and the divisor is greater than dividend.

Press **PRGM** and choice **2: POLYD**.
Press **ENTER** twice.





You can choice between **1: SYNTHEIC** division and **2: LONG** division. As you will decide, hit **ENTER**.

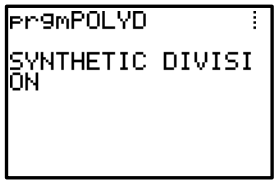




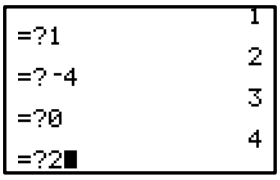
Synthetic Division

Remember! For the synthetic division your dividend must be in the form $(\boldsymbol{x-a})$ only.

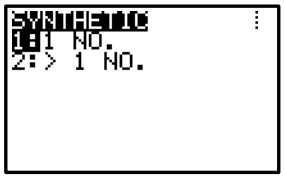
After pressing **ENTER**, you will see the synthetic division screen. Press one more time and show the highest degree of the dividend, type it and press **ENTER**. Then type all coefficients near dividends variables by the order from the leading coefficient to the constant and hit **ENTER** after each of them.

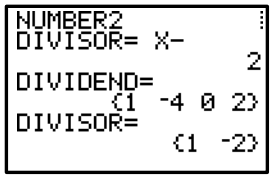




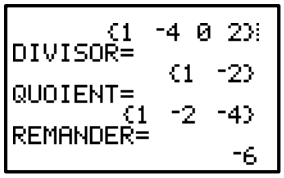


Choice your dividend form. Highlight **1: 1 NO.** and press **ENTER**. Type the coefficient in the divider and press **ENTER**. You can see all dividend and divisor coefficients, mean the dividend is $\boldsymbol{x^3-4x^2+2}$ and the divisor is $\boldsymbol{x-2}$.





When you press **ENTER** and continue to press this key until the end of the process, you can see all other parameters of the divisions: quotient is $\boldsymbol{x^2-2x-4}$ and remainder is $\boldsymbol{-6}$. The final answer looks like: $\frac{x^3-4x^2+2}{x-2}=x^2-2x-4+\frac{-6}{x-2}$.

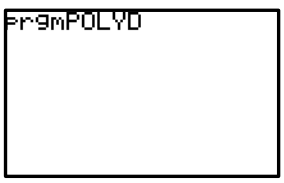


Long Division

The procedure is similar to the rational functions division by synthetic method.

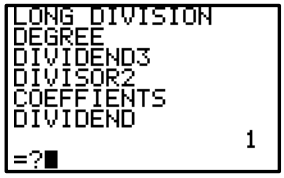
Example: Divide $\frac{5x^3-12x^2+13}{x^2-7}$

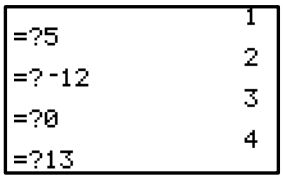


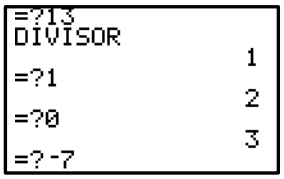


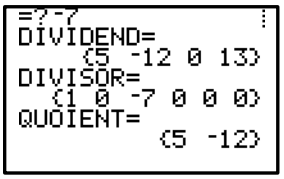


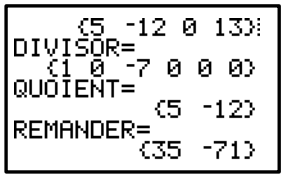












Zeros of the Functions

The graphing calculator is very helpful when determining the zeros of functions (the locations were the graph crosses the *x*-axis --- the roots).

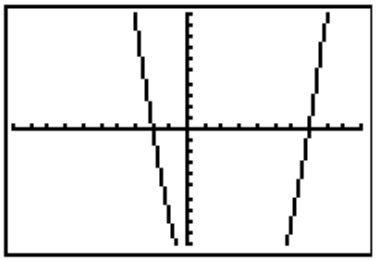
Using the **ZERO** command:

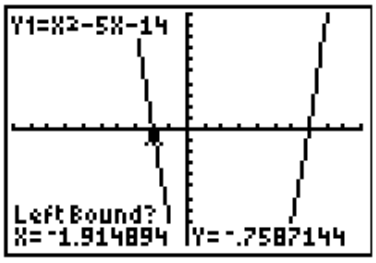
Solve: $x^2 - 5x - 14 = 0$

Since this equation is set equal to zero, the zeros (roots) will be the locations where the graph crosses the *x*-axis (if the roots are real numbers).

1. Set $Y_1 = x^2 - 5x - 14$
2. Use the ZERO command to find the zeros (roots) -- 2nd TRACE (CALC), #2 zero
3. Left bound? Move the spider as close to the zero (root --- where the graph crosses the *x*-axis) as possible. Hit the left arrow to move to the "left" of the zero (root). Hit ENTER. A "marker" ▶ will be set to the left of the zero (root).

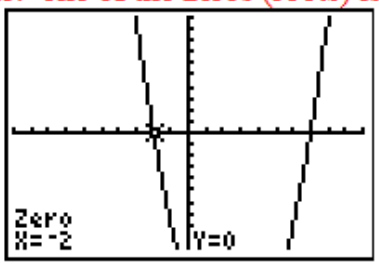
Standard 10 x 10 window

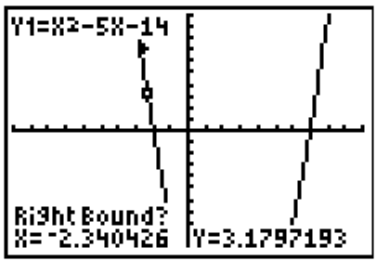


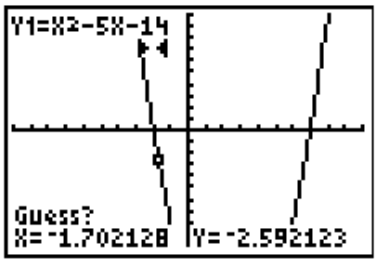


4. Right bound? Move the spider as close to the zero (root --- where the graph crosses the x-axis) as possible. Hit the right arrow to move to the "right" of the zero (root). Hit ENTER. A "marker" ◄ will be set to the right of the zero (root).
5. Guess? Just hit ENTER.
6. Repeat the entire process to find other zeros (roots --- which in this case the second root happens to be $x = 7$).

Answer: one of the zeros (roots) is $x = -2$

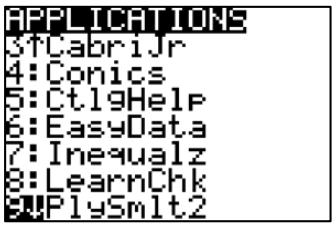






Using APPLICALIONS Tools

Find real zeros for the polynomial function $x^3 - 6x^2 + 11x - 6 = 0$



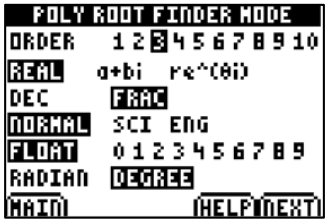
Press **APPS** key and highlight the line **9↓PlySmlt2**, then press **ENTER**.



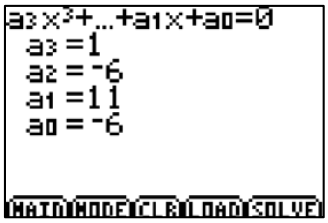
When you will receive this picture, press any key, as written at the screen.



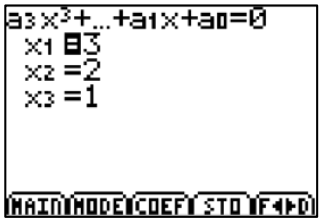
Choose **1: POLY ROOT FINDER** and press **ENTER**.



Because the highest power at the given equation is 3, mean, the equation has 3 roots, highlight the number **3** at the line **ORDER** and press **ENTER**.

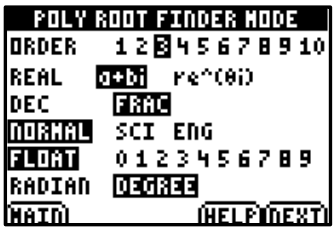


Press **GRAPH** key. Type all given coefficients near the variable. Press **ENTER**.



Press **GRAPH** again. Waiting a small period of the time until you will receive the answer.

Remember: Your zeros of the function are possible to be the complex numbers. In this case, do not forget about the conjugate number. If one of the roots will be in the complex form of $a + bi$, you will have another conjugate root $a - bi$ also.



Attention: In case of possibility of the complex numbers in the answer, your polynomial root finders mode should looks a little different. Other procedure is the same as at the previous example.

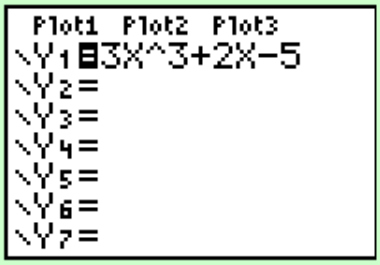
5. Precalculus

Evaluating Functions

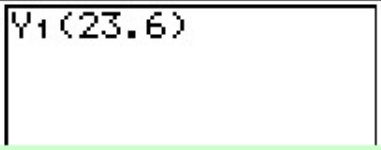
Functions can be evaluated on the graphing calculator by the use of functional notation or by graphing.

1. Evaluating functions using functional notation.

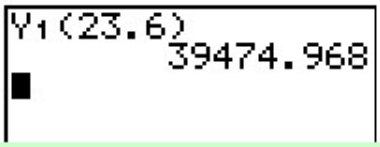
If $f(x) = 3x^3 + 2x - 5$, find $f(23.6)$. Round to 3 decimal places, if needed.



Enter the function in **Y=**.



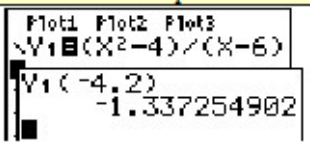
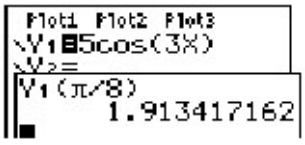
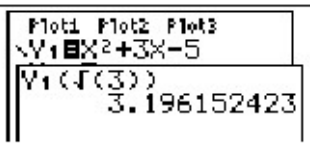
Go to the home screen. Using a functional notation format, enter **Y1(23.6).** [To get **Y1**, go to **VARS**, arrow right to **Y-VARS**, **#1Function**, **#1Y1.**]



Hit **ENTER.**

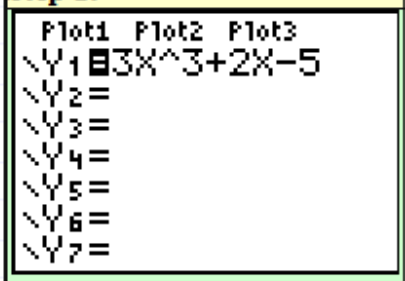
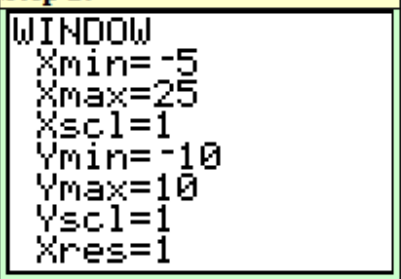
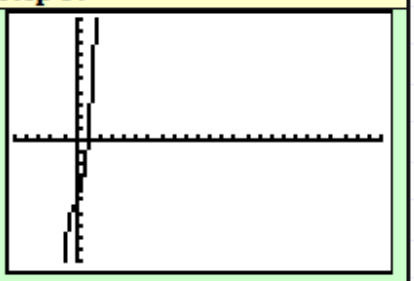
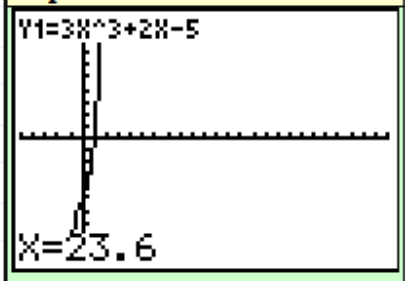
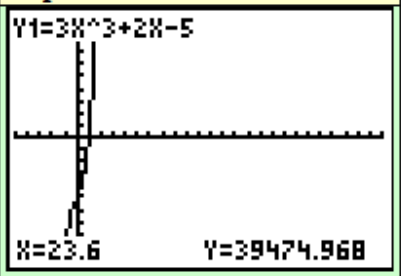
Answer: 39,474.968

Examples:

	Function:	Evaluate:	Answer (round to 3 decimal places if needed)
1.	$f(x) = \dfrac{x^2 - 4}{x - 6}$	$f(-4.2)$	 **Answer: -1.337**
2.	$f(x) = 5\cos 3\theta$	$f\left(\dfrac{\pi}{8}\right)$	Set **MODE** to **Radian.**  **Answer: 1.913**
3.	$f(x) = x^2 + 3x - 5$	$f\left(\sqrt{3}\right)$	 **Answer: 3.196**

2. Evaluating functions by graphing.

If $f(x) = 3x^3 + 2x - 5$, find $f(23.6)$. Round to 3 decimal places, if needed.

Step 1:	Step 2:	Step 3:
		
Enter the function in **Y=**.	Adjust the **WINDOW** so that the point **23.6** will be shown on the *x*-axis.	**Graph**
Step 4:	**Step 5:**	**Step 6:**
		**Answer:** 39,474.068
Hit **TRACE.** Type **23.6** right on the screen. It will automatically appear at the bottom of the graph.	Hit **ENTER.** The answer (the *y*-value) will appear.	

Graphing Piecewise-Defined Functions

Piecewise-defined, or split definition, functions can be graphed on the graphing calculator.

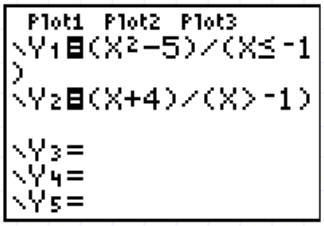
You will need to utilize the relational operators $=, \neq, \leq, <, \geq, >$
found under the TEST menu (2nd MATH).
The format for entering these function can be either
(expression)/(condition) or *(expression)*(condition)*
where parentheses are essential in both situations

Let's examine the differences between the two methods,
see how to enter compound inequalities ($2 < x < 6$),
and discover why these methods work.

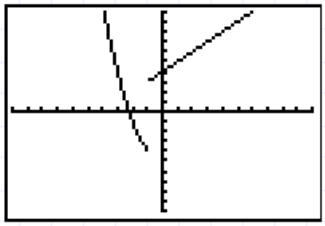
Method 1: *(expression)/(condition)*

When using this method, enter each SECTION of the function into a **separate Y=** area.

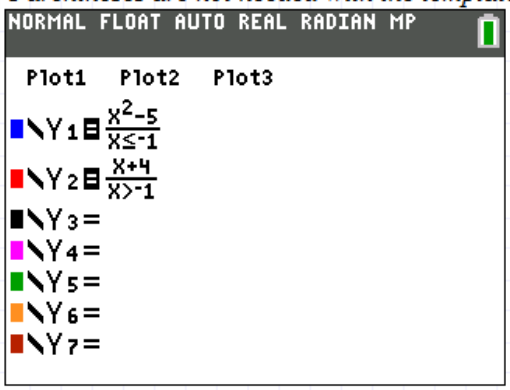
Graph: $f(x) = \begin{cases} x^2 - 5; & x \le -1 \\ x + 4; & x > -1 \end{cases}$

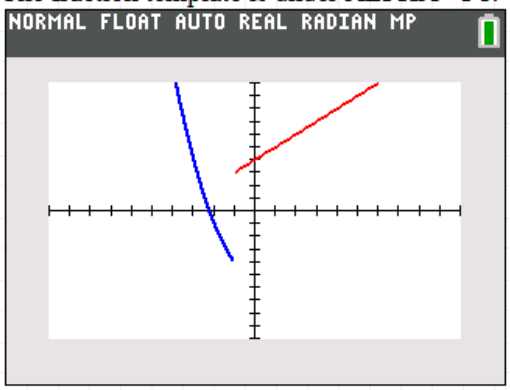


Parentheses needed!



If you use the TI-84+C, you can use the "fraction template" for the "pretty print" display.
Parentheses are not needed with the template. The fraction template is under ALPHA - F1.





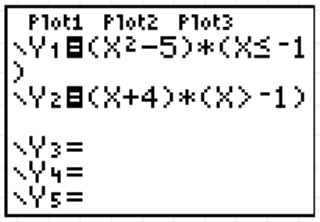
NOTE: Neither method will draw the "open" or "closed" circles at the endpoints of sections of the graph.
You will have to draw the appropriate circles when copying the graph.
"open circle" for < or >
"closed circle" for ≤ or ≥

Advantages of this method:	Disadvantages:
1. This method of entering the function is helpful when trying to find any mistakes as each section can be altered independently.	**1.** When using **TRACE**, you will need to use your up arrow to move between sections of the graph.
2. The graph, while still in **CONNECTED MODE**, will not "connect" the separate sections together. The graph will be a more realistic representation of the graph.	

Method 2: *(expression)*(condition)*

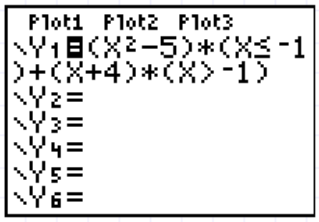
When using this method, enter each SECTION of the function into a **separate Y=** area **OR** enter the ENTIRE function as one statement using + sign to separate the sections.

Graph: $f(x)=\begin{cases} x^2-5; & x\le -1 \\ x+4; & x>-1 \end{cases}$



Entered separately.

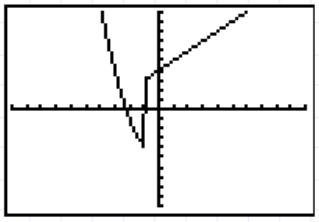
OR

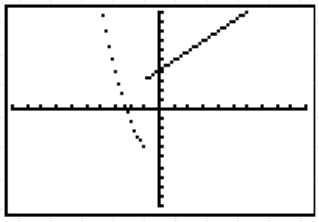


Entered as one statement

The graphs from either of these entries will produce a connected graph.

Unfortunately, DOT MODE is needed with this method to see the actual piecewise functional shape.


Connected MODE


DOT MODE

Advantages of this method:	Disadvantages:
1. This method allows for the function to be entered as one statement. When using **TRACE**, there will be no need to arrow up and down between sections of the graph.	**1.** The graph, while still in **CONNECTED MODE, will** "connect" the separate sections together. This can lead to confusion about the actual graph.
	2. This method requires a change to **DOT MODE** to get a realistic graph.
	3. Entering the function as one statement can be difficult to debug if an error occurs.

Compound inequality: $-1 < x < 2$

$$f(x)=\begin{cases} x^2+1; & x\le -1 \\ x+4; & -1<x<2 \\ 5; & x\ge 2 \end{cases}$$

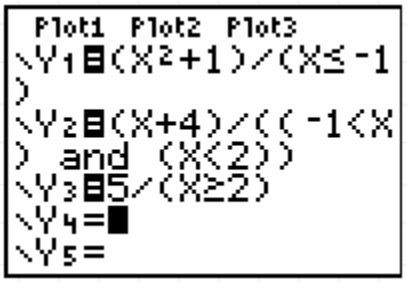
Computers and calculators do not interpret expressions such as $-1 < x < 2$ as we do in mathematics. Both computers and calculators see this expression as **ONLY** $-1 < x$ (and they ignore the second part of the condition). To force them to see the second condition, it it necessary to write the expression:

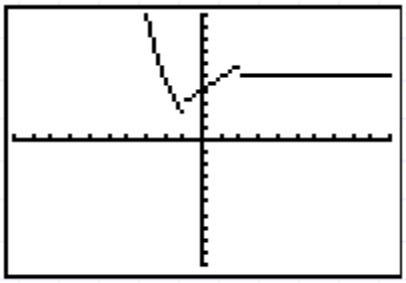
$(-1 < x < 2)$

as

$((-1 < x)$ and $(x < 2))$

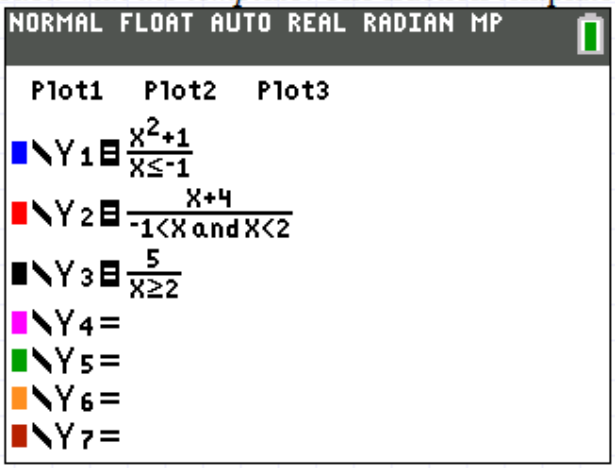
(Find **and** from the catalog or under **TEST (2nd MATH)** arrow right to **LOGIC**).





In **Connected MODE**

If you use the TI-84+C, you can use the "fraction template" for the "pretty print" display. *Parentheses are not needed with the template*. The fraction template is under ALPHA - F1.

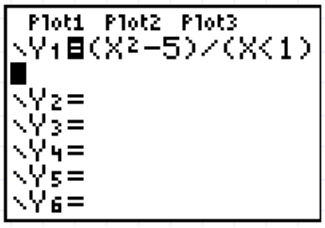


So, what is really going on here???

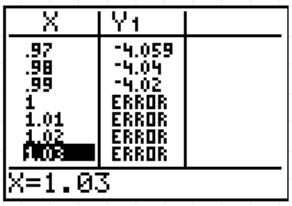
How is this process working?

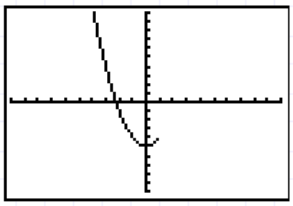
The calculator is doing a Boolean check on these conditions. Remember, Boolean equates a true condition to be a 1 and a false condition to be a 0.

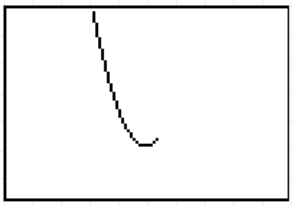
Consider: $f(x) = x^2 - 5; \quad x < 1$



When dealing with the **division method,** if a number is NOT in the interval, $x < 1$, the condition is assigned a value of 0, and the problem becomes division by zero. Since this is not possible, an error is produced at those values and "nothing" is graphed in that area. Since there is NO graph at these locations, there is also NO graphical connection being made, even though the calculator is in **Connected MODE.**

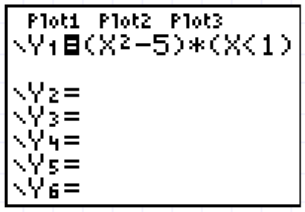


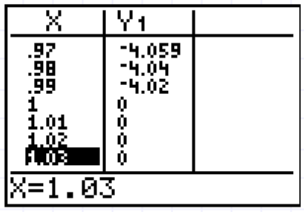


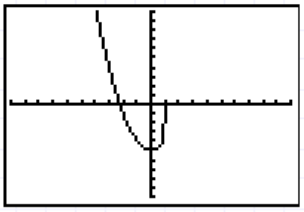


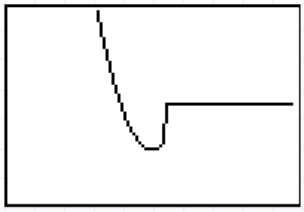
Graph with axes turned off.

When dealing with the **multiplication method,** if a number is NOT in the interval, $x < 1$, the condition is assigned a value of 0, and the problem becomes multiplication by zero. Of course the "graphing" of this 0 value coincides with the x-axis and is not seen when the axes are visible. Because this 0 value IS graphed, the **Connected MODE** kicks in, and the sections of the graph are connected together.







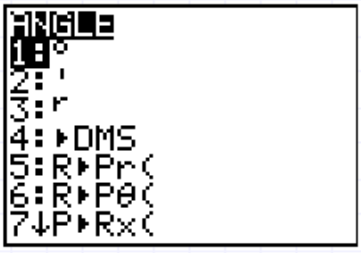


Polar Coordinates

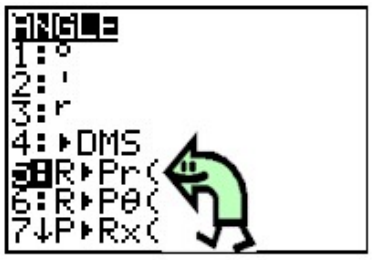
$(x,y) \rightarrow (r,\theta)$ OR $(r,\theta) \rightarrow (x,y)$

Let's start by examining the commands used to convert between rectangular and polar coordinates:

Our primary menu of commands is listed under ANGLE.
Press **2nd APPS (ANGLE)** to open this menu.



Change Rectangular to Polar:



Option #5: **R▶P r (** is the **rectangular to polar** conversion that returns only the **r value.**

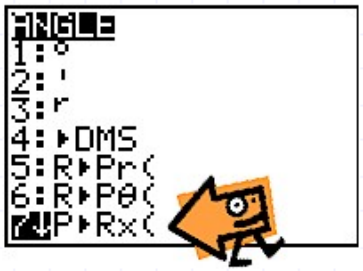
Format: **R▶P r (x,y)**



Option #6: **R▶Pθ (** is the **rectangular to polar** conversion that returns only the **θ value.**

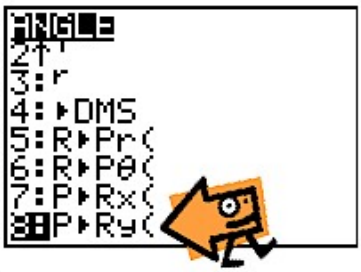
Format: **R▶Pθ (x,y)**

Change Polar to Rectangular:



Option #7: **P▶Rx (** is the **polar to rectangular** conversion that returns the **x value.**

Format: **P▶Rx (x,y)**



Option #8: **P▶Ry (** is the **polar to rectangular** conversion that returns the **y value.**

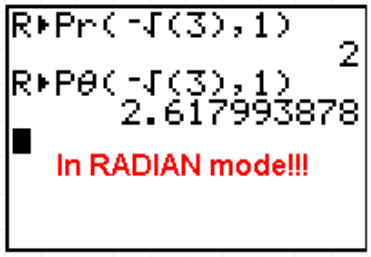
Format: **P▶Ry (x,y)**

Examples:

1. Convert $(-\sqrt{3},1)$ to polar form.

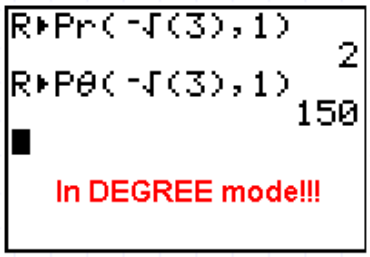
We need to find the corresponding (r,θ) values for the given (x,y) values.

Options **#5: R▶P r (** and **#6: R▶Pθ (** give:



NOTE: $\frac{5\pi}{6} \approx 2.617993878$

Answer: (2, 2.618) or $\left(2,\frac{5\pi}{6}\right)$



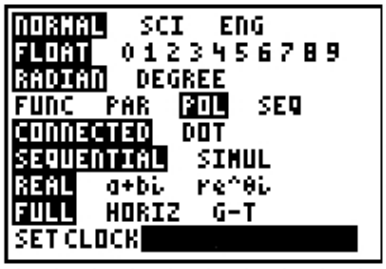
Answer: (2, 150°)

Polar Graphing

To graph an equation in polar form, set the calculator MODE to polar (POL).→

If you want to see your **TRACE** in terms of r and θ, change **FORMAT (2nd ZOOM)** to **PolarGC.**

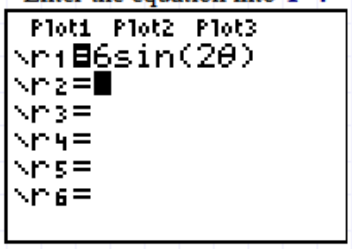




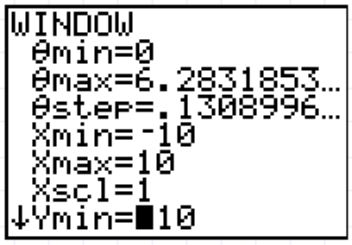
Examples:

1. Graph $r = 6\sin(2\theta)$

Enter the equation into **Y=:**



Choose **ZOOM #6 ZStandard**



Choosing **#6 ZStandard** will set the window as follows:

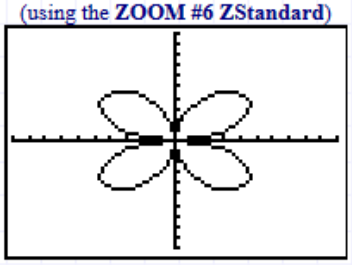
$\theta\min = 0$

$\theta\max = 2\pi$

$\theta\text{step} = \frac{\pi}{24}$

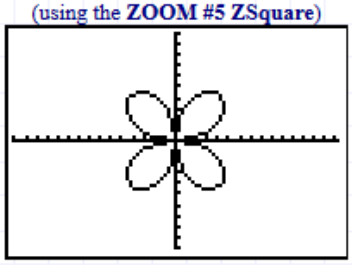
with the x and y values in the normal -10 to10 configuration

Graph
(using the **ZOOM #6 ZStandard**)



Because the Standard viewing window has a 3 to 2 aspect ratio (it is not evenly divided on both axes)., the graphs may appear slightly distorted.

Graph
(using the **ZOOM #5 ZSquare**)



To prevent distortion, choose **ZOOM #5 ZSquare.**

Choosing **#5 ZSquare** will also set the window as follows:

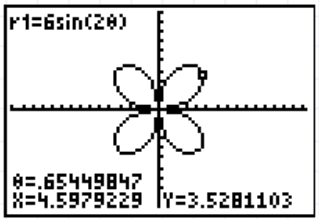
$\theta\min = 0$

$\theta\max = 2\pi$

$\theta\text{step} = \frac{\pi}{24}$

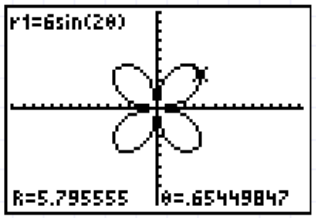
but with the x and y values in the -15.16129 to 15.16129 configuration.

Trace



Notice that x and y are displayed. The calculator **FORMAT** is set to the default value of **RectGC**.

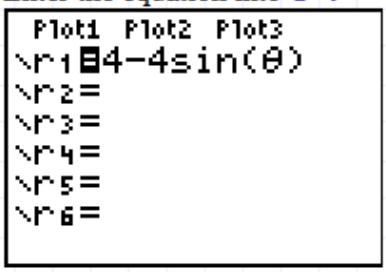
Trace



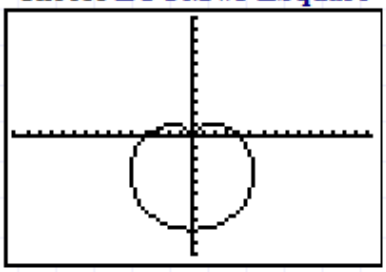
Notice that r and θ are displayed. The calculator **FORMAT** is set to **PolarGC.**

2. Graph $r = 4 - 4\sin\theta$

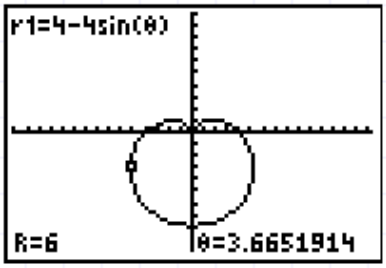
Enter the equation into **Y=:**



Choose **ZOOM #5 ZSquare**



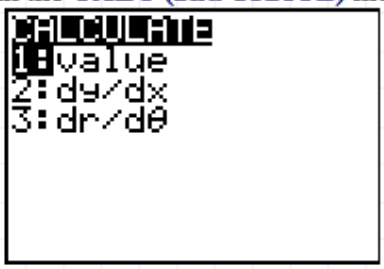
Trace



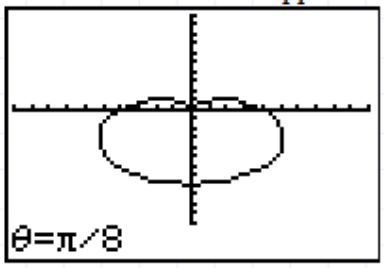
3. Find r from the graph screen of $r = 4 - 4\sin\theta$ when $\theta = \pi/8$

You can find the value of a function from the graph screen in two ways:

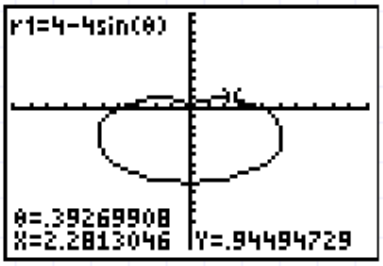
Method 1: -- by using the **VALUE** function in the **CALC (2nd TRACE)** menu:



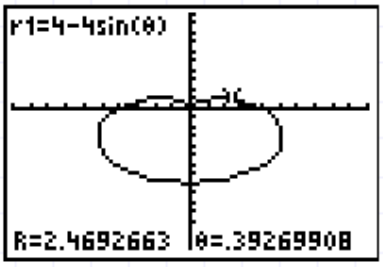
This screen will appear....



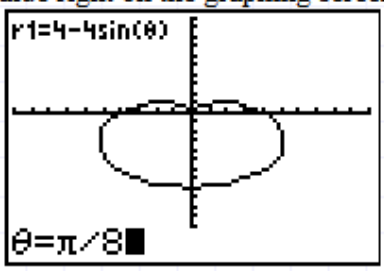
If the **FORMAT (2nd ZOOM)** is set to **RECTGC** you will see the x and y coordinates...



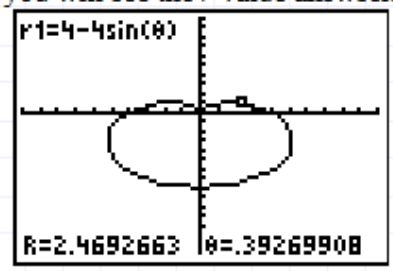
If the **FORMAT** is set to **POLARGC** you will see the r and theta coordinates...



Method 2: -- simply hit **TRACE** and enter the theta value right on the graphing screen.



If the **FORMAT** is set to **POLARGC**, you will see the r value answser.



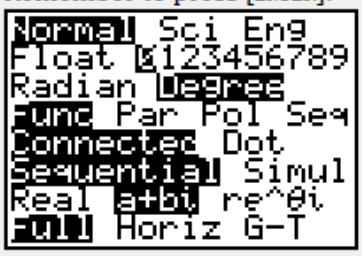
Complex Numbers in Polar and Rectangular Forms

Rectangular Display Mode

Rectangular mode means you want *answers* in $a + bi$ form, whether you use polar or rectangular form when entering your expressions.

Once only, you need to tell the TI-83/84 that you want results in rectangular mode.

[MODE] [▼ 6 times] [▶] [ENTER] selects a+bi mode.
Remember to press [ENTER].



[2nd MODE *makes* QUIT] returns to home screen.

For complex numbers in rectangular form, the other mode settings don't much matter.

Polar Display Mode

"Polar form" means that the complex number is expressed as an absolute value or modulus r and an angle or argument θ. There are four common ways to write polar form: $r\angle\theta$, $re^{i\theta}$, r cis θ, and $r(\cos\theta + i\sin\theta)$.

Polar mode on your calculator means that **you want *answers* in a polar form**, even if you enter expressions in rectangular form. Here's how to set polar mode for display.

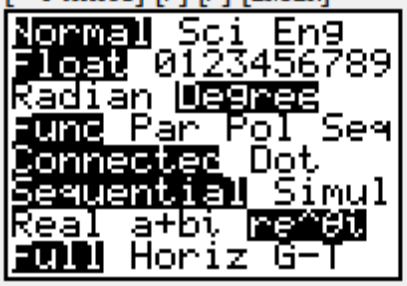
Since polar mode involves an angle, select degree or radian mode.

Tell the calculator that you want results in polar mode.

Caution: Degree mode is shown here by way of example. Make sure you select Radian mode if that's what you want.

[MODE]] [▼] [▼]. Then cursor to Radian or Degree and press [ENTER].

[▼ 4 times] [▶] [▶] [ENTER]



Then [2nd MODE *makes* QUIT] to return to the home screen.

Your calculator will display polar format differently, depending on whether you selected degree mode or radian mode:

Polar Display in Degrees	Polar Display in Radians
`re^(θi)` with θ in degrees. Example: 3−4*i* displays as `5e^(-53.13010235i)`.	`re^(θi)` with θ in radians. Example: 3−4*i* displays as `5e^(-.927295218i)`.

Entering Numbers

You can enter numbers in rectangular form or polar form, regardless of how you have set the display mode. You can even mix the two forms in one expression.

Rectangular Form for Input

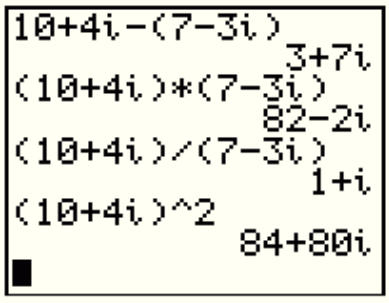
Enter numbers just as you see them. For example, here's 8−3*i*. Engineers, use *i* instead of *j*.	Find `i` in yellow above the decimal point. Enter 8 [-] 3 [`2nd` . *makes* `i`].

Remember to distinguish between the **negative-number key [(-)]** and the **subtract key [−]**. Use the subtract key for numbers with interior minus like 7−3*i* and 2*i*−11; use the negative-number key for numbers with leading minus like −2*i* and −7+3*i*.

Entering Expressions

Even though a complex number is a single number, it is written as an addition or subtraction and therefore you need to put parentheses around it for practically any operation. The illustration shows correct methods for subtraction, multiplication, division, and squaring.

Try these operations without parentheses and you'll see that you get wrong answers.



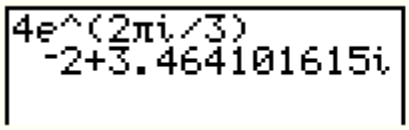
Polar Form for Input

Here's how to enter the number 4∠120° or $4e^{120°i}$ in your calculator. Note that 120° = 2π/3 radians.

Overview:	r `e^` (θ*i*) with angle in radians even if the calculator is in degree mode; the *i* is required.

Details:

Enter the absolute value or modulus, r.	4
Enter the separators between r and θ.	Press [2nd LN *makes* e^]. The display shows 4e^(.
Enter the angle or argument, θ.	Since 120° = 2π/3 radians, you must enter 2π*i*/3. (Not 2π/3*i*: remember the order of operations.) Press 2 [2nd ^ *makes* π] [2nd . *makes* i] [÷] 3. Caution: The angle must be in radians, even if the calculator is in degree mode, and the imaginary symbol *i* is required.
Enter the closing parenthesis.	[)] [ENTER]



Here's what you get if you enter the same number when the TI-83/84 is set for rectangular (a+bi) display.

Converting to Polar or Rectangular Form

Your TI-83/84 will automatically convert all answers to polar or rectangular form, depending on how you set the display format. But you can convert a particular answer without changing the mode. The conversion command (to **Rect** or to **Polar**) comes at the end of the command line, never in the middle.

To convert an answer to rectangular form:

Enter the number or expression, then ▸Rect.	[MATH] [▸] [▸] [6]	2e^(πi/3)*2.5e^(πi/6)▸Rect 5i

To convert an answer to polar form:

Enter the number or expression, then ▸Polar.	[MATH] [▸] [▸] [7] The calculator will display the angle (part of the exponent on e) in radians or degrees according to how you set the mode. Be careful to interpret the answer in the correct measure! This example shows the same calculation in radian mode and then in degree mode.

```
(10+4i)-(5-8i)▸P
olar
13e^(1.17600520…
(10+4i)-(5-8i)▸P
olar
13e^(67.3801350…
```

Finding the Angle

You can find just the angle (or argument) for a complex number. The angle will be in radians or degrees, according to the calculator mode.

Example:

What's the angle for the complex number −16+47*i*? To begin with, since the number is in quadrant 2 (negative real part, positive imaginary part), the angle must be between 90° and 180° or between about 1.7 and 3.1 radians.

Select the angle function.	[MATH] [▸] [▸] [4]
Enter the number.	[(-)] 16 [+] 47 [2nd . *makes i*]
Enter the closing parenthesis and find the answer, about 108.8° or 1.8989 radians depending on your calculator mode.	[)] [ENTER]

```
angle(-16+47i)
      108.7998852
```

Finding the Absolute Value r

Let's find r, the absolute value or modulus, of the number −16+47*i*

Let's find r, the absolute value or modulus, of the number −16+47*i*.

Select the abs function.	[MATH] [▸] [1]
Enter the number.	[(-)] 16 [+] 47 [2nd . *makes i*]
Enter the closing parenthesis and find the answer, about 49.649.	[)] [ENTER]

```
abs(-16+47i)
      49.64876635
```

De Moivre's Theorem

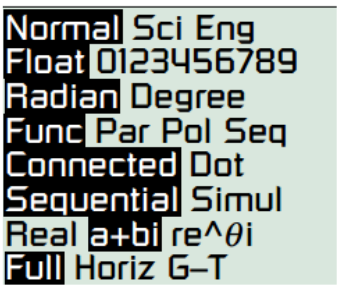
Let $z = r(\cos\theta + i\sin\theta)$ and let n be a positive integer. Then

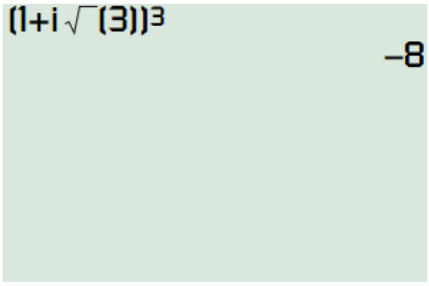
$z^n = \left[r(\cos\theta + i\sin\theta)\right]^n = r^n(\cos n\theta + i\sin n\theta)$.

Example:

Find $(1+\sqrt{3}i)^3$ using De Moivre's Theorem

Because we have the complex number, change the **MODE** to $a + bi$ form. Then press **2ND** and **QUIT**. Type all given data correctly and press **ENTER**.





Binomial Theorem

Binomial Theorem
(or Binomial Expansion Theorem)

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k}b^k$$

Most of the syntax used in this theorem should look familiar. The $\binom{n}{k}$ notation is just another way of writing a combination such as ${}_nC_k$ (read "*n choose k*").

$$\binom{n}{k} = {}_nC_k = \frac{n!}{(n-k)!k!}$$

The Binomial Theorem can also be written in its expanded form as:

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

Remember that $\binom{n}{k} = {}_nC_k = \frac{n!}{(n-k)!k!}$ and that $\binom{n}{0} = 1$ and $\binom{n}{n} = 1$.

Example 1: **Expand** $(x+2)^5$.

Let $a = x$, $b = 2$, $n = 5$ and substitute.
(Do not substitute a value for k.)

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

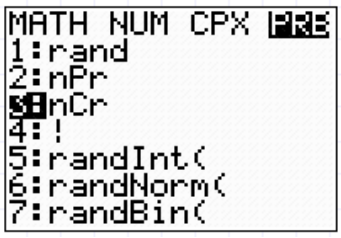
$$(x+2)^5 = \sum_{k=0}^{5} \binom{5}{k} x^{5-k} 2^k$$

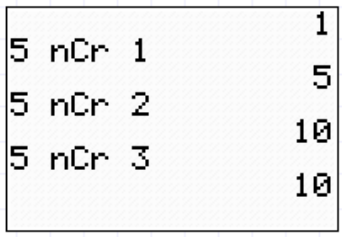
$$(x+2)^5 = \binom{5}{0} x^5 2^0 + \binom{5}{1} x^{5-1} 2^1 + \binom{5}{2} x^{5-2} 2^2 + \binom{5}{3} x^{5-3} 2^3 + \binom{5}{4} x^{5-4} 2^4 + \binom{5}{5} x^{5-5} 2^5$$

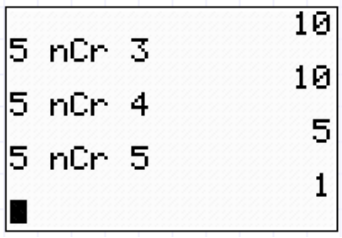
Now, grab your graphing calculator to find those combination values.

Method 1: Use the graphing calculator to evaluate the combinations on the **home screen.**

Remember: Enter the top value of the combination FIRST. Then hit **MATH** key, arrow right (or left) to **PRB** heading, and choose **#3 nCr.** Now, enter the bottom value of the combination.

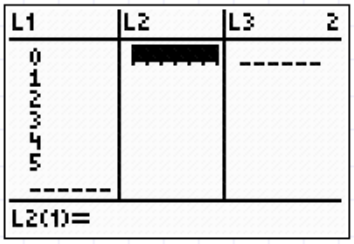




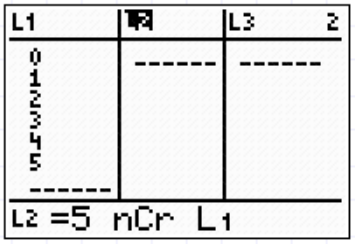


$$(x+2)^5 = x^5 + 5x^4 \cdot 2 + 10x^3 \cdot 2^2 + 10x^2 \cdot 2^3 + 5x^1 \cdot 2^4 + 2^5$$
$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

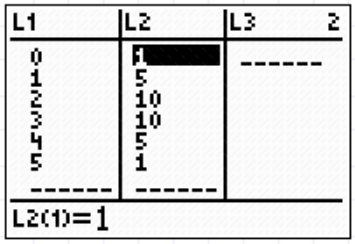
Method 2: Use the graphing calculator to evaluate the combinations **under the lists.**



In L1, enter the values 0 through the power to which the binomial is raised, in this case 5.



In L2, enter the combination formula, using the power of the binomial as the starting value, and the entries from L1 as the ending values.



The coefficients from the combinations will appear in L2.

$$(x+2)^5 = x^5 + 5x^4 \cdot 2 + 10x^3 \cdot 2^2 + 10x^2 \cdot 2^3 + 5x^1 \cdot 2^4 + 2^5$$
$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

Finding a Particular Term in a Binomial Expansion

The r^{th} term of the expansion of $(a+b)^n$ is:

$$\binom{n}{r-1} a^{n-(r-1)} b^{r-1}$$

Example 2: **Find the 5^{th} term of** $(3x-4)^{12}$.

Let $r = 5$, $a = (3x)$, $b = (-4)$, $n = 12$ and substitute.

$$\binom{n}{r-1} a^{n-(r-1)} b^{r-1}$$

$$\binom{12}{5-1} (3x)^{12-(5-1)} (-4)^{5-1}$$

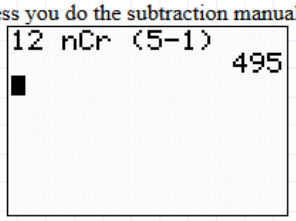
$$= \binom{12}{4} (3x)^8 (-4)^4$$

$$= 495 \cdot 6561x^8 \cdot 256$$

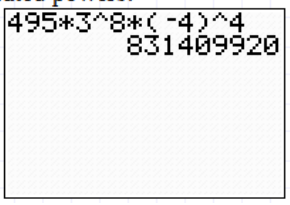
$$= 831409920x^8$$

Grab your calculator.

Be sure to include those parentheses!
(unless you do the subtraction manually)



Be sure to raise each entire parentheses to the indicated powers!

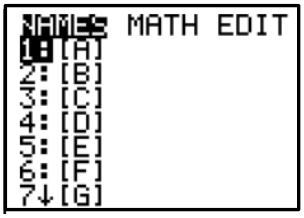


Working with Matrices

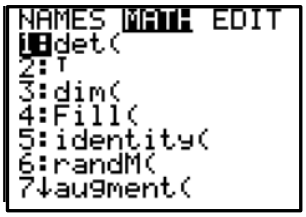
Matrices are rectangular arrays of elements.
The ***dimension*** of a matrix is the number of rows by the number of columns.

Inputting/Editing Matrices:

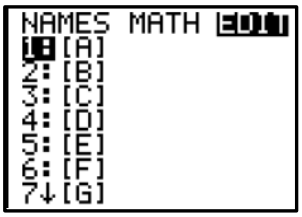
Before we can work with matrices, we must first input them into the calculator. To enter the Matrix Editor, press [2nd] → [MATRX] (above the [x^{-1}] key). The TI-83/84 calculators contain predefined matrix variables labeled [A] through [J].



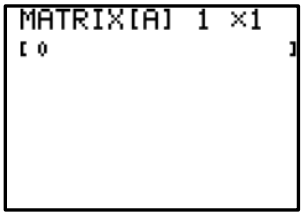
The "NAMES" column in the Matrix Editor is where we will choose a matrix after it has been created. The "EDIT" column looks just like the "NAMES" column, except this is where we input or edit a matrix. The "MATH" column contains special commands that only work with matrices, some of which we will discuss later.



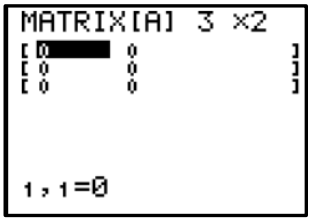
Inputting matrices into the TI-83/84 is easy. Press the right arrow key twice over to the "EDIT" column. Use the [▲] and [▼] arrow keys to select a matrix name. Let's use [A] since it is already selected, so press [ENTER].



Since we are creating a new matrix, the calculator displays a default 1x1 matrix. Let's say we want a 3x2 matrix instead. Type: [3] → [ENTER] → [2] → [ENTER].



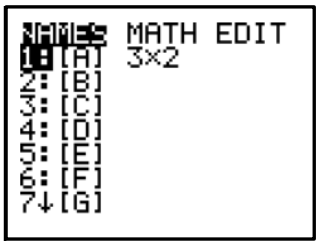
You should now have this screen. As you can see, we have a 3x2 matrix filled with zeros. To input your own matrix, type in each element of the matrix row-by-row. That is, to enter the matrix $\begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 8 & 7 \end{bmatrix}$, type [4] → [ENTER] → [2] → [ENTER] → [1] → [ENTER] → [3] → [ENTER] → [8] → [ENTER] → [7] → [ENTER].



And this is what you should have. Before we can use this matrix, we need to first exit the Matrix Editor. To do this, do <u>not</u> press [CLEAR]! Pressing [CLEAR] will remove the selected number. Instead, press [2nd] → [QUIT] (above [MODE]) to return to the Home screen.



Return to the Matrix Editor ([2nd] → [MATRX]). Notice that [A] now has the dimension of our matrix beside it. This is how we know that a matrix has already been entered into the calculator. To edit this matrix, right-arrow-key over to "EDIT", select the matrix name ([A] in this case) and press [ENTER]. The stored matrix will be displayed for editing. Remember to always press [2nd] → [QUIT] to exit the Editor!

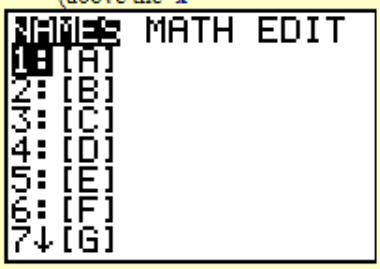


Adding Matrices - matrices must be of the ***same dimension*** to be added.

Add: $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} + \begin{bmatrix} 2 & 2 & 4 \\ 3 & 3 & 2 \end{bmatrix}$

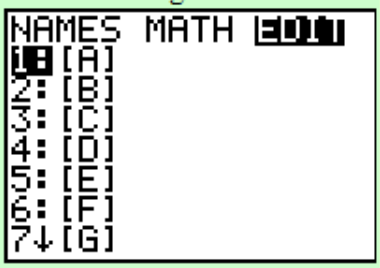
First Enter the Matrices (one at a time):

Step 1: Go to **Matrix** (above the x^{-1} key)

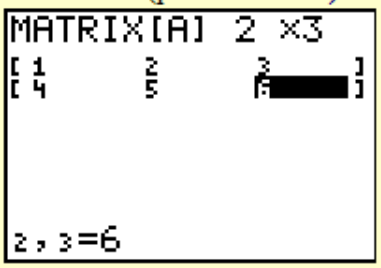


If dimensions appear next to the names of the matrices, such as 3x3, a matrix is already stored in the calculator. You may save it by moving to a new name, or overwrite it.

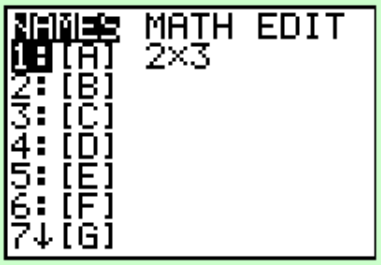
Step 2: Arrow to the right to **EDIT** to allow for entering the matrix.



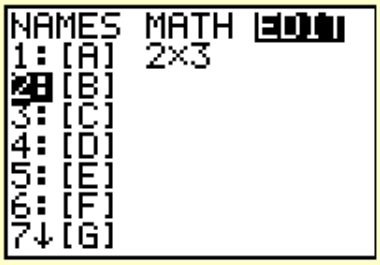
Step 3: Type in the dimensions (size) of your matrix and enter the elements (press **ENTER**).



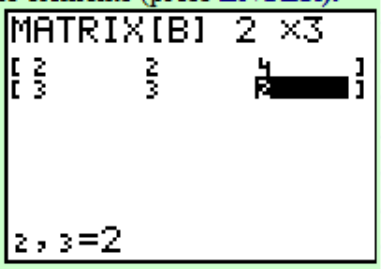
Step 4: Repeat this process for the second matrix



Step 5: Arrow to the right to **EDIT** and choose a new name.

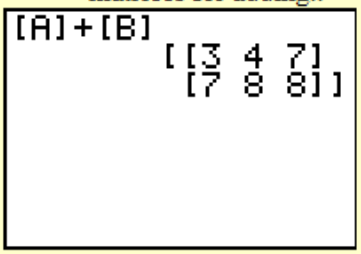


Step 6: Type in the dimensions (size) of your matrix and enter the elements (press **ENTER**).



Now, add:

Step 7: Return to the home screen. Go to **Matrix** to get the names of the matrices for adding..



The answer to the addition, as seen on the calculator screen, is =

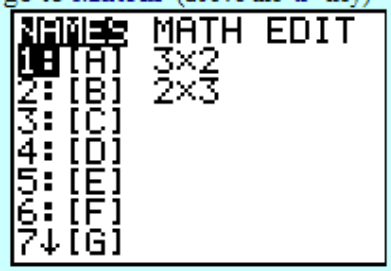
$\begin{bmatrix} 3 & 4 & 7 \\ 7 & 8 & 8 \end{bmatrix}$

Multiplying Matrices - for multiplication to occur, the ***dimensions*** of the matrices must be related in the following manner: ***m x n*** **times** ***n x r*** **yields** ***m x r***

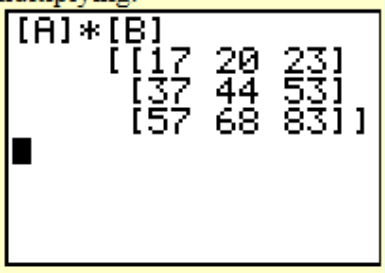
Multiply: $\begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix} \bullet \begin{bmatrix} 3 & 4 & 7 \\ 7 & 8 & 8 \end{bmatrix}$

First Enter the Matrices (one at a time) as shown above:

Step 1: Once the matrices are entered, you should see their dimensions in residence when you go to **Matrix** (above the x^{-1} key)



Step 2: Return to the home screen. Go to **Matrix** to get the names of the matrices for multiplying.



The product, as seen on the calculator screen, is =

$\begin{bmatrix} 17 & 20 & 23 \\ 37 & 44 & 53 \\ 57 & 68 & 83 \end{bmatrix}$

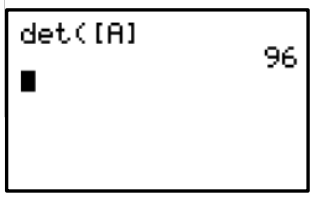
Determinant of the Matrix

Find the determinant of the matrix $[A] = \begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$

Enter this matrix as matrix [A] in the calculator: $\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$

To calculate the determinant of [A], enter the Matrix menu and ▸ arrow key over to "MATH". Select the first command, "det(", and press ENTER to paste it to the Home screen. Select matrix [A] from "NAMES", press ENTER to paste it to the screen, and ENTER again to run the command. That's it! The determinant of this matrix is 96.

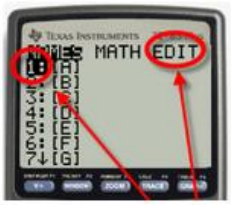




Inverse of the Matrix

Find the inverse of the matrix $[A] = \begin{bmatrix} 2 & 3 & -4 \\ 1 & -2 & 0 \\ 3 & 0 & 1 \end{bmatrix}$

Step 1. Open the **MATRIX** menu by press **2ND** and X^{-1} keys. Use the arrow key to highlight **EDIT** and then **1:[*A*]**. Press **ENTER**.



Step 2. Indicate the size of the matrix. Enter the number of rows and columns, and press **ENTER**.



Step 3. Add elements of the matrix followed by pressing **ENTER** after each element. When you will finish, press **ENTER** also.



Step 4. Enter $[A]^{-1}$ on the calculator. Find $[A]$ by going to the **MATRIX** menu (as Step 1) and pressing **ENTER** with **NAMES** and **A** highlighted. Now hit X^{-1} key and again press **ENTER**.



Step 5. Success but with a problem. This example has the hard reading answer.



Step 6. Fix the answer. Press **MATH** key and highlight **1:▷Frac** and press **ENTER**.



Step 7. Receive the final answer. Press **ENTER**. You can transform your matrix by the following way

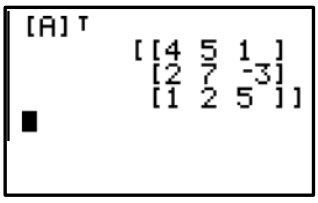
$$\begin{bmatrix} \frac{2}{31} & \frac{3}{31} & \frac{8}{31} \\ \frac{1}{31} & -\frac{14}{31} & \frac{4}{31} \\ -\frac{6}{31} & -\frac{9}{31} & \frac{7}{31} \end{bmatrix} = \frac{1}{31}\begin{bmatrix} 2 & 3 & 8 \\ 1 & -14 & 4 \\ -6 & -9 & 7 \end{bmatrix}$$



Transpose Matrix

Let's calculate the transpose of $\begin{bmatrix} 4 & 2 & 1 \\ 5 & 7 & 2 \\ 1 & -3 & 5 \end{bmatrix}$. If it is not there already, input this matrix as [A] in the calculator. After pasting [A] to the Home screen from the "NAMES" column, go back to the "MATH" column in the Matrix menu and select the second command. Press ENTER to paste the command to the screen, and then ENTER again to run the command.





Solving System of Equations by Different Matrix Methods

1. Using Inverse of a Matrix

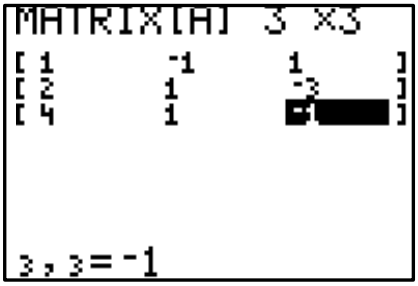
For the operation by this method use the formula: $X = A^{-1} \cdot B$

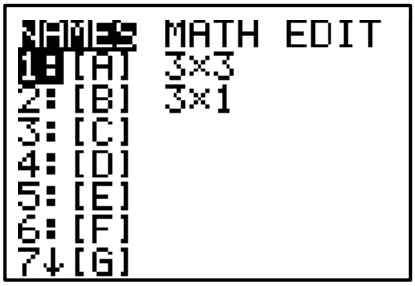
Solve the system of equations $\begin{cases} x - y + z = 3 \\ 2x + y - 3z = 5 \\ 4x + y - z = 11 \end{cases}$

Let the matrix $[A] = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 4 & 1 & -1 \end{bmatrix}$ and the matrix $[B] = \begin{bmatrix} 3 \\ 5 \\ 11 \end{bmatrix}$.

Step 1. Enter all equations.

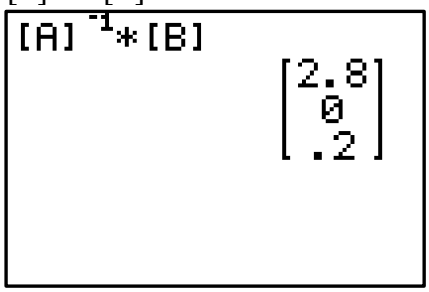






Step 2. Use the formula.

Choose the matrix $[A]$ and find it inverse $[A]^{-1}$. Then provide the multiplication of two matrices $[A]^{-1} \cdot [B]$.



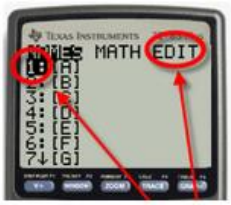
2. Row Reduction Echelon Form

Solve the system of equations $\begin{cases} 2x + 3y - 4z = 4 \\ x - 2y + 3 = 0 \\ 3x + z = 6 \end{cases}$

Rewrite given equation and create the matrix

$$\begin{cases} 2x + 3y - 4z = 4 \\ x - 2y = -3 \\ 3x + z = 6 \end{cases} \Rightarrow \begin{bmatrix} 2 & 3 & -4 & 4 \\ 1 & -2 & 0 & -3 \\ 3 & 0 & 1 & 6 \end{bmatrix}.$$

Step 1. Open the **MATRIX** menu by press **2ND** and X^{-1} keys. Use the arrow key to highlight **EDIT** and then **1:[*A*]**. Press **ENTER**.



Step 2. Indicate the size of the matrix. Enter the number of rows and columns, and press **ENTER**.



Step 3. Add elements of the matrix followed by pressing **ENTER** after each element. When you will finish, press **ENTER** also.



Step 4. Reduce the row at the entered matrix. Exit all menus by pressing **2ND** and **QUIT**. Press **2ND** and X^{-1}, go to the **MATH** menu and move cursor to **B↓rref(** . Press **ENTER**.



Step 5. Finish to enter rref ([*A*]) on the calculator. Find [*A*] by going to the **MATRIX** menu (as step 1) and highlight **NAMES** and ***A***. Close the parentheses and press **ENTER**.





Step 6. Success, but with a problem. This example is hard to read.



Step 7. Fix your answer. Going to the **MATRIX** menu and highlight **1:▷Frac** and hit **ENTER**.



Step 8. Receive the final answer. Press **ENTER**.



Partial Fractions Decomposition

The factory-equipped calculator TI-84 do not have the special menu for the partial fractions operation. Moreover, decomposition of the partial fraction is possible by

the transformation of the partial fraction equations to the matrix and additional solution as the matrix.

Remember: The answer of the given problem will be at the matrix form also. Really, it will be numerators of your partial fractions and all denominators you should receive by the manual calculation.

1. Solution through the matrix methods

Example:

Write the partial fraction decomposition: $\frac{1}{(x+5)^2(x-1)}$

The beginning of the procedure is the same as during the manual calculation of the partial fraction decomposition.

$$\frac{1}{(x+5)^2(x-1)} = \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{c-1}$$

$$1 = A(x+5)(x-1) + B(x-1) + C(x+5)^2$$

$$1 = Ax^2 + 5Ax - Ax - 5A + Bx - B + Cx^2 + 10Cx + 25C$$

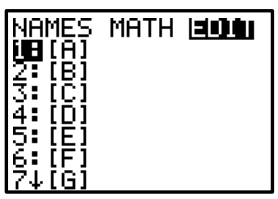
$$1 = x^2(A+C) + x(4A + B + 10C) + (-5A - B + 25C)$$

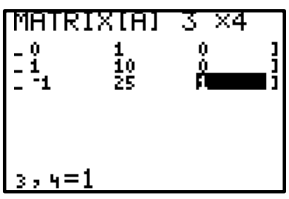
$$\begin{cases} 1A + 0B + 1C = 0 \\ 4A + 1B + 10C = 0 \\ -5A - 1B + 25C = 1 \end{cases}$$

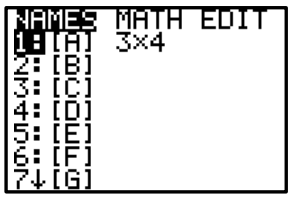
Transform this system of equation to the matrix

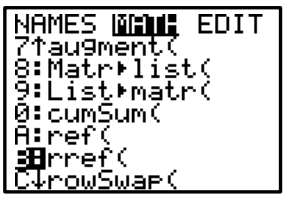
$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 4 & 1 & 10 & 0 \\ -5 & -1 & 25 & 1 \end{bmatrix}$$

Solve this matrix by using the graphing calculator by inverse the matrix or row reducing echelon form methods (in the book use the row reduced echelon method).



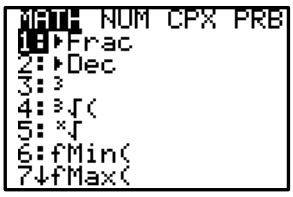


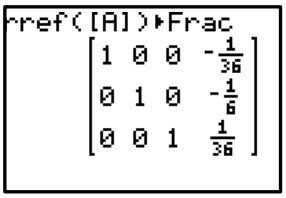












The variables $\begin{cases} A = -\frac{1}{36} \\ B = -\frac{1}{6} \\ C = \frac{1}{36} \end{cases}$

The partial fraction looks like:

$$\frac{1}{(x+5)^2(x-1)} = \frac{-\frac{1}{36}}{x+5} + \frac{-\frac{1}{6}}{(x+5)^2} + \frac{\frac{1}{36}}{x-1}$$

2. Solution through the downloadable software

Attention: Do not try to download any software from the internet to the calculator by yourself! You can destroy your calculator.

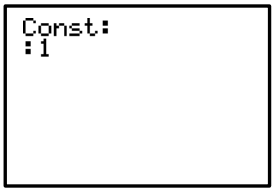
Example:

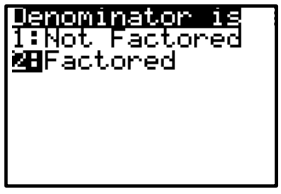
Write the partial fraction decomposition: $\frac{1}{(x+5)^2(x-1)}$

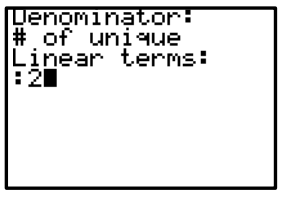
This procedure is very logical, quick and simple. After the downloading required software as you pressing APPS key, you receiving additional row: **9: PartFrac**. Choice it and hit **ENTER**. The calculator will ask you a lot of question about the given fraction. After the answer for each question, hit **ENTER**.

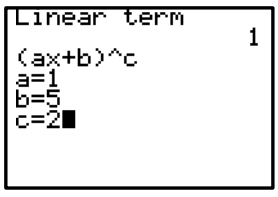


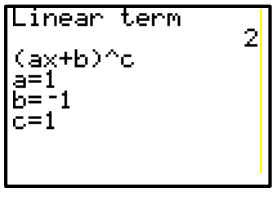


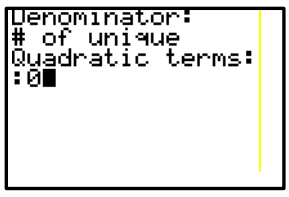


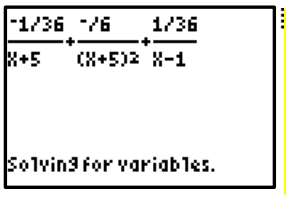












Because this program is not the factory-created, this is the order of the error codes happened through the possible mistakes during the partial fraction decomposition:

Error codes:

The size of the program is already pretty big, thus I spared the strings required to describe the errors and gave every error a code. Here are the meanings:

Code 1: The numerator is zero. The numerator of the fraction cannot be zero, because the whole fraction would be zero.
Code 2: You did not define enough factors. It is required to have at least 1 repeated or two unrepeated factors.
Code 3: The last factor is neither linear nor quadratic. All factors must be linear or quadratic. Review the section on repeated factors.
Code 4: "Bad Factors". This happens if the program could not solve the system of equations. A factors was most probably entered more than once. Review repeated factors.
Code 5: The last factor is a constant. A factor cannot be a constant. Multiply one of the other factors with this constant and enter it as one factor.
Code 6: You tried to repeat a factor more than three times. The program does not support such factors.

Limitations of the program:

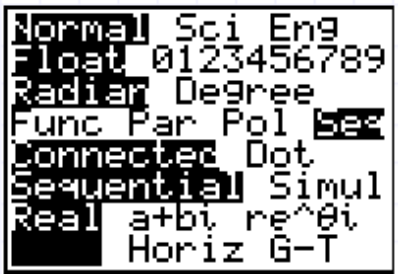
The problems are:

- Improper fraction will be evaluated incorrectly. An answer is shown, but it is most probably incorrect. Thus make sure that the order of "X" is higher in the enumerator.
- If a factor is repeated the calculations will say: "BAD FACTORS... ERROR! CODE: 4" this means, that the program could not split the fraction. Check, whether you factored all quadratic polynomials.
- Long calculation times. The times range from 10 seconds for two linear factors to minutes for many factors. Normal text-book fractions should however be solved fairly quickly (faster than doing all the calculations by hand).
- The precision with many factors is not perfect. Again text-book problems should be OK.

Working with Sequences

(in both **Seq** and **Func** Modes)

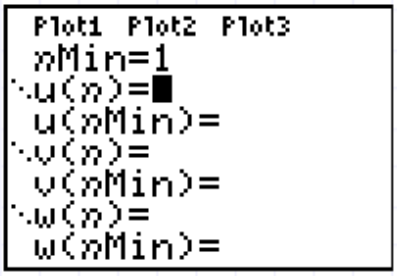
In Seq MODE:



To work with sequences in the **Seq** mode:

- Press MODE key
- Choose Seq in the fourth line
- Hit ENTER to highlight Seq
- Leave other settings as default (on the left side)

Press Y= and notice the difference in this screen.



To define a sequence, you must specify:

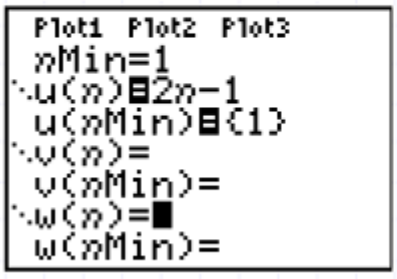
- *n*Min - where you will start counting the numbers in the sequence (most likely this value is 1)
- u(*n*) - the pattern for the sequence (use X,T,θ,*n* key to enter the pattern)
- u(*n*Min) - the first number in the sequence

(When you enter the first number in your sequence, notice the list braces that appear. These braces will allow for the entry of more than one term if needed.)

Example 1: Consider the sequence defined as $a_n = 2n - 1$.

List the first 6 terms of the sequence and find the 105th term.

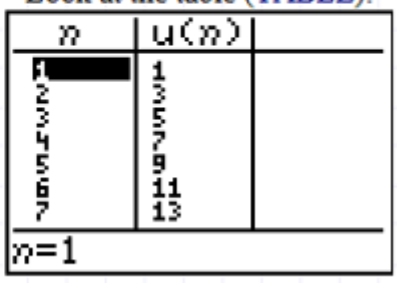
Enter the information in Y=:



Set the table (TBLSET):



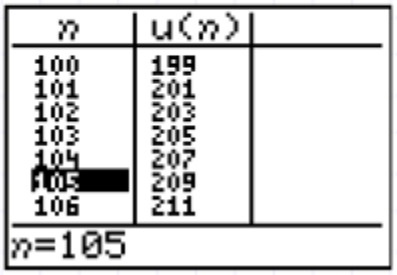
Look at the table (TABLE):



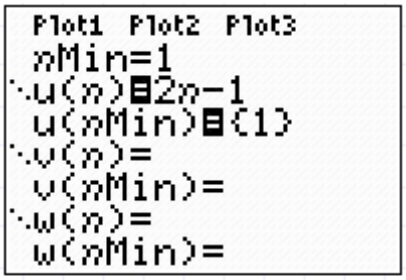
The first 6 terms are easily visible in the table.

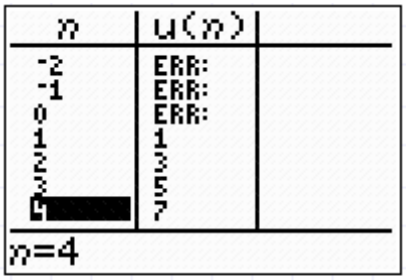
To find the 105th term, reset the TblStart to a value near (or at) the desired term. Read the answer from the table.

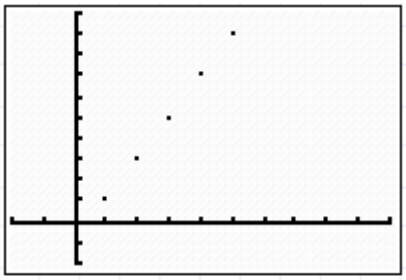




If you **GRAPH** when in **seq**uence mode, even if your calculator is set in **connected mode**, you will only see plots of points. Remember that a sequence is a listing of numbers without other values in between. Notice, in the center calculator screen below, how the **seq**uence mode table shows answers only for *n* values of 1 or larger. Remember that the domain of a sequence is the set of natural numbers {1, 2, 3, ...}.







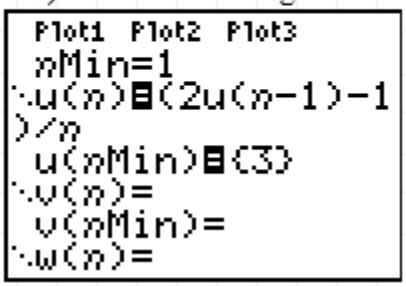
If you express this sequence as a function and return to **func**tion mode, you will see a connected graph (a line in this case), rather than the plotted points.

Example 2: (**Recursive sequence**) List the first 7 terms of the sequence:

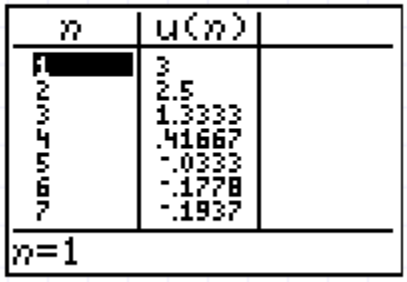
$$a_1 = 3$$

$$a_n = \frac{2a_{n-1} - 1}{n}$$

Enter the recursive formula information. Enter the **u** by using the alpha letter **u** (2nd 7). Note the starting value of 3.

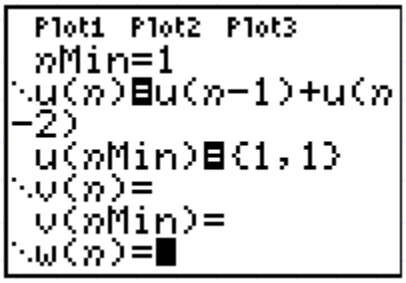


View the terms of the sequence in the table.

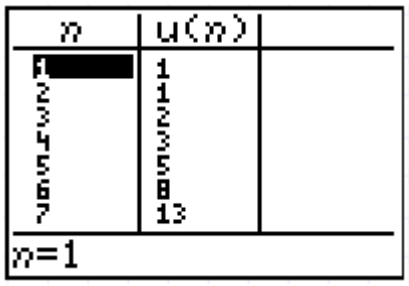


Example 3: Generate the Fibonacci Sequence (where each term, after the first two terms, is determined by adding the two previous terms).

Enter the pattern for the sequence. Notice how the u(*n*Min) displays the first two terms using the list braces.

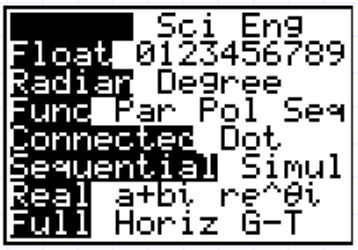


Go to the table to view the Fibonacci sequence



In Func MODE:

Certain sequences can be examined in **Func** mode. Consider the following examples:

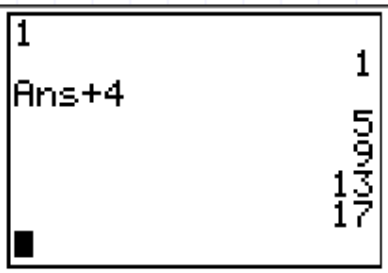


Example 4: Arithmetic and geometric sequences can be quickly generated in Func mode if the common difference or common ratio, respectively, are known, along with the starting value.

List the first five terms of an arithmetic sequence whose first term is 1 and whose common difference is 4.

A "quick - see it on the screen" method:

- Enter the first term on the home screen.
- Press ENTER to register the first term.
- Press + 4 (the expression Ans+4 will appear).
- Press ENTER.
- Continuing to press ENTER will generate successive terms in this sequence.

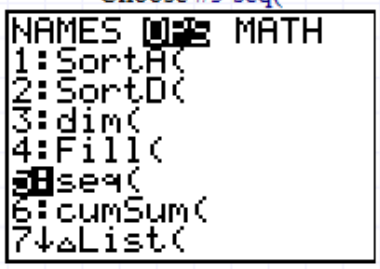


Unfortunately, finding specific terms with this method, such as the 100th, will require that you press ENTER and count very carefully - not always an easy process.

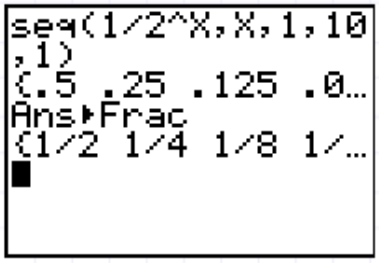
Example 5: If an expression/formula is known for the generation of a sequence, the terms can be listed in Func mode using the seq(function.

List the first 6 terms of the sequence defined by: $a_n = \frac{1}{2^n}$

2nd STAT (LIST) → OPS
Choose #5 seq(



Using seq(, type the formula, variable, starting value for n, last value of n needed, increment. (For ease, X may be used as the variable -- or type N using the alpha key.) Arrow to the right to see more of the terms of the sequence.
If you need fractional answers, utilize the MATH - #1▶Frac

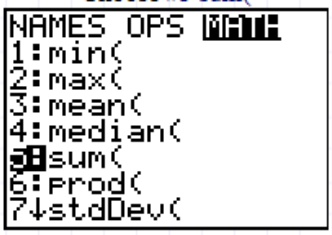


This method of listing terms of a sequence will be limited to 999 terms of the sequence at any one time.

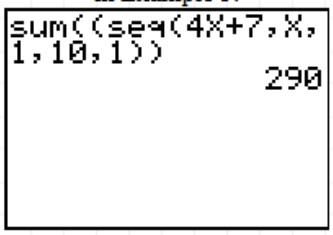
Example 6: If an expression/formula is known for the generation of a sequence, the sum of a specific number of terms can be found in Func mode.

Find the sum of the first 10 terms of the sequence defined by: $a_n = 4n + 7$

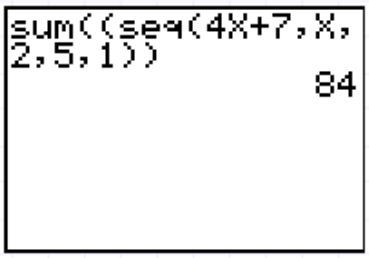
2nd STAT (LIST) → MATH
Choose #5 sum(



Notice that sum is followed by the entry of the sequence as shown in Example 5.



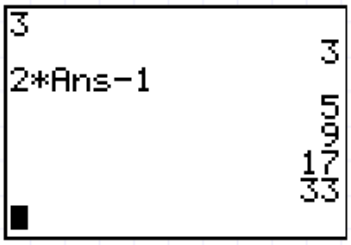
Find the sum of the second through fifth terms of the sequence: $a_n = 4n + 7$



Example 7: Sequences represented by a recursive formula can be generated in Func mode.

List the first 4 terms of the sequence defined by: $a_1 = 3$, $a_{n+1} = 2a_n - 1$

- Type the starting value and press ENTER.
- Use the ANS function to create the formula used to generate the sequence. Type 2*ANS-5 and press ENTER.
- Now, simply press ENTER to create successive terms of the sequence.



Evaluating Summation Notation

(**Func**tion Mode)

General summations:

A shorthand notation is used to represent sums with more than a few terms. This shorthand notation uses the Greek letter sigma (which denotes sum). If we have a rule such as $f(n)$ which assigns the value $f(n)$ to each integer n in the set of integers $\{j, j+1, j+2, \ldots k\}$, we can represent the sum as:

$$\sum_{n=j}^{k} f(n) = f(j) + f(j+1) + f(j+2) + \ldots + f(k)$$

On the graphing calculator:

We will be using the **sum** feature and the **seq** feature to work with a general summation. On the calculator, we will interpret a summation as follows:

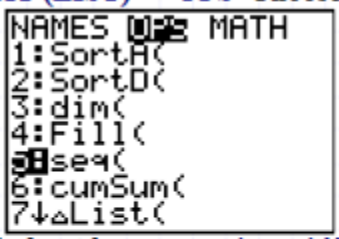
$$\sum_{n=j}^{k} f(n) = sum(seq(f(n), n, 1, k, 1))$$

2nd STAT (LIST) → MATH #5 sum



The format for **sum** is **sum(** *list* **)**
where *list* will be the terms of the rule
$f(j), f(j+1), f(j+2), \ldots, f(k)$

2nd STAT (LIST) → OPS Choose #5 seq(

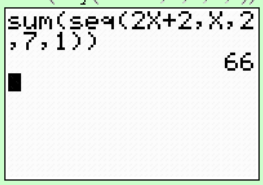


The format for **seq:** *expression, variable, starting value, ending value, increment.*

While sequences have a domain of natural numbers, {1, 2, 3, 4, ...}, in **Func** mode, the **sequence** command will accept integer domain values less than one. (In **Seq** mode, however, the **sequence** command will accept only integer domain values of 1 or greater.)

Example 1: Evaluate: $\sum_{k=2}^{7} 2k+2$

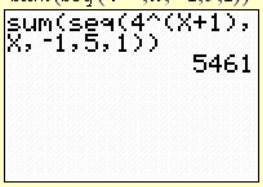
$sum(seq(2x+2, x, 2, 7, 1))$



notice that this sum starts with $k = 2$

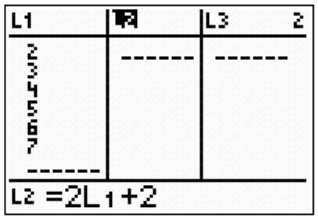
Example 2: Evaluate: $\sum_{r=-1}^{5} 4^{r+1}$

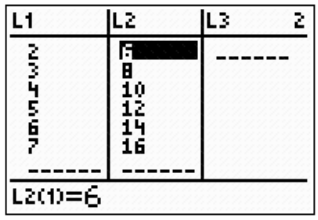
$sum(seq(4^{x+1}, x, -1, 5, 1))$



(For ease of entry, **X** was used as the variable in both examples above.
If you wish to use the designated variable, type it into the calculator using the alpha key.)

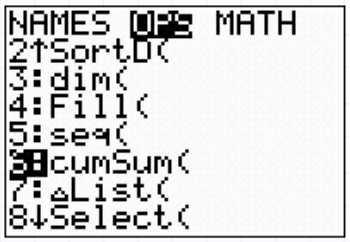
Another approach to ***Example 1****:*
Type the number of terms needed into **L1**.
Type the rule into **L2**.
Add the answers manually on the **HOME** screen.



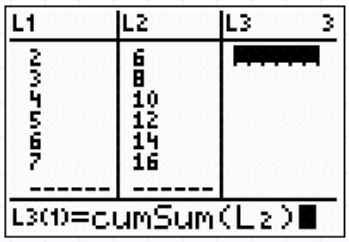


Or, have the calculator do the adding. If you do not want to add manually on the home screen, you can use the **cumulative summation feature (cumSum)** to add **L2**.

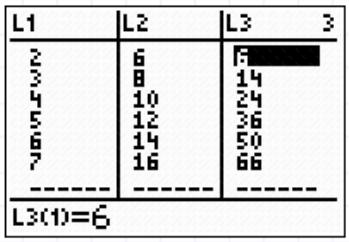
Remember that Example 1 starts with $k = 2$.



To find **cumSum(** , go to **LIST** (**2nd STAT**), arrow to **OPS** heading, and choose **#6 cumSum(.**



Enter **cumSum(L2)** as the pattern to determine **L3.**



The entry in column **L3** corresponding to the value 7 in **L1** will be the sum.

Summations and Sequences:

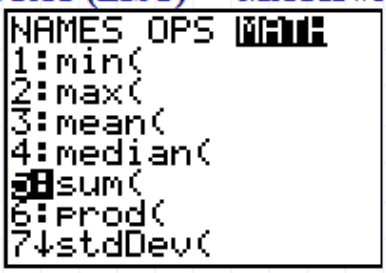
Sums, such as $\sum_{j=1}^{n} a_j$, where $j = 1$, may represent the summation of a sequence, called a series.

Remember, sequences have a domain of natural numbers, {1, 2, 3, 4, ...}.
In the sequence related problems below, the starting value in the summation will be greater than or equal to 1.

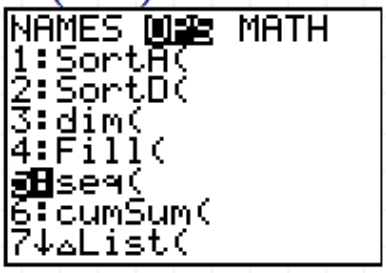
Again, we will be using the **sum** and **seq** features of the calculator to find the summations. The summation is interpreted as:

$$\sum_{j=1}^{n} a_j = \text{sum}(\text{seq}(a_j, j, 1, n, 1))$$

2nd STAT (LIST) → MATH #5 sum

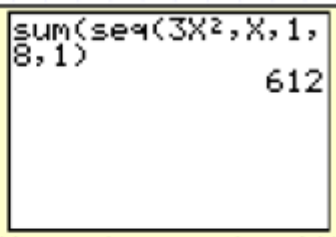


2nd STAT (LIST) → OPS Choose #5 seq(



Example 3:

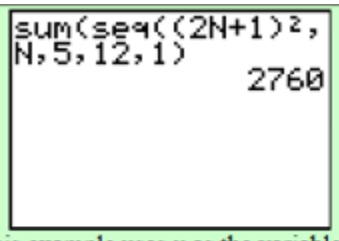
Evaluate: $\sum_{j=1}^{8} 3j^2$



This example uses x as the variable, instead of using j. The x is simply easier to enter into the calculator.

Example 4:

Evaluate: $\sum_{n=5}^{12}(2n+1)^2$



This example uses *n* as the variable -- obtained by engaging the alpha key.

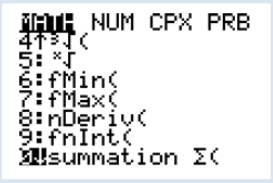
Example 5:

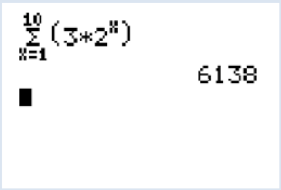
Find the sum of geometric sequence

$\sum_{x=1}^{10} 3\cdot 2^x$

Step1. Working in **MATH** menu. Choose **0↓summation ∑)** and hit **ENTER**.

Step 2. Type all parameters of the summation by the calculator and press **ENTER**.





Infinite Series

An **infinite series** is an expression such as: $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ where the dots imply that an infinite number of terms will be added.

Partial Sums:

$S_1 = 1$

$S_2 = 1+\frac{1}{2}=\frac{3}{2}$

$S_3 = 1+\frac{1}{2}+\frac{1}{4}=\frac{7}{4}$

and so on.

To find the sum of an infinite series, we exam the *partial sums*. The sum of the series will be the limit of the partial sums (that is, the number that the partial sums are approaching, if, in fact, they do approach a number).

If the limit exists, the series is said to *converge*.
If the limit does not exist the series is said to *diverge*.

The sum of an infinite series is the limit of the partial sums.

$$\sum_{n=1}^{\infty} a_n = \lim_{N\to\infty} S_n = \lim_{N\to\infty} \sum_{n=1}^{N} a_n$$

On the graphing calculator:

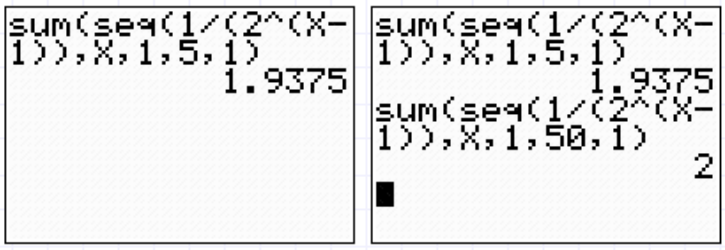
Method 1:

An infinite series can be examined as you would examine a finite series. Utilize the **sum** feature and the **seq** features. This time, however, we will "choose" an ending value for the summation. We are looking for an idea of where the sum is heading. Let's examine our first series:

$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ where the pattern is $\frac{1}{2^{n-1}}$

- Enter the series as we did finite series.
- Examine 5 partial sums.
- Recall the entry and examine 50 partial sums.
- The limit approaches 2.

Choose whichever numbers you wish.

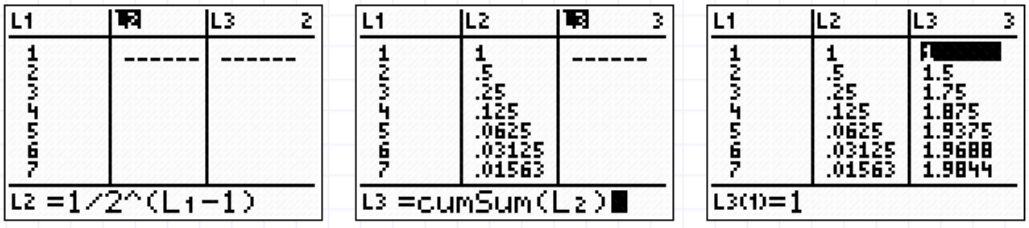


Method 2:

An infinite series can also be examined using lists. Let's examine our first series again:

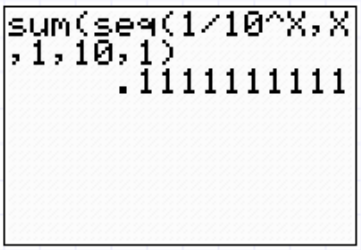
$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$ where the pattern is $\frac{1}{2^{n-1}}$

Enter the number of terms you wish to examine in **L1.** Enter the formula into **L2**. Enter the cumulative sum into **L3. cumSum** is found **2nd LIST - OPS - #6 cumSum**

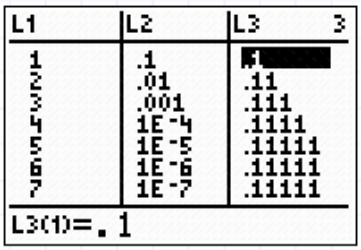


Now, examine what is happening to the partial sums in **L3.**
Increase the number of entries if needed for further examination.

Example: Evaluate: $\sum_{k=1}^{\infty} \frac{1}{10^k}$



Method 1: Using **sum** and **seq** with 10 partial sums.



Method 2: Using the list with 10 partial sums

Examining the partial sums shows that the limit is approaching .1111111111 which is 1/9.

Conics

Any equations of conics

Start with the general quadratic equation:

$$ax^2 + bxy + cy^2 + dx + ey + f = 0$$

Regroup on the y values:

$$cy^2 + (bx + e)y + (ax^2 + dx + f) = 0$$

Solve for y in terms of x using the quadratic formula:

$$y = \frac{-(bx + e) \pm \sqrt{(bx + e)^2 - 4 * c * (ax^2 + dx + f)}}{2c}$$

Enter this equation into Y1.

Go to the Home Screen and store numbers into A, B, C, D, E, and F.

This equation will allow you to graph any conic in any position.

Circle

A. center at the origin:	**B.** center at (h, k):
$x^2 + y^2 = r^2$	$(x-h)^2 + (y-k)^2 = r^2$

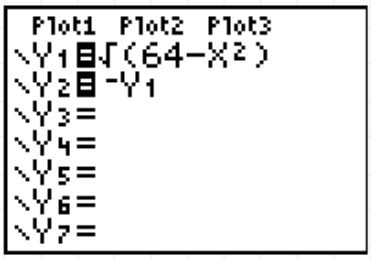


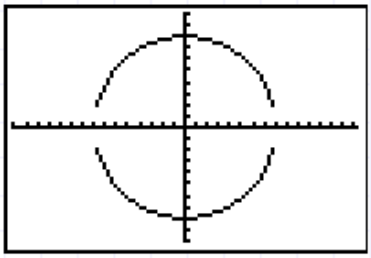
Example A: Graph: $x^2 + y^2 = 64$

Solution:

- Enter the equation into Y= by solving for y.

  $y = \pm\sqrt{64 - x^2}$
- Enter the positive square root into Y1.
- Enter the negative square root into Y2, or enter the negation of Y1.
- If you choose ZOOM #6 (the standard window), the graph will appear to be an ellipse rather than a circle due to the 3/2 aspect ratio of the viewing screen (the standard viewing screen is not a square).
- Choose ZOOM#5 ZSquare to create a viewing window where the units on both axes are the same length.





NOTE:

* You may notice that the "vertical" edges of the graph may not appear in the viewing window. The viewing screen cannot graph points that fall between pixels.

* You may notice that the TRACE function will not move automatically between the two sections of the graph because the positive and negative square roots were graphed as two separate equations. The up arrow can be used to move between the two sections of the graph. Also, the cursor will disappear if it is moved beyond the domain for which x is defined.

Example B: Graph: $(x-2)^2+(y-1)^2=1$

Solution:

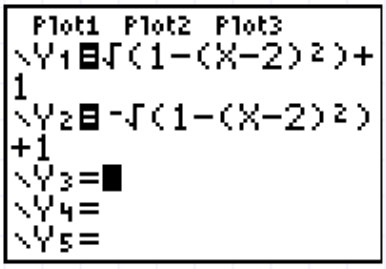
- Enter the equation into Y= by solving for y.

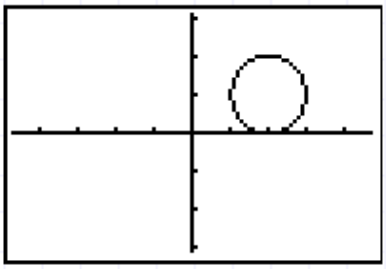
$$(y-1)^2=1-(x-2)^2$$

$$y-1=\pm\sqrt{1-(x-2)^2}$$

$$y=\pm\sqrt{1-(x-2)^2}+1$$

- Enter the equation with the positive square root into Y1.
- Enter the equation with the negative square root into Y2. You cannot simply negate Y1 to obtain Y2 in this problem.
- Choose ZOOM#5 ZSquare to create a viewing window where the units on both axes are the same length.

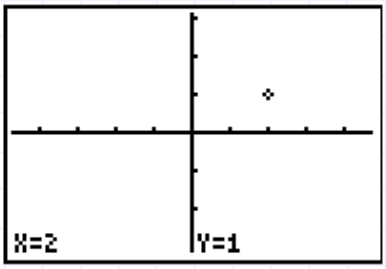


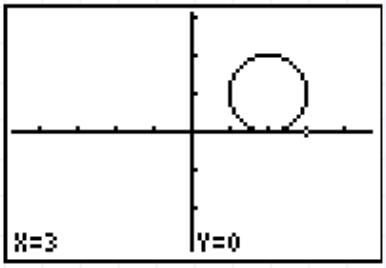


Solution Using DRAW:

(please follow steps in order)

- Press ZOOM#4 (ZDecimal) to go to a graphing screen.
- To graph the circle, press 2nd PRGM (DRAW) #9 Circle.
- Move the cursor to the "*h*" value of +2 by using the arrows.
- Move the cursor to the "*k*" value of +1 by using the arrows.
- Press ENTER to set the point for the center of the circle.
- Move the cursor the length of the radius (1) away from the center. Keep track of the values at the bottom of the window.
- When you hit ENTER, the circle will be automatically drawn.
- You may need a larger window if your radius is large. (Zoom Out)





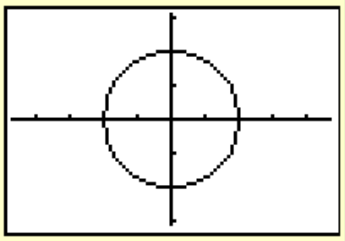
Tidbit:

You can use a "list" approach to deal with the plus and minus square roots:

Graph: $x^2+y^2=4$ which becomes $y=\pm\sqrt{4-x^2}$

Enter: $Y1=\{1,-1\}*\sqrt{4-x^2}$

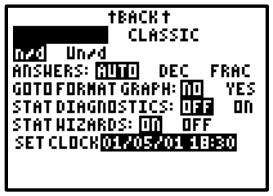
Tracing still requires arrowing up and down between entries.



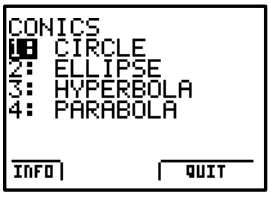
Another way for graphing the cirle

Select **APP** by pressing the **APPS** key and choice 4:Conics. If **APP** does not behave this script, press **MODE** and ensure the window setting is **AUTO**. Then press **2ND** and **QUIT**.

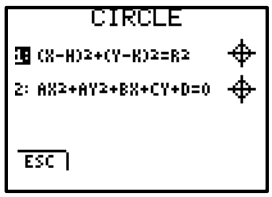




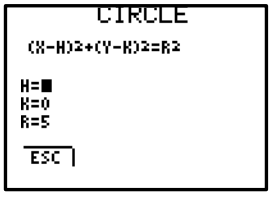
At the main menu select the **1:CIRCLE**. Then press **ENTER**.

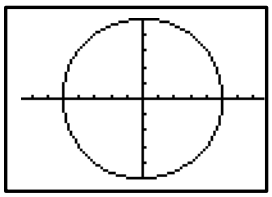


Celect the type of the equation of the circle. Choice **1** and hit **ENTER**.

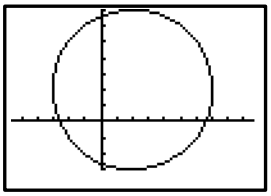


Change to $\boldsymbol{H = 0}$, $\boldsymbol{K = 0}$ and $\boldsymbol{R = 5}$. Press **ENTER**. Press **GRAPH**.

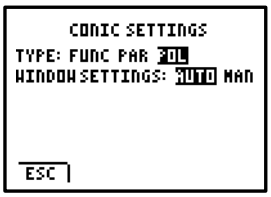


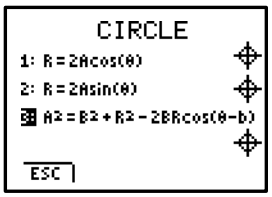


Change to $\boldsymbol{H = 2}$, $\boldsymbol{K = 2}$ and $\boldsymbol{R = 5}$. Press **ENTER**. Press **GRAPH**. The new circle with the center at (2, 2) is displayed.

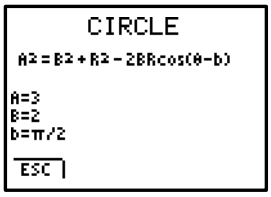


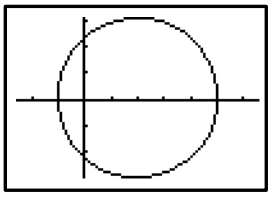
Circle in polar form. Change the **MODE** to the polar form **POL** and press **ENTER**. Continue all previous procedure. The conic settings screen became to change. Choice **3** and press **ENTER**.





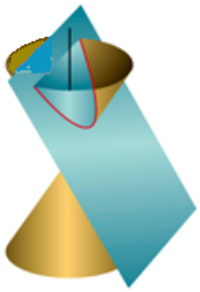
Choice constants as $\boldsymbol{A = 2}$, $\boldsymbol{B = 3}$ and $\boldsymbol{C = \frac{\pi}{2}}$ and press **ENTER**. After that the constant C changes to $\boldsymbol{C = 1.5707963267949}$ which is the value of the $\frac{\pi}{2}$. Press **GRAPH**.





Parabola

A. Center at origin:	B. Center at $(h,\ k)$:
$x^2 = 4py$	$(x-h)^2 = 4p(y-k)$
$y^2 = 4px$	$(y-k)^2 = 4p(x-h)$



Example: Graph the equation of a parabola

$$4x^2 - 24x - 40y - 4 = 0$$

Solution:

$4x^2 - 24x = 40y + 4$ *Move the constant and unpaired linear term to the right*

$4(x^2 - 6x) = 40y + 4$ *Complete the square*

$4(x^2 - 2 \cdot 3x + 9) = 4 \cdot 9 + 40y + 4$ *Add the square to the both sides*

$4(x-3)^2 = 40y + 40$

$\frac{4(x-3)^2}{40} = \frac{40y+40}{40}$ *Divide both sides by the coefficient of the unpaired linear variable*

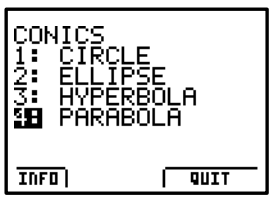
$\frac{1}{10}(x-3)^2 = y + 1$

$10(y+1) = (x-3)^2$

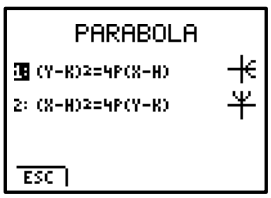
Another method for graphing a parabola

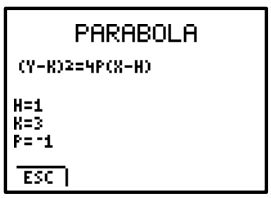
Rectangular coordinates.

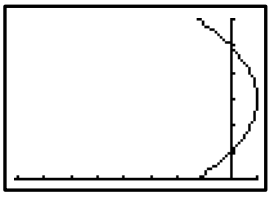
Choice the rectangular mode **FUNC** and follow all procedure by the similar way as the previous example.



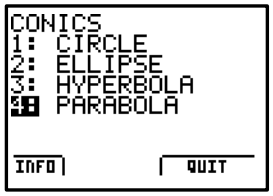




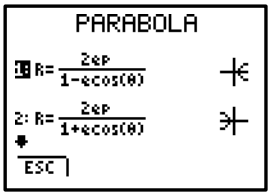




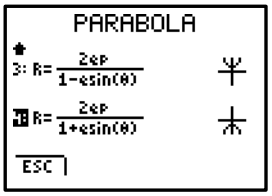
Similarly, when you start to work in polar coordinate **POL** mode.

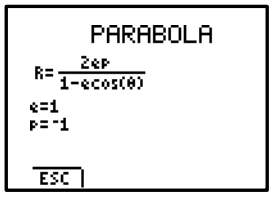


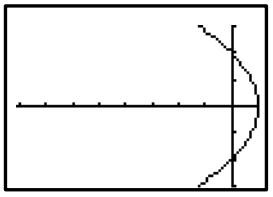




or







Ellipse

A. center at the origin:	**B.** center at (h, k):
$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$
$\frac{y^2}{a^2}+\frac{x^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}+\frac{(x-h)^2}{b^2}=1$



Example: Graph: $\frac{(y+3)^2}{9}+\frac{(x-2)^2}{4}=1$

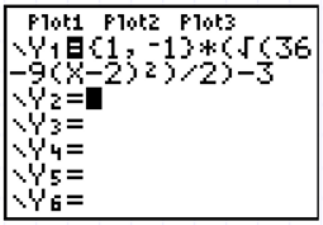
Solution:

- Solve the equation for y.

$\frac{(y+3)^2}{9}+\frac{(x-2)^2}{4}=1$

$4(y+3)^2+9(x-2)^2=36$

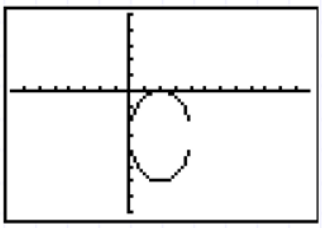
$4(y+3)^2=36-9(x-2)^2$



$$(y+3)^2 = \frac{36-9(x-2)^2}{4}$$

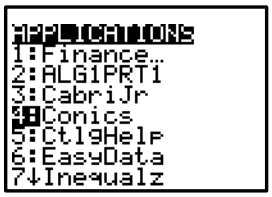
$$y+3 = \pm\sqrt{\frac{36-9(x-2)^2}{4}}$$

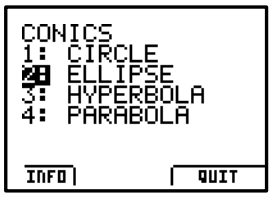
$$y = \frac{\pm\sqrt{36-9(x-2)^2}}{2} - 3$$

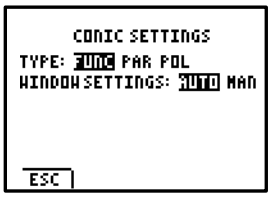


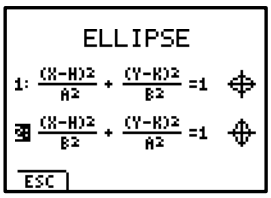
Another method for graphing an ellipse:

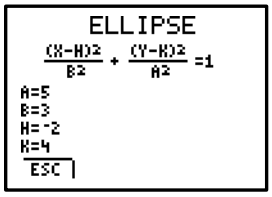
Rectangular coordinates **FUNC** mode

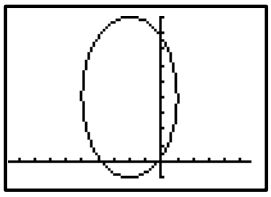






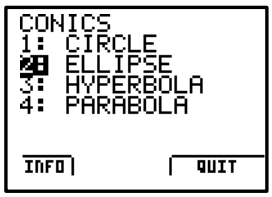


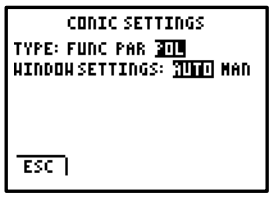


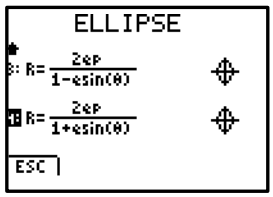


Polar coordinates **POL** mode

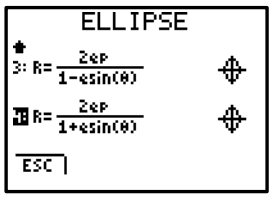


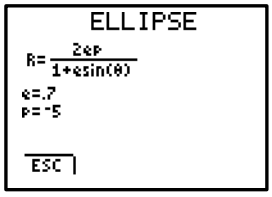


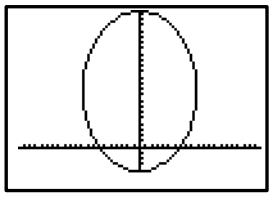




or







Hyperbola

A. center at the origin:	**B.** center at (h, k):
$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$
$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$



Example: Graph: $\dfrac{(x-1)^2}{9}-\dfrac{(y-2)^2}{9}=1$

Solution:

- Solve the equation for y.

$$\frac{(x-1)^2}{9}-\frac{(y-2)^2}{9}=1$$

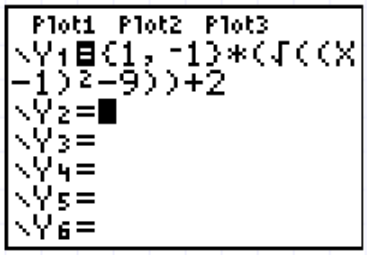
$$(x-1)^2-(y-2)^2=9$$

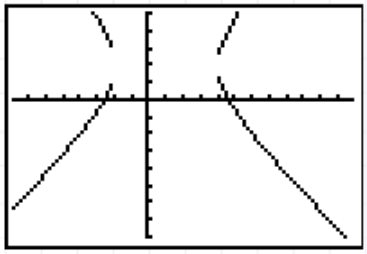
$$(x-1)^2-9=(y-2)^2$$

$$\pm\sqrt{(x-1)^2-9}=y-2$$

$$\pm\sqrt{(x-1)^2-9}+2=y$$

- Enter the equation with the positive square root into Y1.
- Enter the equation with the negative square root into Y2. You cannot simply negate Y1 to obtain Y2 in this problem.
- OR enter using the {list} approach as shown at the right.
- If you choose ZOOM #6 (the standard window), remember that the viewing window has a 3/2 aspect ratio (the standard viewing screen is not a square).
- Choose ZOOM#5 ZSquare to create a viewing window where the units on both axes are the same length.
- To see more of the graph, choose ZOOM #3 (Zoom Out).





NOTE:

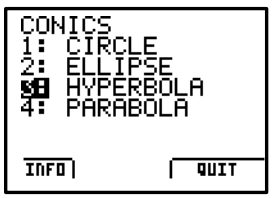
* You may notice that the "vertical" edges of the graph may not appear in the viewing window. The viewing screen cannot graph points that fall between pixels.

* You may notice that the TRACE function will not move automatically between the two sections of the graph because the positive and negative square roots were graphed as two separate equations. The arrow keys can be used to move between the two sections of the graph. Also, the cursor will disappear if it is moved beyond the domain for which x is defined.

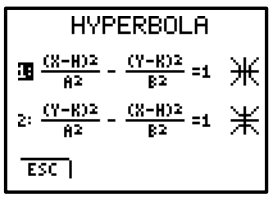
Another method for graphing a hyperbola:

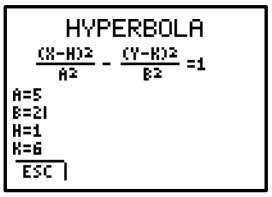
Rectangular coordinates **FUNC** mode

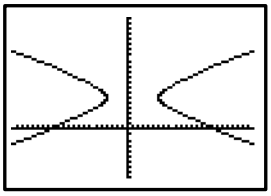




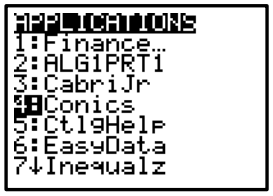


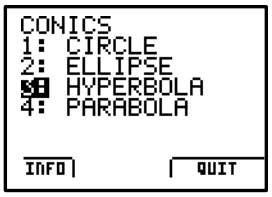




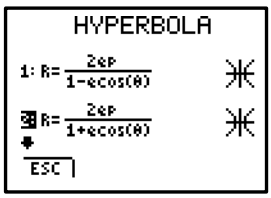


Polar coordinates **POL** mode

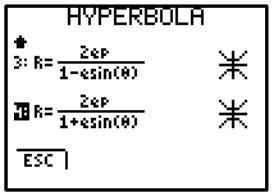


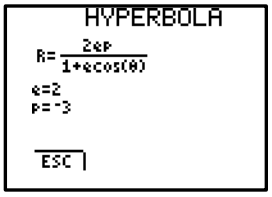


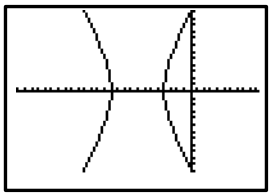




or







Vectors

TI-84 calculator can do operations on lists of numbers. Though it doesn't know about vectors as such, you can consider a list of two or three numbers to be the components of a vector in the plane or in space.

Addition of two vectors (downloadable software)

If two vectors are issued from the same initial point, which is the origin point $(0, 0)$ and we have the destination point coordinates of the each vector, we can operate with this software.

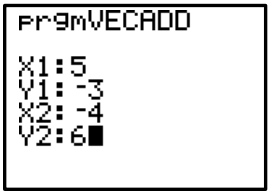
Press **PRGM** and choice **5:VECADD**.
Press **ENTER** twice.



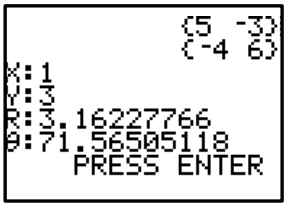




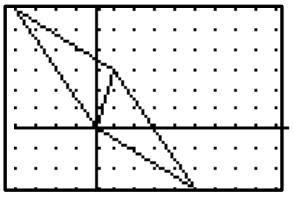
Type coordinates of the destination points for each vector. Let for the first vector (x_1, y_1) **X1: 5** and **Y1:-3**; for the second vector (x_2, y_2) **X2: -4** and **Y2:6**. After typing the each value, hit **ENTER**.



When we press **ENTER** again, we receive coordinates of the destination point for the resulted vector, the length of the resulted vector and the angle θ between two given vectors.



When we press **ENTER** as prescribed at the previous screen, we can see the graph of the given two vectors and the resulted vector.



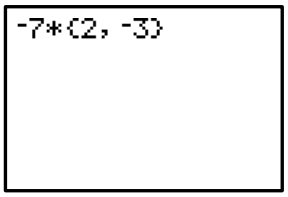
Multiplication by a scalar

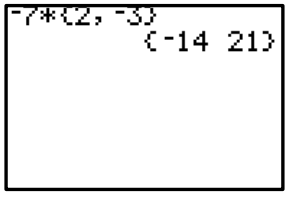
Use curly braces **{ }** around the list of components. To get a left or right curly brace, press the **2ND** key and then the left or right parenthesis.

Example:

Suppose vector **a** is [2,–3] and you want to display the vector –7**a**.

Enter the scalar. Press **(-)** and **7**. Type the multiplication sign ×. Enter the vector's value {2, −3}. Press **ENTER.**





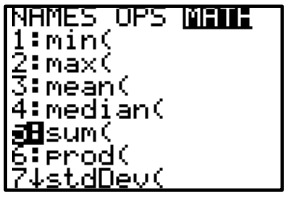
Dot product

The dot product of two vectors **u** and **v** is formed by multiplying their components and adding. In the plane, $\boldsymbol{u} \cdot \boldsymbol{v} = u_1 v_1 + u_2 v_2$; in 3D coordinates it's $u_1 v_1 + u_2 v_2 + u_3 v_3$.

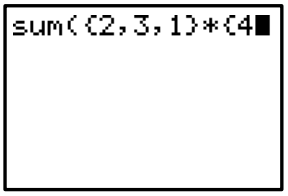
If you tell the TI-84 to multiply two lists, it multiplies the elements of the two lists to make a third list. The sum of the elements of that third list is the dot product of the vectors.

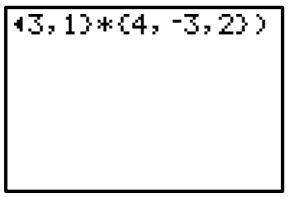
Example:
If **u** = [2, 3, 1] and **v** = [4, −3, 2], find $\boldsymbol{u} \cdot \boldsymbol{v}$.

Step 1. Invoke the summation function. Press **2ND** and **LIST**. Highlight **MATH** and **5:sum(** . Hit **ENTER**.

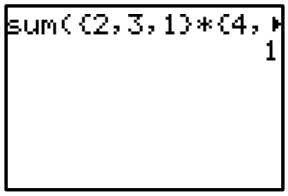


Step 2. Press **ENTER**. Type all problem's data and requirement.





Step 3. Press **ENTER**.



The length of a vector

You know that the length or magnitude of vector **v** is found by

$$||\boldsymbol{v}|| = \sqrt{v_1^2 + v_2^2}$$

You know also that the dot product of a vector with itself is

$$\boldsymbol{v} \cdot \boldsymbol{v} = v_1 v_1 + v_2 v_2 = v_1^2 + v_2^2$$

which is the square of the length of **v**. Therefore

$$||\boldsymbol{v}|| = \sqrt{\boldsymbol{v} \cdot \boldsymbol{v}}$$

With the TI-84, there's no need to enter the vector twice to find its length. Instead, just square the list of components — the calculator interprets this as squaring every component. Then take the square root of the sum.

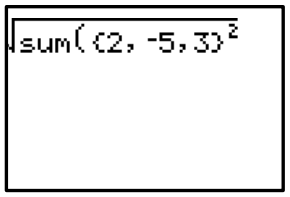
Example:
Find the length of vector **a** = [2,−5,−3].

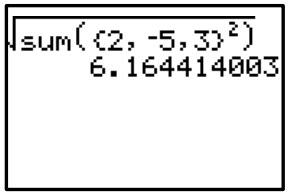
Step 1. Set up the square root by press **2ND** and $\sqrt{\ }$. Involve the summation function by the same way, as at the previous example.





Step 2. Type the function's value and requirement, and hit **ENTER**.

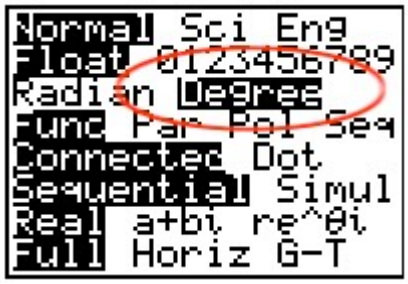


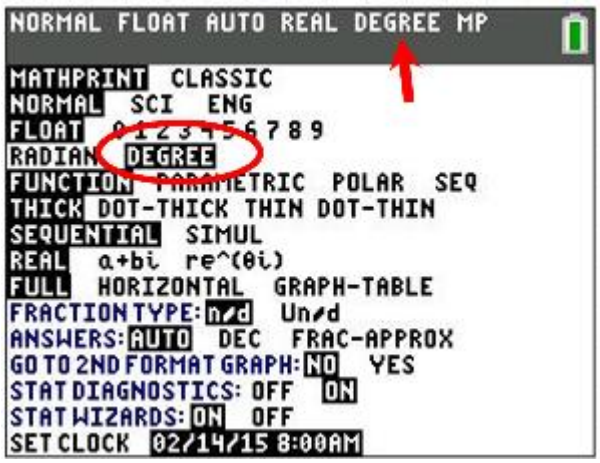


6. Trigonometry

Working with Trigonometric Ratios

When first learning about the Trigonometric Ratios, you will be working in **Degree** mode. The calculator will default to **Radian** mode, so start by setting mode to Degree.



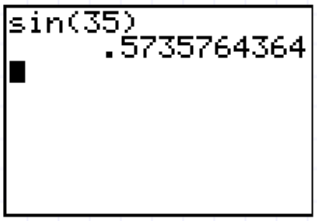


Note: If the calculator is set in **Radian** mode and you wish to find an angle in **Degrees,** you can insert a degree symbol after the number of degrees in the angle. The degree symbol is found under **2nd APPS (Angle) #1 °**. Only that one entry will be calculated in **Degrees --** all others will remain in **Radian** mode.

Find sin 35°

Set **MODE** to **Degree**.

Type entry on the Home Screen.
Hit ENTER.



- You may be asked to round this answer to a specified value.
- If not asked to round, state the full screen value as the answer.
- If further calculations are needed, keep this full screen value and continue working.

Find $\angle A$ to the *nearest degree*, given $\cos A = 3/5$

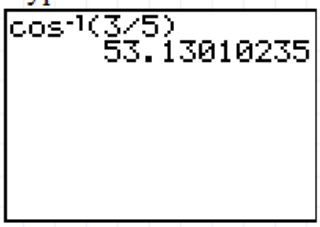
To find the number of degrees in the angle, you will need to use the calculator's inverse trigonometric functions. You will find these inverse functions ($\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$) "above" the keys for sin, cos, and tan.

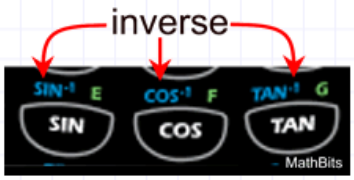
Find $\angle A$ to the *nearest degree*, given $\cos A = 3/5$

To find the number of degrees in the angle, you will need to use the calculator's inverse trigonometric functions. You will find these inverse functions ($\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$) "above" the keys for sin, cos, and tan.

Set **MODE** to **Degree**.

On the Home Screen, hit 2nd cos to get the $\cos^{-1}$. Type 3/5. Hit ENTER.





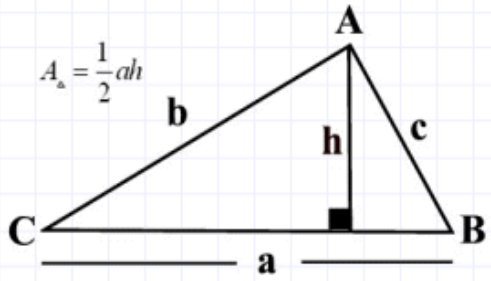
- Rounding answer to the *nearest degree* gives 53°.

Trigonometry Formulas for the Area of Triangle and the Area of Parallelogram

The formula for the area of a triangle is

$$A_{\triangle} = \frac{1}{2}bh$$

where b stands for the base and h stands for the altitude (height) drawn to that base.



(lettering the diagram is of no importance to the formula)

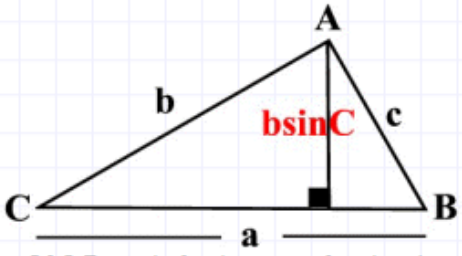
By using trigonometry in the right triangle (*on the left side of the diagram*), we find:

$$\sin C = \frac{h}{b}$$

Multiplying by b, gives $h = b\sin C$.

Substituting this new value of h into the area formula gives the trig area of triangle formula:

$$A_{\triangle} = \frac{1}{2}ab\sin C$$



SAS Formula for the area of a triangle where the pattern is to use "two sides and the sine of the included angle".

The diagonal of a parallelogram divides the parallelogram into two congruent triangles. Consequently, the area of a parallelogram can be thought of as doubling the area of one of the triangles formed by a diagonal. This gives the trig area formula for a parallelogram:

$$A_{\square} = ab\sin C$$

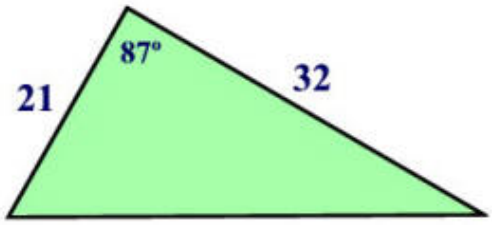
Example 1

Given the triangle at the right, find its area, to the *nearest hundredth*.

$$A_{\triangle} = \frac{1}{2}ab\sin C$$

$$A_{\triangle} = \frac{1}{2} \cdot 21 \cdot 32 \cdot \sin 87°$$

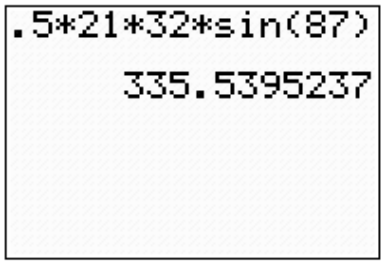
$$A_{\triangle} = 335.5395237 = 335.54 \text{ sq.units}$$



Be careful!!! When using your graphing calculator, be sure that you are in DEGREE Mode, **or** that you are using the degree symbol if in RADIAN Mode.

Degree Mode:



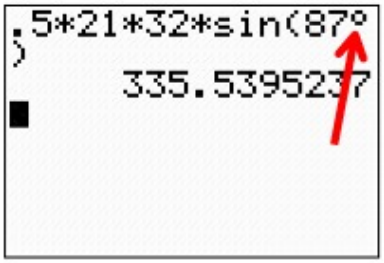


Radian Mode:



Find degree symbol under ANGLE (2nd APPS)





Example 2

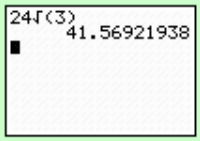
Given the parallelogram at the right, find its area to the *nearest hundredth.*

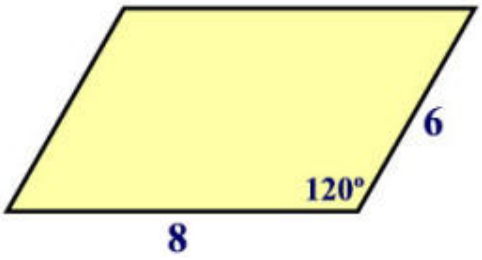
Again!!! Be sure that you are in DEGREE Mode, **or** that you are using the degree symbol if in RADIAN Mode.

If this problem had asked for an **EXACT answer**, do **not** use your calculator, as the calculator rounds the value for sin 120°. It will be necessary to use the 30°- 60°- 90° reference triangle in Quadrant II. The EXACT ANSWER will be

$$A_{\square} = 6 \cdot 8 \cdot \sqrt{3}/2 = 24\sqrt{3} \text{ sq.units}$$

Check to see that the exact answer yields the calculator decimal answer.

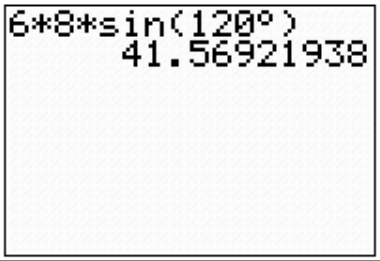




$$A_{\square} = ab\sin C$$

$$A_{\square} = 6 \cdot 8 \cdot \sin 120°$$

$$A_{\square} = 41.56921938 = 41.57 \text{ sq.units}$$



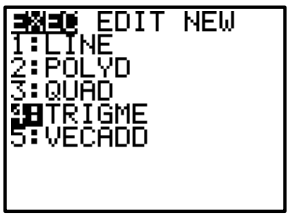
Another method for finding parameters of the triangles

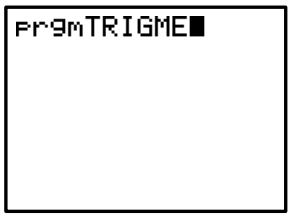
(downloadable software)

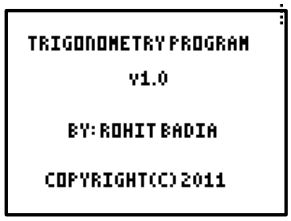
Attention! This program proposed the operation in degree mode only!

Press **PRGM** and choice **4:TRIGME.**
Press **ENTER** three times.

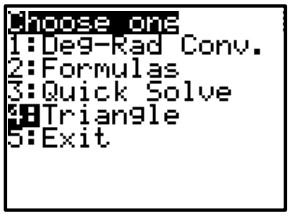


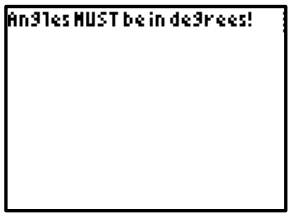




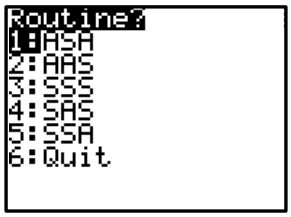


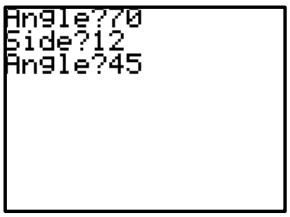
Choice **4:Triangle** and hit **ENTER**.
You will receive the warning message "**Angles MUST be in degrees**". Press **ENTER**.



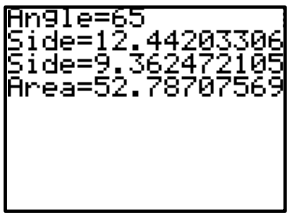


Choice the one of the proposed combinations of sides and angles. Let choice **1:ASA**, mean angle – side – angle and press **ENTER**. When you will finish to put all given values, press **ENTER**.





You will receive the measures of the missing angle and sides, and the area of the given triangle.



All other proposed combinations of the sides and angles of the triangle works by the similar method.

Degrees, Radians, and Trigonometry Angle Entries

Remember: **The calculator defaults to Radian measure.**

Working with Degrees:

There are two ways to engage "Degrees" when working with the calculator:

1. Set the MODE to Degree and all further calculations will be in degrees.
2. If in Radian MODE, use the Degree Symbol (2nd APPS (Angle) and select #1 °). The degree status will be applied to that one calculation only. All other calculations will return to radian mode.

Converting Degrees to Radians:

Find the radian measure of an angle whose measure is 500°.

Method 1: (in Radian or Degree mode) Convert by multiplying by $\frac{\pi}{180}$.	Mode set to Radian 500*(π/180) 8.72664626 500° 8.72664626
Method 2: (in Radian mode) With the mode set to Radian, type 500°. Notice the degree symbol. Hit Enter.	

NOTE: If in Degree mode, you can force an answer to radians by using 2nd APPS(Angle) #3 r.

In Degree mode,	but .....
$\sin(\pi)$ gives the answer 0.0548036651 (WRONG!!!)	$\sin(\pi^r)$ gives the correct answer of 0.

Converting Radians to Degrees:

Find the degree measure of an angle whose measure is $\frac{3\pi}{2}$.

Method 1: (in Radian or Degree mode) Convert by multiplying by $\frac{180}{\pi}$.	Mode set to Degree (3π/2)*(180/π) 270 (3π/2)ʳ 270
Method 2: (in Degree mode) In Degree mode, the r symbol can be used to convert radians to degrees. The r is found 2nd APPS(Angle) #3 r. With the mode set to Degree, type $(3\pi/2)^r$	

Converting Degrees to Radians:

Find the radian measure of an angle whose measure is 500°.

Method 1: (in Radian or Degree mode) Convert by multiplying by $\frac{\pi}{180}$.	Mode set to Radian 500*(π/180) 8.72664626 500° 8.72664626
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NOTE: If in Degree mode, you can force an answer to radians by using 2^{nd} APPS(Angle) #3 r.

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Method 2: (in Degree mode) In Degree mode, the r symbol can be used to convert radians to degrees. The r is found 2^{nd} APPS(Angle) #3 r. With the mode set to Degree, type $(3\pi/2)^r$	

Convert 57° 45' 17" to decimal degrees:

In either Radian or Degree Mode: Type 57° 45' 17" and hit Enter.

° is under Angle (above APPS) #1
' is under Angle (above APPS) #2
" use ALPHA (green) key with the quote symbol above the + sign.

Answer: 57.75472222

Convert 57° 45' 17" to radians:

With the mode set to Radian: Type 57° 45' 17" ° and hit Enter.

** Note the use of the additional degree symbol at the end. Without it, conversion would be to decimal degrees, even though the mode is set to radians.

Convert 48.555° to degrees, minutes, seconds:

Type 48.555 ►DMS Answer: 48° 33' 18"

The ►DMS is #4 on the Angle menu (2^{nd} APPS). This function works even if Mode is set to Radian.

Convert 48.555° to degrees, minutes, seconds:

Type 48.555 ►DMS Answer: 48° 33' 18"

The ►DMS is #4 on the Angle menu (2^{nd} APPS). This function works even if Mode is set to Radian.

Given cos A = .0258. Find <A expressed in degree, minutes, seconds.

With the mode set to Degree: Type $\cos^{-1}(.0258)$. Hit Enter.
Engage ►DMS Answer: 88° 31' 17.777"
(Be careful here to be in the correct mode!!)

Find sin 57° 45' 17":

Enter sin(57° 45' 17" °). This problem is clearly dealing with degrees. The entry shown here works even if MODE is set to Radian. Answer: .8457717984
If MODE is set to Degree, the degree symbol at the end is not needed.

Find cos(-250° 21'):

Enter cos (-250° 21' °) when in Radian mode. Answer: -.3362735421
If MODE is set to Degree, the degree symbol at the end is not needed.

Find sec(25° 40'):

There is no key for secant. But, since $\sec x = \frac{1}{\cos x}$, enter: 1/cos(25° 40' °) when in Radian mode. If MODE is set to Degree, the degree symbol at the end is not needed.
Answer: 1.109472591

Find $\cos\left(\frac{7\pi}{9}\right)$:

Enter directly: $\cos(7\pi/9)$ when in Radian mode. Answer: -.7660444431
If in Degree mode, enter $\cos((7\pi/9)^r)$.

Find $\csc\left(\frac{5\pi}{9}\right)$:

There is no key for cosecant. But, since $\csc x = \frac{1}{\sin x}$, enter: $1/\sin(5\pi/9)$ when in Radian mode. Answer: 1.015426612

In Degree mode, enter $1/\sin((5\pi/9)^r)$.

Another method of the degree – radian conversion

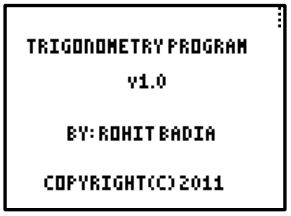
(downloadable software)

Press **PRGM** and choice **4:TRIGME.**
Press **ENTER** three times.

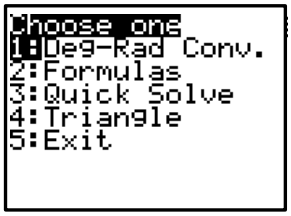


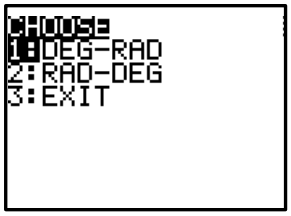




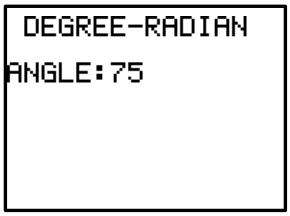


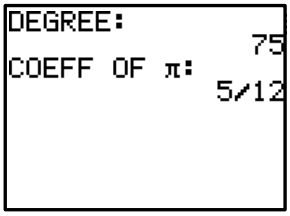
Choice **1:Deg-Rad Conv.** and click **ENTER**. For the degree-radian conversion select **1:Deg-Rad** and press **ENTER**.





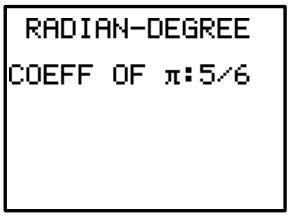
Let we need to convert to radians 75^0 angle. Type **75** at the next screen and hit **ENTER**.

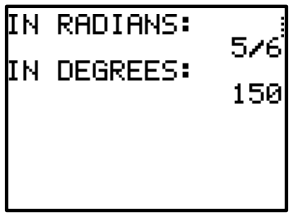




For the radian-degree conversion select **2: Rad-Deg** and press **ENTER**. Let we need to convert $\frac{5}{6}\pi$ radians to degree. Type **5/6** and hit **ENTER**.





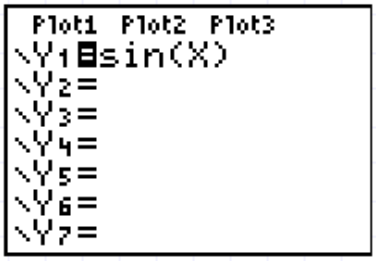


Graphing Basic Trigonometric Functions

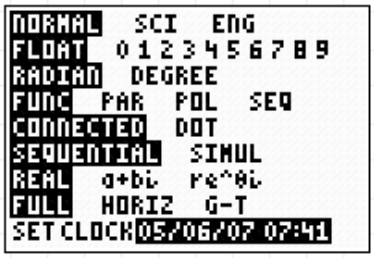
This page concentrates on graphing in Radian mode.

Graph $y = \sin(x)$

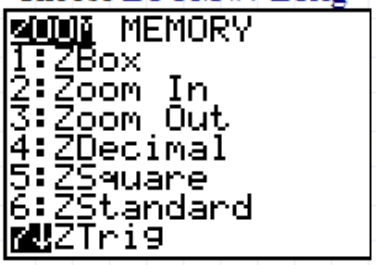
Enter the function.



Set the MODE to Radian.

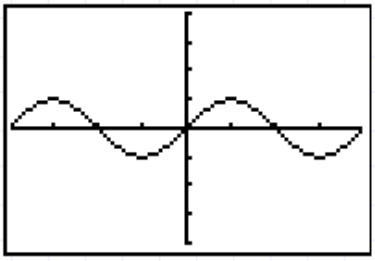


Choose ZOOM #7 ZTrig



Choosing ZOOM #7 ZTrig will set the x interval from $-2\pi \le x \le 2\pi$, set the y interval from $-4 \le y \le 4$, and set the increment at $\pi/2$.

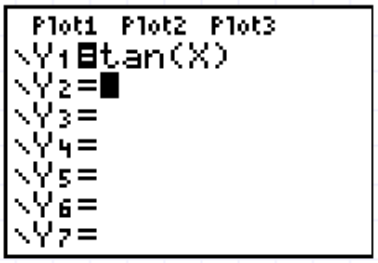
Depending upon your desired graph, you may wish to further adjust these values under WINDOW.



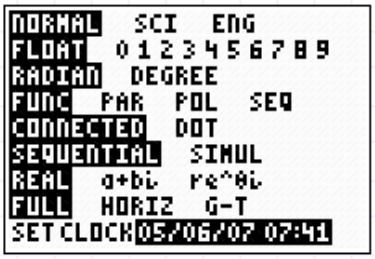
NOTE: If you do a ZOOM #7 ZTrig while in DEGREE mode, the x interval will be set from -352.5° to 352.5°, with the y interval from -4 to 4, and the increment set at 90°.

Graph $y = \tan(x)$

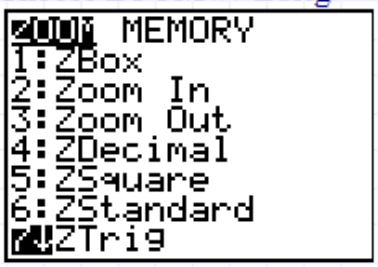
Enter the function.

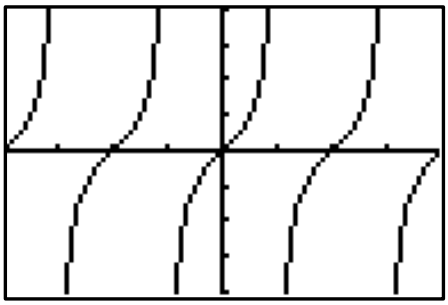


Set the MODE to Radian.



Choose ZOOM #7 ZTrig



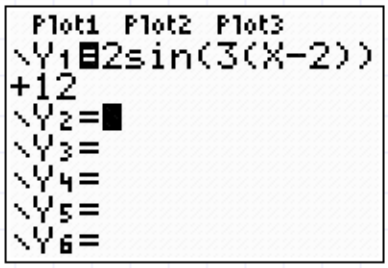


Graph $y = 2\sin(3(x - 2)) + 12$

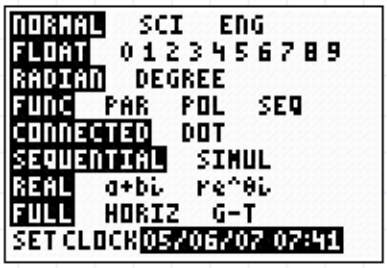
This is a sinusoidal equation of the form $y = A\sin(B(x - C)) + D$, where A is the amplitude, B is the frequency, C is the horizontal shift and D is the vertical shift. The period of this graph will be $2\pi/B$.

When graphing such equations, it may be necessary to adjust the window to "see" the graph. If ZOOM #7 Trig is used with this graph, the screen will appear empty. The vertical shift tells you that the graph has been "raised" by 12 units. Adjust the yMax and yMin accordingly. The yMin has been set to 0 for this example to establish a reference point when viewing the graph.

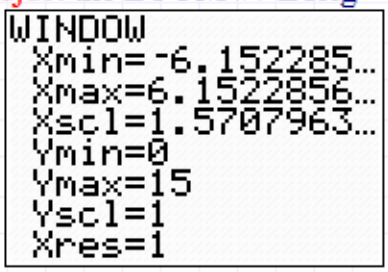
Enter the function.



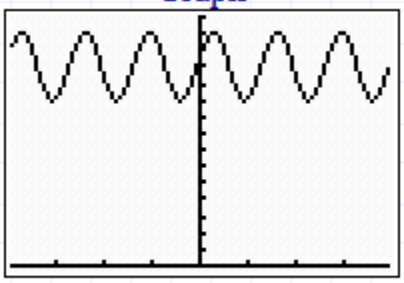
Set the MODE to Radian.



Adjust the ZOOM #7 ZTrig



Graph



Adjust the WINDOW further for additional examinations of the graph.

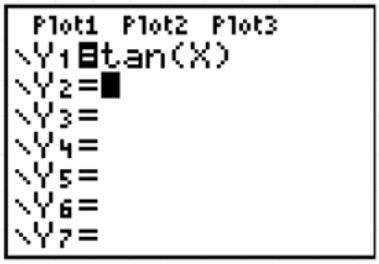
Asymptotes - Am I Really Seeing Them?

When graphing various trigonometric functions on the TI-84+, it may appear that asymptotes are also being graphed. Are these really asymptotes?
(**ANSWER**: No. They are not asymptotes.)

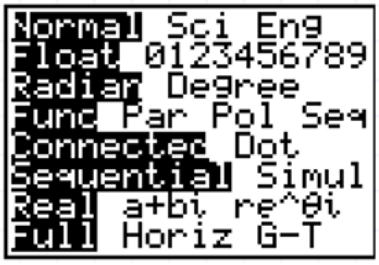
Let's examine some trigonometric graphs in **Radian** MODE:

Graph $y = \tan(x)$

Enter the function.



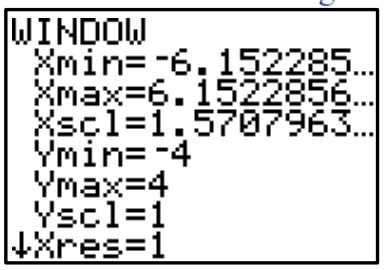
Set the MODE to Radian.



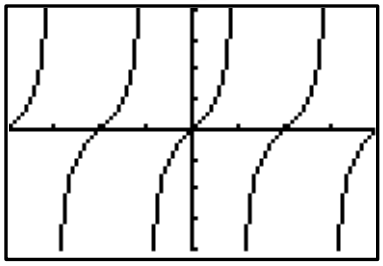
Choose ZOOM #7 ZTrig



Choose ZOOM #7 ZTrig



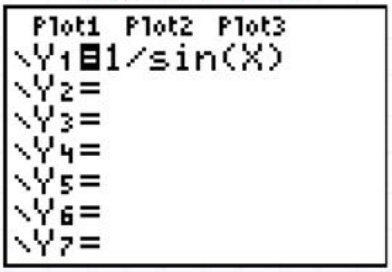
Graph



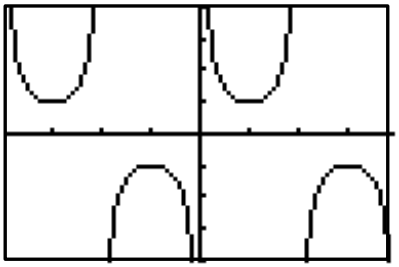
Graph $y = \csc(x)$

This same problem occurs with the graph of $y = \csc(x)$ in **Radian** MODE.

Enter the function.



Graph using ZOOM #7 ZTrig



If I use DEGREE MODE, I don't see these "supposed" asymptotes also.

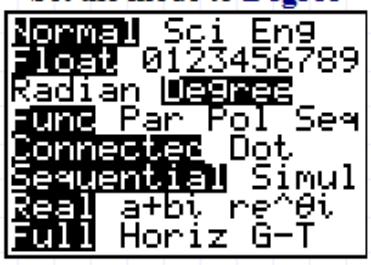
Let's examine these same two functions in **DEGREE** Mode.

$y = \tan(x)$

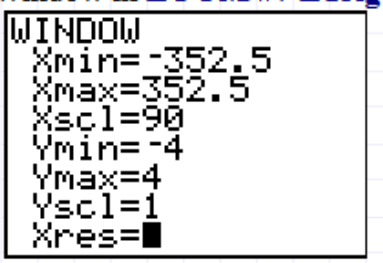
$y = \csc(x)$

Still in **connected mode.**

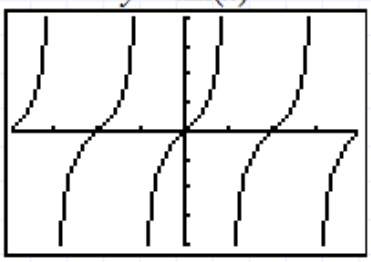
Set the mode to **Degree**



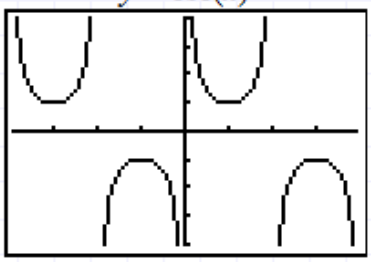
Window in **ZOOM #7 ZTrig**



$y = \tan(x)$



$y = \csc(x)$

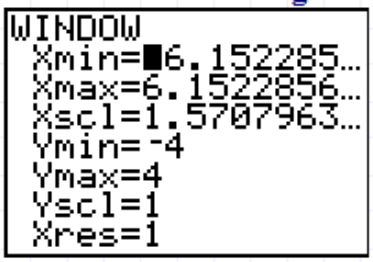


Notice -- there are NO "supposed" asymptotes, or "heartbeat" lines, in these graphs.

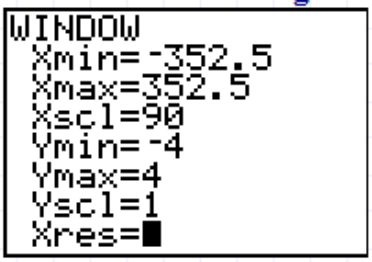
So, why am I not seeing "heartbeat" lines in Degree mode?

The answer can be found by examining the windows created by **ZTrig**.

RADIAN - ZTrig



DEGREE - ZTrig



Note: The **Degree Mode** uses 90 as the **Xscl**. The **Radian Mode** does not truly use $\frac{\pi}{2}$ as **Xscl.** It expresses this value as an approximate decimal.

The **Radian Mode window** does not truly use $\frac{\pi}{2}$ as **Xscl.**
It expresses this value as an approximate decimal.

In **Radian** mode, the calculator is never graphing the undefined points on the graphs. The decimal approximations for these locations are slightly "off" -- meaning that the graph is not undefined at these "close" approximations and consequently points **are** plotted. When a point is plotted, the **connected mode** engages and connects adjacent points.

The **Degree Mode window** uses 90° as the **Xscl**.

In **Degree mode**, the calculator attempts to graph the actual undefined points. Such an attempt produces an "error" and nothing is plotted for those *x*-values. Consequently, a break in the graph appears (a non-plotting), preventing the **connected mode** from connecting adjacent points, which prevents the creation of the "heartbeat" line.

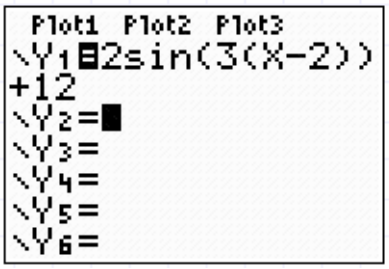
Graphing Sinusoidal Functions

Graph $y = 2\sin(3(x - 2)) + 12$

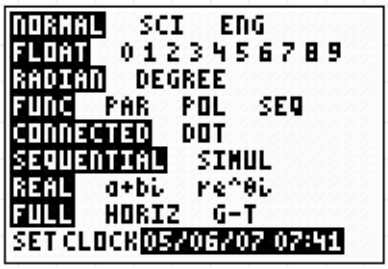
This is a sinusoidal equation of the form $y = A\sin(B(x - C)) + D$, where A is the amplitude, B is the frequency, C is the horizontal shift and D is the vertical shift. The period of this graph will be $\frac{2\pi}{B}$.

When graphing such equations, it may be necessary to adjust the window to "see" the graph. If ZOOM #7 Trig is used with this graph, the screen will appear empty. The vertical shift tells you that the graph has been "raised" by 12 units. Adjust the yMax and yMin accordingly. The yMin has been set to 0 for this example to establish a reference point when viewing the graph.

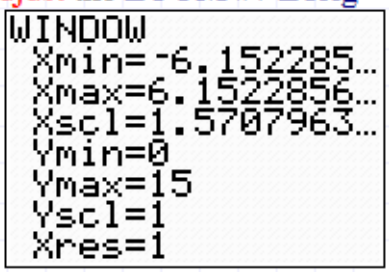
Enter the function.



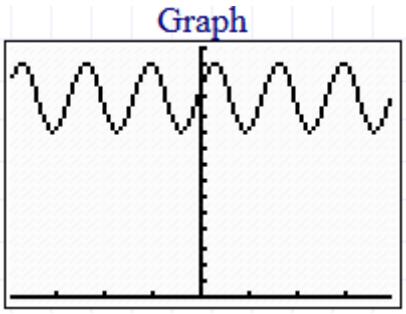
Set the MODE to Radian.



Adjust the ZOOM #7 ZTrig



Graph



Adjust the WINDOW further for additional examinations of the graph.

Inverse Trigonometric Functions

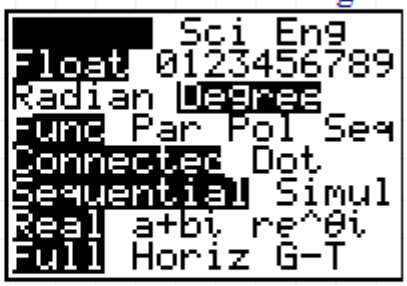
Inverse trigonometric functions are also referred to as arcfunctions. The notation may be:

$\sin^{-1}(0.5)$ or $\arcsin(0.5)$

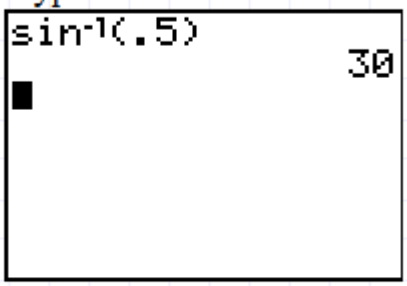
where each is interpreted as "the angle whose sine is 0.5".

Find $\sin^{-1}(0.5)$ in degrees

Set the MODE to Degree.



Type on the Home Screen



The angle whose sine is 0.5 is 30°. But it could also be 150°, 390°, 510°, ...
If the problem was stated as *Arc*sin(0.5), where the A was capitalized,
it would have been asking for the function *Arc*sin in the interval [-90°, 90°]
instead of the multi-valued relation *arc*sin.
When the notation is $\sin^{-1}(0.5)$, you will need to read the context of the problem.

Find $\sec^{-1}(3/2)$ in degrees

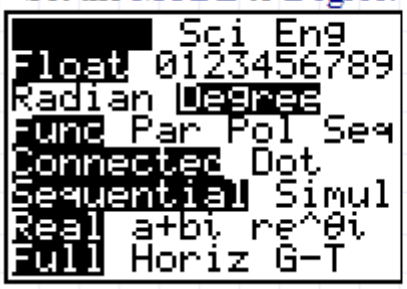
Be careful when working with the inverses of the reciprocal trig functions. Their computations may not be what you expect them to be. Observe:

Arc notation	**Exponent notation**
$\sec x = \dfrac{1}{\cos x} \Rightarrow \operatorname{arcsec} x = \arccos\left(\dfrac{1}{x}\right)$	$\sec^{-1} x = \cos^{-1}\left(\dfrac{1}{x}\right)$
$\csc x = \dfrac{1}{\sin x} \Rightarrow \operatorname{arcsin} x = \arcsin\left(\dfrac{1}{x}\right)$	$\csc^{-1} x = \sin^{-1}\left(\dfrac{1}{x}\right)$
Be careful with cotangent inverse! If we try to use this same technique with cotangent inverse, stating $\cot x = \dfrac{1}{\tan x} \Rightarrow \operatorname{arccot} x = \arctan\left(\dfrac{1}{x}\right)$	*Another complete formula for cotangent inverse can be:* $\cot^{-1} x = \begin{cases} \tan^{-1}\left(\dfrac{1}{x}\right), & x > 0 \\ \tan^{-1}\left(\dfrac{1}{x}\right) + \pi, & x < 0 \\ \dfrac{\pi}{2}, & x = 0 \end{cases}$
We get a formula that does not yield all possible results. For example, unlike the inverses of secant and cosecant, cotangent inverse deals with angles whose cotangent may be zero. It is preferable to state a more universal formula that works in all cases, such as :	**OR** use the previous formula

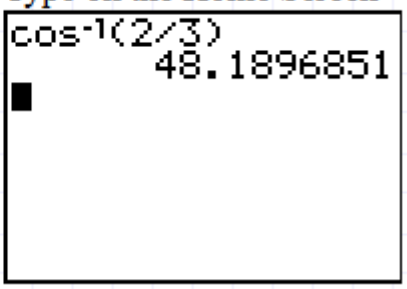
$\operatorname{arccot} x = \frac{\pi}{2} - \arctan x$	$\cot^{-1} x = \frac{\pi}{2} - \tan^{-1} x$

S O L U T I O N: **Find $\sec^{-1}(3/2)$ in degrees.**

Set the MODE to Degree.

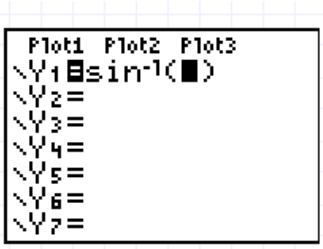


Type on the Home Screen



Graph the inverse trigonometric functions

The graphing calculator is programmed to graph arcsin (x), arccos (x) and arctan (x) as functions (thus the domains are limited to ensure that the graphs pass the vertical line test for functions).

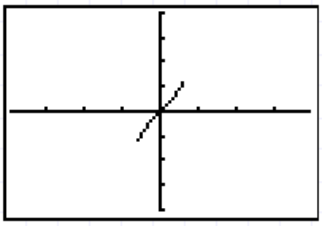


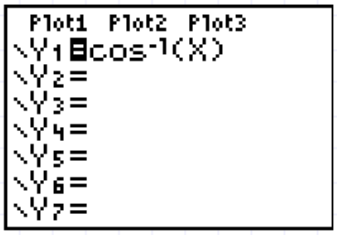
$f(x) = \sin^{-1}(x)$

$f(x) = \arcsin(x)$

Domain: [-1,1]

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



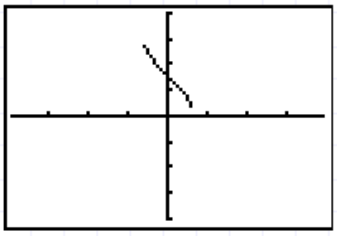


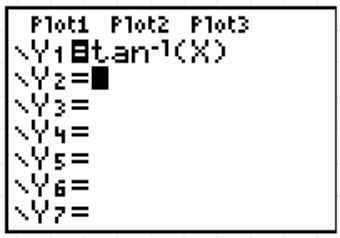
$f(x) = \cos^{-1}(x)$

$f(x) = \arccos(x)$

Domain: [-1,1]

Range: $[0, \pi]$



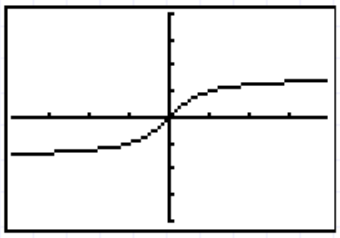


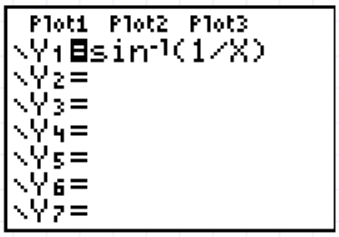
$f(x) = \tan^{-1}(x)$

$f(x) = \arctan(x)$

Domain: $(-\infty, \infty)$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



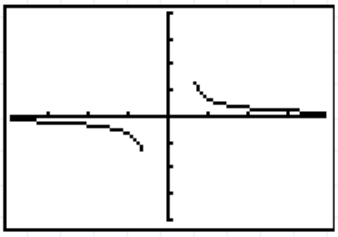


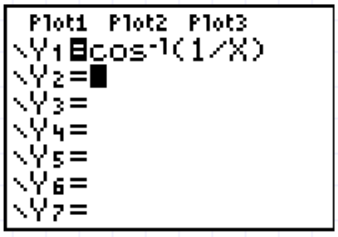
$f(x) = \csc^{-1}(x)$

$f(x) = \operatorname{arccsc}(x)$

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right], y \neq 0$



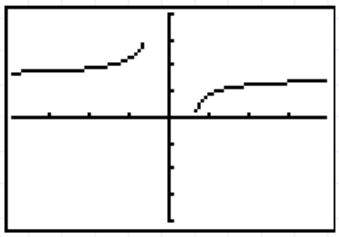


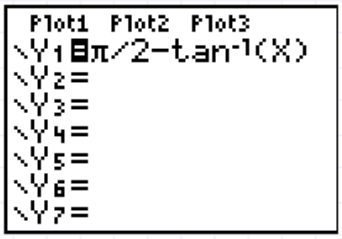
$f(x) = \sec^{-1}(x)$

$f(x) = \operatorname{arcsec}(x)$

Domain: $(-\infty, -1] \cup [1, \infty)$

Range: $[0, \pi], y \neq \frac{\pi}{2}$



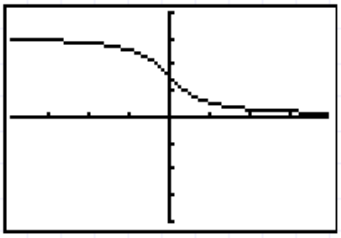


$f(x) = \cot^{-1}(x)$

$f(x) = \operatorname{arccot}(x)$

Domain: $(-\infty, \infty)$

Range: $(0, \pi)$



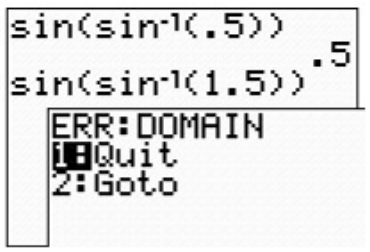
Composition with Inverse Trigonometric Functions

The graphs of the compositions of a trigonometric function with its inverse can yield some interesting results. It is understood that a function and its inverse, when composed, return the original starting value -- they UNDO one another. So, why does this "appear" to NOT be true when working with trigonometric functions on the graphing calculator?

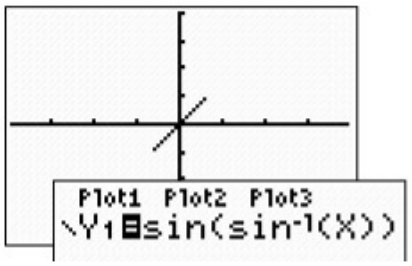
Let's do some investigating:



The calculator is set to Radian mode.



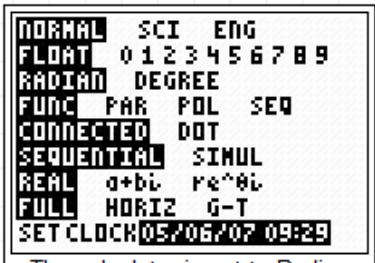
The composition of a value within the limited domain of the sine inverse **function**, [-1,1], returns the starting value, just as expected. If we, however, choose a value outside the domain of the sine inverse function, an error message is returned.



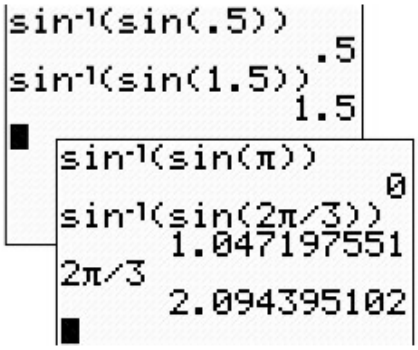
The graph of the composition returns the identity line, y = x, for the domain of the sine inverse function from -1 to 1. Our expected result of returning the starting value holds true as long as we remain within the domain of the sine inverse function.

Conclusion 1: The calculator is seeing "sine inverse" to be ONLY the **principal sine inverse function**, and as such limits its domain to [-1,1]. All other domain values are seen as producing an error. Consequently, $\sin(\sin^{-1}(x))$, which starts with $\sin^{-1}(x)$, is limited to accepting only values within the domain of the "function" $\sin^{-1}(x)$, which is [-1,1]. We see only these x-values being plotted which creates only a "segment" of the expected identity line $y = x$.

Let's REVERSE the order of the composition and see what happens:

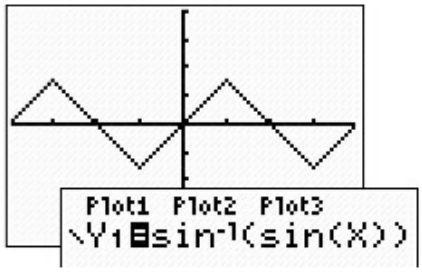


The calculator is set to Radian mode.



No error message this time, but the "starting value" is not always being returned.

The sin(x) function is now the starting function in the composition. It has a domain of all Real values and always returns a result between -1 and 1 inclusive, which happens to be within the domain of the sine inverse function. The sine inverse function will always return a value, but that value will be within its range of $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.



Why are we seeing so much more? Since sin(x) is producing values in the range [-1,1], the sine inverse of these values can always yield a result and we see points plotted across the entire graph.

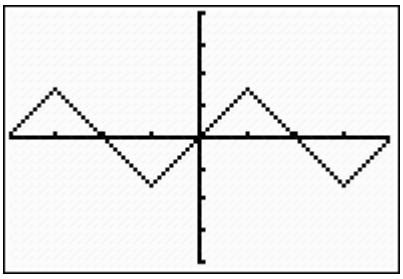
But this is not the expected y = x graph. The range of sine inverse is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$ and it returns values only within this range.

Conclusion 2: When the "starting" function in the composition is sine ($\sin^{-1}\sin(x)$), all numerical evaluations of the composition are possible, but the results will be restricted to the range of the inverse function.

When sin(x) produces positive values, the **inverse function** maps those values to $\left(0,\frac{\pi}{2}\right]$ in the first quadrant (or on the y-axis). When sin(x) produces negative values, the inverse function maps those values to $\left[-\frac{\pi}{2},0\right)$ in the fourth quadrant (or on the y-axis)

When sin(x) = 0, $\sin^{-1}(x) = 0$.

Consequently, the graph takes on the appearance of straight line segments alternating between positive and negative slopes, due to the restricted range of sine inverse to $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$.

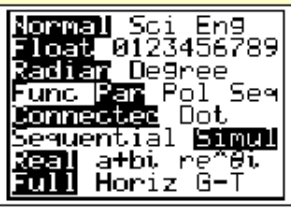


Unit Circle Investigation

Listed below are two investigations that will allow you to investigate the workings of the unit circle and the relationship between the unit circle and the trigonometric graphs.

Animated display of the relationship between the unit circle and the creation of the trigonometric curves

- Set the **MODE** to Radian, Parametric (Par), and Simultaneous (Simul).



- Set the **WINDOW** to:

$\text{Tmin} = 0$

$\text{Tmax} = 2\pi$

$\text{Tstep} = 0.1$

$\text{Xmin} = -2$

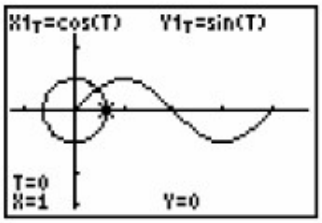
$\text{Xmax} = 7.4$

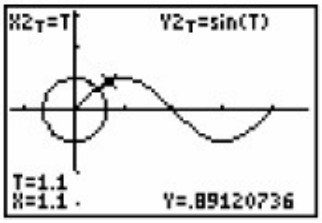
$\text{Xscl} = \frac{\pi}{2}$

$\text{Ymin} = -3.1$

$\text{Ymax} = 3.1$

$\text{Yscl} = 1$





Use the up and down arrows to move between the unit circle and the trig curve.

- Enter the **equations:**

$X_{1T} = \cos T$

$Y_{1T} = \sin T$

$X_{2T} = T$

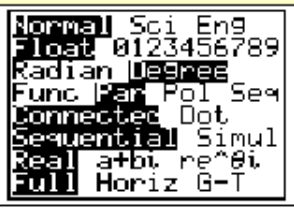
$Y_{2T} = \sin T$

- Press **GRAPH.** The Unit Circle and the Trigonometric graph will be drawn simultaneously. Trace between the unit circle and the trig curve to see the relationships between the graphs.

- A further exploration can be demonstrated by changing $Y_{2T} = \cos T$ and again tracing between the graphs.

Investigate the Unit Circle more closely:

- Set the **MODE** to Degree, Parametric (Par), Sequential and clear the equations.



- Set the **WINDOW** to:

Tmin = 0

Tmax = 360

Tstep = 10

Xmin = –3

Xmax = 3

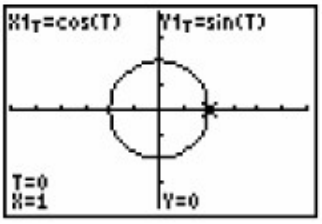
Xscl = 0.5

Ymin = –2

Ymax = 2

Yscl = 0.5

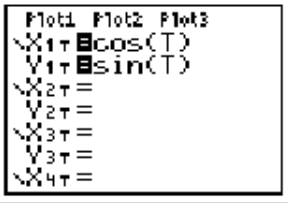
Note: The Tstep controls the increment of angle rotation.



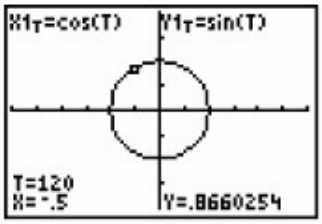
- Enter the **equations:**

$X_{1T} = \cos T$

$Y_{1T} = \sin T$



- Press **TRACE** and arrow to the right. The spider will work its way around the unit circle examining the angles as it progresses.



Working with Reciprocal Trigonometry Functions

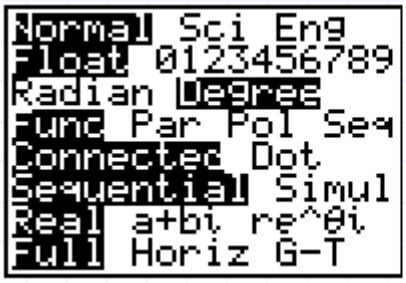
There are no specific keys or menu items on the graphing calculator for the secant, cosecant and cotangent functions. Consequently, you will have to use your knowledge of these functions when working with them on the graphing calculator.

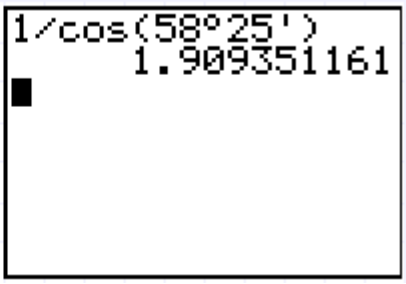
$$\csc x = \frac{1}{\sin x} \qquad \sec x = \frac{1}{\cos x} \qquad \cot x = \frac{1}{\tan x} \quad \text{or} \quad \cot x = \frac{\cos x}{\sin x}$$

Find the numerical value of sec 58º 25'

First, remember that: $\sec 58°25' = \frac{1}{\cos 58°25'}$

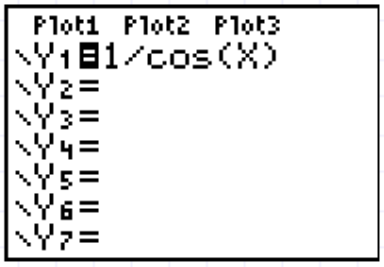
With the calculator set to Degree mode, enter the expression. Remember that the symbols for degrees and minutes are found under 2nd APPS (ANGLE).



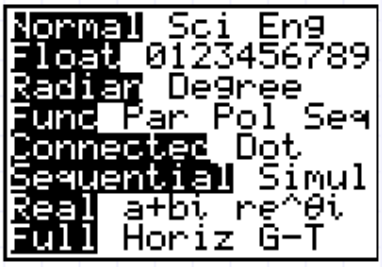


Graph sec(x)

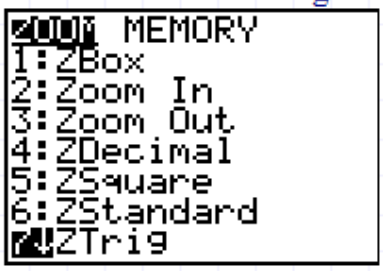
Enter the function.

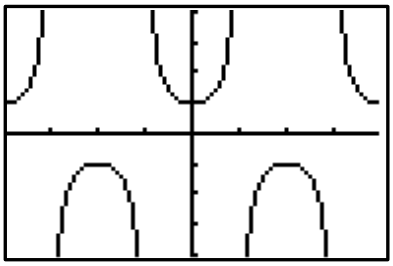


Set the MODE to Radian.



Choose ZOOM #7 ZTrig





Solving Trigonometric Equations

Remember: The calculator defaults to Radian measure.

Problem: Find all values of x for which $\cos x = \frac{1}{2}$ on the interval $0 \le x \le 2\pi$.

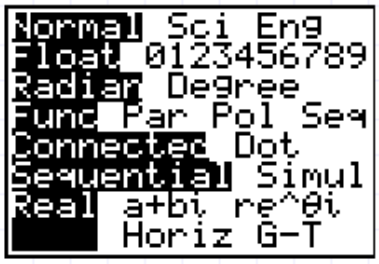
Since this problem states the interval in terms of radians, the answer will also need to be expressed in radians.

Let's look at two possible solution methods. The first method will work ONLY with Radians. The second method will work with Degrees and then convert back to Radians for the final answer. Pick the method with which you feel most comfortable.

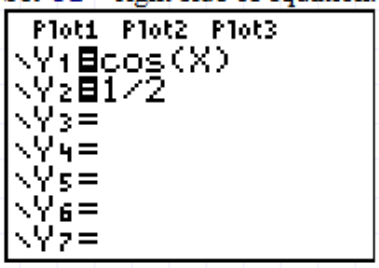
Method Using Only Radian:

You are going to graph each side of the equation and look for all of the points of intersection within the given interval.

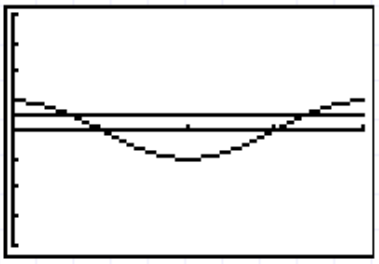
Set the MODE to Radian



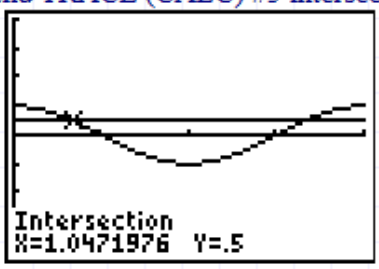
Set Y1 = left side of equation.
Set Y2 = right side of equation.



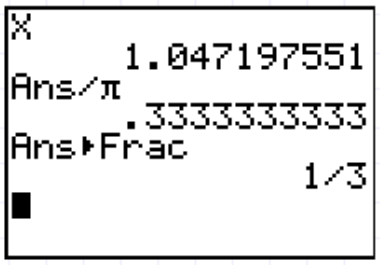
Start by hitting ZOOM #7 ZTrig.
Adjust WINDOW for Xmin = 0
and Xmax = 2π



Use the INTERSECT option.
2nd TRACE (CALC) #5 intersect



To express this answer in terms of π,
go to Home Screen and hit x ENTER.
Divide by π. Convert to fraction.
MATH #1 ▶Frac. ENTER.

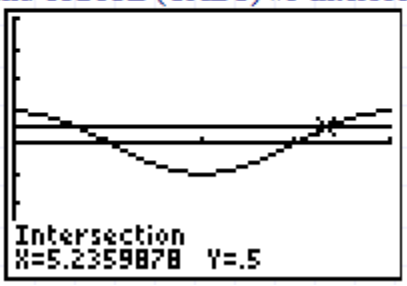


Now, find the second point of intersection!

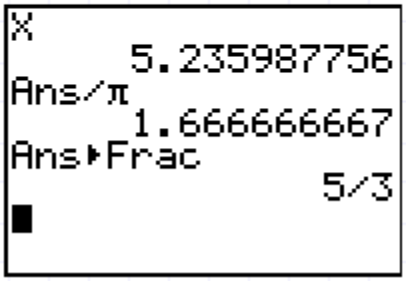
Answer:

$\frac{1}{3}\pi$ or $\frac{\pi}{3}$.

Use the INTERSECT option **again.**
2nd TRACE (CALC) #5 intersect



To express this answer in terms of π,
go to Home Screen and hit x ENTER.
Divide by π. Convert to fraction.
MATH #1 ▶Frac. ENTER.



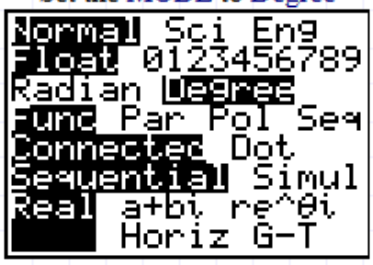
Answer:

$\frac{5}{3}\pi$ or $\frac{5\pi}{3}$.

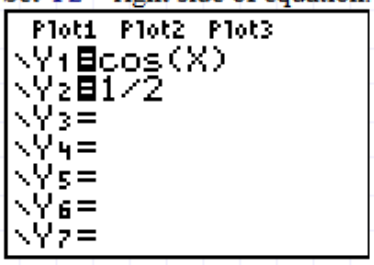
Method Using Degree and Converting Back to Radian:

Again, you are going to graph each side of the equation and look for all of the points of intersection within the given interval. For this method, you must be aware that the interval in degrees is $0 \le x \le 360$.

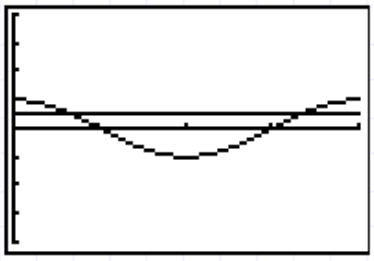
Set the MODE to Degree



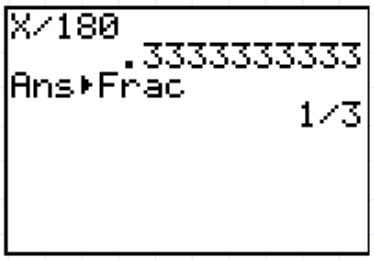
Set Y1 = left side of equation.
Set Y2 = right side of equation.



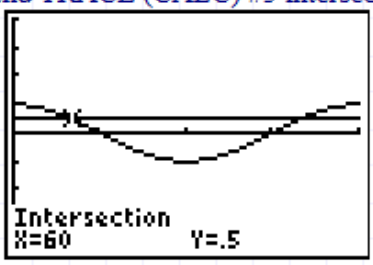
Start by hitting ZOOM #7 ZTrig.
Adjust WINDOW for Xmin = 0 and Xmax = 360



To convert to radians (in terms of π), go to Home Screen and hit x/180 ENTER. Convert to fraction. MATH #1 ▶Frac. ENTER.



Use the INTERSECT option.
2nd TRACE (CALC) #5 intersect

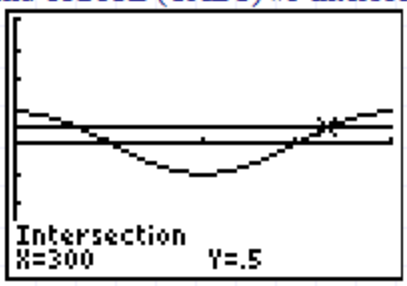


Now, find the second point of intersection!

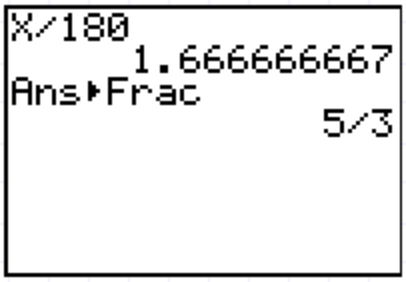
Answer:

$\frac{1}{3}\pi$ or $\frac{\pi}{3}$.

Use the INTERSECT option **again.**
2nd TRACE (CALC) #5 intersect



To convert to radians (in terms of π), go to Home Screen and hit x /180 ENTER. Convert to fraction. MATH #1 ▶Frac. ENTER.



Answer:

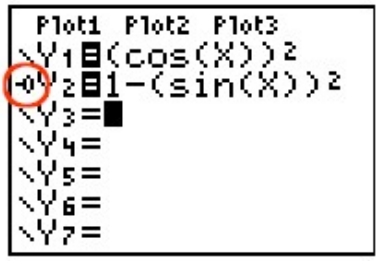
$\frac{5}{3}\pi$ or $\frac{5\pi}{3}$.

Verifying Trigonometric Identities

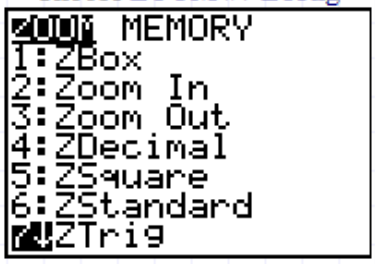
If you need to determine if an equation is a trigonometric identity, you can use your graphing calculator. A "formal proof" of a trigonometric identity is done algebraically. The graphing calculator, however, can be used to "check" your work.

Determine if $\cos^2 x = 1 - \sin^2 x$ is a trig identity:

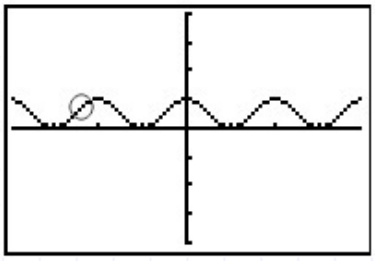
Enter the left side in Y1.
Enter the right side in Y2.
Set the bubble animation for the second graph.



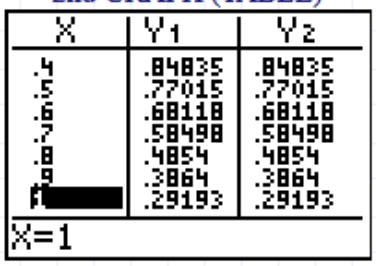
Choose ZOOM #7 ZTRig



If the bubble runs over the same graph as the first plotting, the equation is a trig identity.



2nd GRAPH (TABLE)



X	Y1	Y2
.4	.84835	.84835
.5	.77015	.77015
.6	.68118	.68118
.7	.58498	.58498
.8	.4854	.4854
.9	.3864	.3864
1	.29193	.29193

X=1

The TABLE can also be used to check to see if the results are the same from both graphs. Arrow up and down to see if the values listed under columns Y1 and Y2 remain the same. Adjust your table through 2nd WINDOW (TBLSET) to see more varied values.

Answer: Yes, **$\cos^2 x = 1 - \sin^2 x$ is a trig identity.**

Trigonometry Formulas

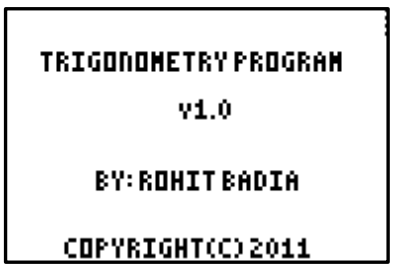
(downloadable software)

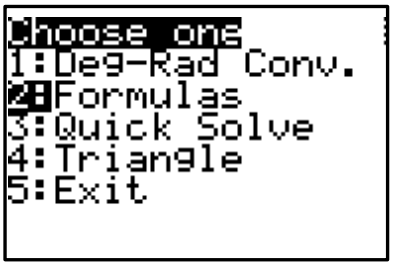
By using this software, you can operate by these formulas during the Mathematics Tests and Exams. Now you do not need to memorize it but **you must know HOW and WHERE use these formulas.**

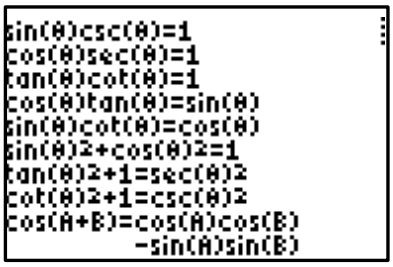
This software starts by pressing to **PRGM** key and continue by operating with the **ENTER** key all time you need to change the screen picture.

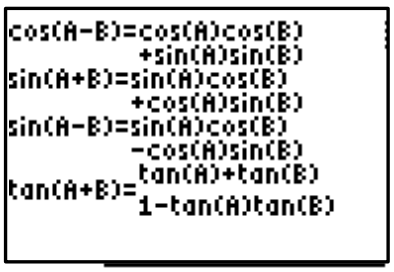

EXEC EDIT NEW
1:LINE
2:POLYD
3:QUAD
4:TRIGME
5:VECADD

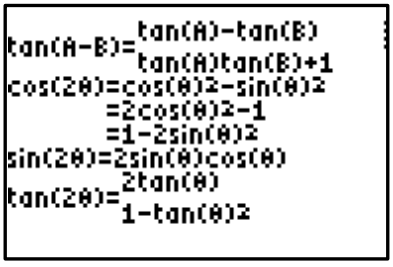

prgmTRIGME

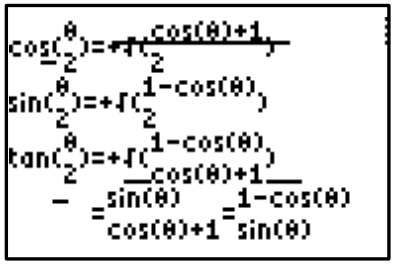

TRIGONOMETRY PROGRAM
v1.0
BY:ROHIT BADIA
COPYRIGHT(C)2011

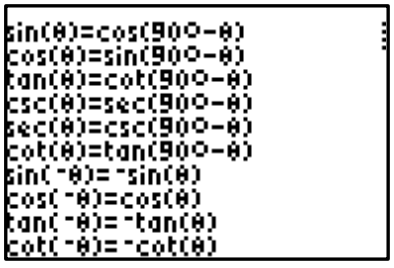

Choose one
1:Deg-Rad Conv.
2:Formulas
3:Quick Solve
4:Triangle
5:Exit

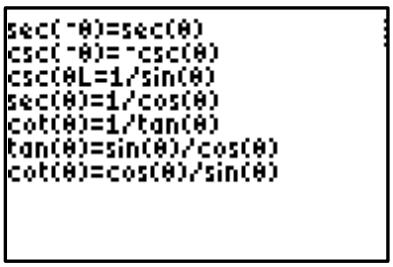

sin(θ)csc(θ)=1
cos(θ)sec(θ)=1
tan(θ)cot(θ)=1
cos(θ)tan(θ)=sin(θ)
sin(θ)cot(θ)=cos(θ)
sin(θ)²+cos(θ)²=1
tan(θ)²+1=sec(θ)²
cot(θ)²+1=csc(θ)²
cos(A+B)=cos(A)cos(B)
-sin(A)sin(B)


cos(A-B)=cos(A)cos(B)
+sin(A)sin(B)
sin(A+B)=sin(A)cos(B)
+cos(A)sin(B)
sin(A-B)=sin(A)cos(B)
-cos(A)sin(B)
tan(A+B)=(tan(A)+tan(B))/(1-tan(A)tan(B))


tan(A-B)=(tan(A)-tan(B))/(tan(A)tan(B)+1)
cos(2θ)=cos(θ)²-sin(θ)²
=2cos(θ)²-1
=1-2sin(θ)²
sin(2θ)=2sin(θ)cos(θ)
tan(2θ)=2tan(θ)/(1-tan(θ)²)


cos(θ/2)=±√((cos(θ)+1)/2)
sin(θ/2)=±√((1-cos(θ))/2)
tan(θ/2)=±√((1-cos(θ))/(cos(θ)+1))
=sin(θ)/(cos(θ)+1)=(1-cos(θ))/sin(θ)


sin(θ)=cos(90°-θ)
cos(θ)=sin(90°-θ)
tan(θ)=cot(90°-θ)
csc(θ)=sec(90°-θ)
sec(θ)=csc(90°-θ)
cot(θ)=tan(90°-θ)
sin(-θ)=-sin(θ)
cos(-θ)=cos(θ)
tan(-θ)=-tan(θ)
cot(-θ)=-cot(θ)


sec(-θ)=sec(θ)
csc(-θ)=-csc(θ)
csc(θL=1/sin(θ)
sec(θ)=1/cos(θ)
cot(θ)=1/tan(θ)
tan(θ)=sin(θ)/cos(θ)
cot(θ)=cos(θ)/sin(θ)

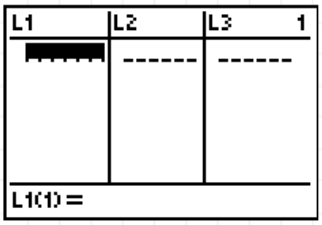
7. Statistics and Probabilities

Basic Commands

Consider the data set: {15, 22, 32, 31, 52, 41, 11}

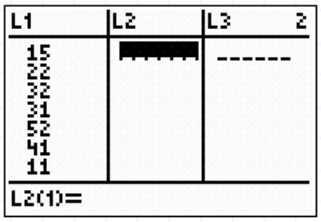
Entering Data:

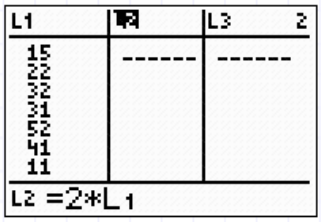
Data is stored in ***Lists*** on the calculator. Locate and press the **STAT** button on the calculator. Choose **EDIT.** The calculator will display the first three of six lists (columns) for entering data. Simply type your data and press **ENTER.** Use your arrow keys to move between lists.

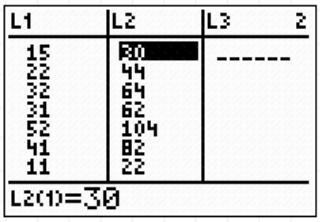


Data can also be entered from the home screen using set notation -- **{15, 22, 32, 31, 52, 41, 11} → L1** (where → is the **STO** key)

Data can be entered in a second list based upon the information in a previous list. In the example below, we will double all of our data values in **L1** and store them in **L2.** If you arrow up ONTO **L2**, you can enter a formula for generating **L2.** The formula will appear at the bottom of the screen. Press **ENTER** and the new list is created.

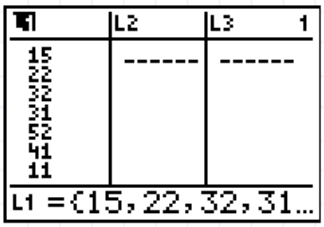






Clearing Data:

To clear all data from a list: Press **STAT.** From the **EDIT** menu, move the cursor up **ONTO** the name of the list (**L1**). Press **CLEAR.** Move the cursor down. ***NOTE:*** The list entries will not disappear until the cursor is moved down. (**Avoid** pressing **DEL** as it will delete the entire column. If this happens, you can reinstate the column by pressing **STAT #5 SetUpEditor**.)

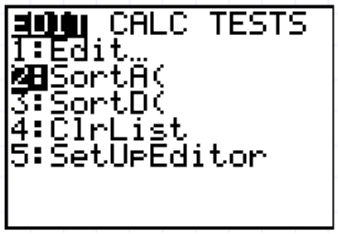


You may also clear a list by choosing option **#4** under the **EDIT** menu, **ClrList. ClrList** will appear on the home screen waiting for you to enter which list to clear. Enter the name of a list by pressing the **2nd** button and the yellow **L1** (above the **1**).

To clear an individual entry: Select the value and press **DEL**.

Sorting Data: (helpful when finding the mode)

Locate and press the **STAT** button. Choose option **#2, SortA(.** Specify the list you wish to sort by pressing the **2nd** button and the yellow **L1** list name. Press **ENTER** and the list will be put in ascending order (lowest to highest). **SortD** will put the list in descending order.

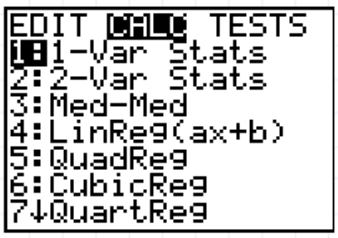


One Variable Statistical Calculations:

Press the **STAT** button. Choose CALC at the top. Select **1-Var Stats.** Notice that you are now on the home screen. Specify the list you wish to use by choosing the **2nd** button and the list name:

1-Var Stats L1

Press **ENTER** and view the calculations. Use the down arrow to view all of the information.



$\bar{X}$ = mean

$\sum X$ = the sum of the data

$\sum X^2$ = the sum of the squares of the data

Sx = the sample standard deviation

σ_x = the population standard deviation

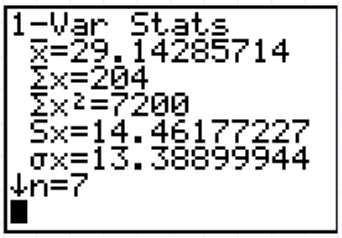
n = the sample size (# of pieces of data)

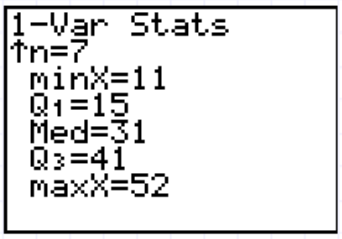
$\min X$ = the smallest data entry

Q_1 = data at the first quartile

med = data at the median (second quartile)

Q_3 = data at the third quartile



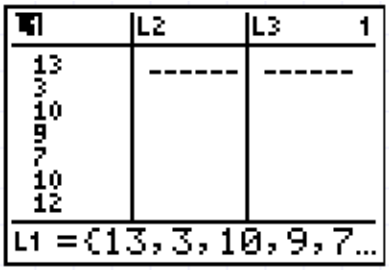


Mean, Mode, Median

Given the data set:{13, 3, 10, 9, 7, 10, 12, 8, 6, 3, 9, 6, 11, 5, 9, 13, 8, 7, 7}.
Find the mean, median and mode.

Enter the data into a list.

(See Basic Commands for entering data.)



Find the Mean and Median:

Method 1: *(fast and easy)*

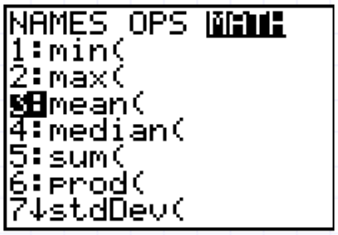
Press **2nd MODE (QUIT)** to return to the home screen.

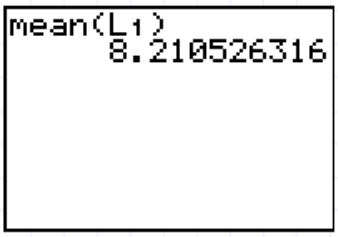
Press **2nd STAT (LIST).** Arrow to the right to **MATH.**

Choose option **#3: mean(** if you want the mean.
Choose option **#4: median(** if you want the median.

Your choice will appear on the home screen waiting for you to tell it which list to use.

Remember the List names appear on the face of the calculator above the number keys 1-6.





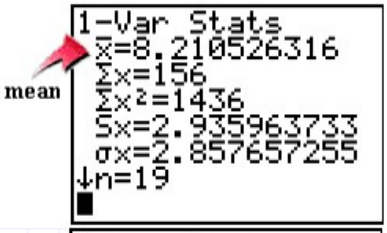
Find the Mean and Median:

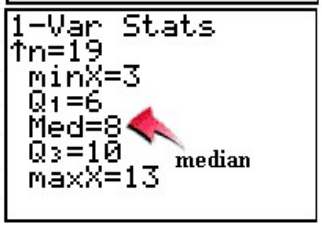
Method 2: *(a bit more sophisticated)*

Press **STAT.** Arrow to the right to **CALC.** Now choose option **#1: 1-Var Stats.**

When **1-Var Stats** appears on the home screen, tell the calculator the name of the list you are using (such as: **1-Var Stats L1)**
Press **ENTER.**

Arrow up and down the screen to see the statistical information about the data.





Find the Mode:

(While there is no specific calculator function to find the mode, the calculator is helpful in ordering the data so that you can find the mode easily.)

Sort the data into ascending or descending order to help find the mode.

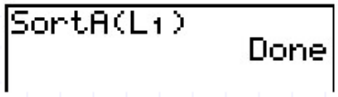
STAT, #2 SortA(, and specify **L1**, or the list you are using.

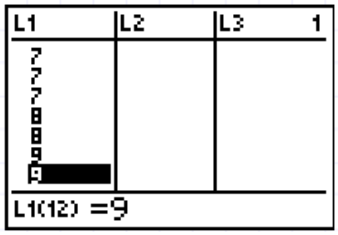
Look at the list (**STAT, #1 EDIT**).

Examine the data to see which value(s) appear(s) most often.

The mode for this data set is 7 and 9. Each of these values appears 3 times in the data set.







Also see the example on finding mean, median and mode with grouped data.

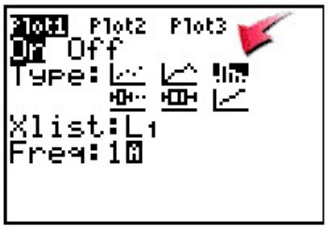
Histograms

Given the data set: {13, 3, 10, 9, 7, 10, 12, 8, 6, 3, 9, 6, 11, 5, 9, 10 13, 8, 7, 7}.
Create a histogram representing this data.

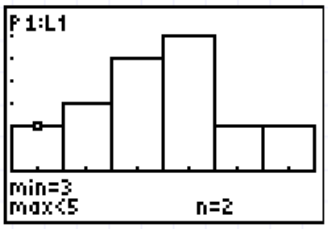
1. **CLEAR** out the graphs under **y** = (or turn them off).

2. Enter the data into the calculator lists. Choose **STAT, #1 EDIT** and type in entries. (See Basic Commands for entering data.)

3. To plot a histogram:
Press **2nd STATPLOT** and choose **#1 PLOT 1.** You should see the screen at the right. Be sure the plot is **ON**, the histogram icon is highlighted, and that the list you will be using is indicated next to **List.** **Freq: 1** means that each piece of data will be counted one time.



4. Controlling the graphical display of a histogram:
To see the histogram, press **ZOOM** and **#9 ZoomStat.** (**ZoomStat** automatically sets the window to an appropriate size to view all of the data.) Press the **TRACE** key to see on-screen data about the histogram. The spider will jump from bar to bar showing the range of values contained within each bar and the number of entries from the list (**n**) that fall within that range.



- Under your **WINDOW** button, the Xscl value controls the width of each bar beginning with **Xmin.** Choosing **ZoomStat** will automatically adjust **Xmin, Xmax, Ymin, Ymax,** and **Xscl.** (If you wish to see EACH piece of data as a separate interval, set the **Xscl** to 1.)
- **Integer values for Xscl** will be the easiest to read.
- If you wish to adjust your own viewing window, remember that **(Xmax-Xmin)/Xscl** must be less than or equal to 47 for the histogram to be seen in the viewing window.
- A value that occurs on the edge of a bar is counted in the bar to the right.

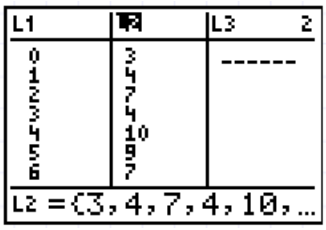
Frequency Tables

From a Frequency Table:

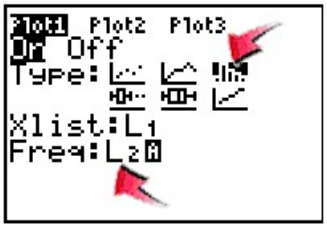
X	0	1	2	3	4	5	6	7	8	9	10
f	3	4	7	4	10	9	7	3	6	2	4

prepare a histogram representing this data.

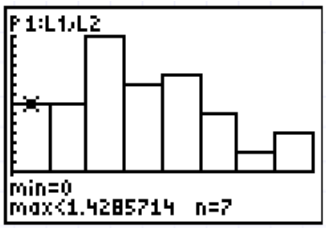
1. Enter the data values in **L1.** Enter their frequencies in **L2,** being careful that each data value and its frequency are entered on the same horizontal line. (See Basic Commands for entering data.)



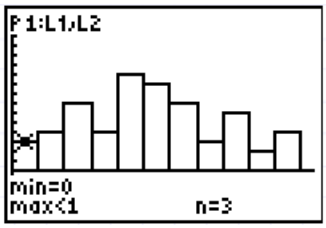
2. Activate the histogram. Press **2nd STATPLOT** and choose **#1 PLOT 1.** You will see the screen at the right. Be sure the plot is **ON**, the histogram icon is highlighted, and that the list you will be using is indicated next to **Xlist.** When using a Frequency Table set **Freq: L2** so that the number of times the data values appear will be determined by the numbers appearing in **L2.**



3. To see the histogram, press **ZOOM** and **#9 ZoomStat.** Press the **TRACE** key to see on-screen data about the histogram. The screen to the right shows the histogram developed directly from the **ZoomStat** choice of increments. Not so nice increments!



4. Adjusting the **Xscl** value to 1 (under **WINDOW**), gives a better representation of the data in this example. Much nicer increments!



Cumulative Frequency Histogram

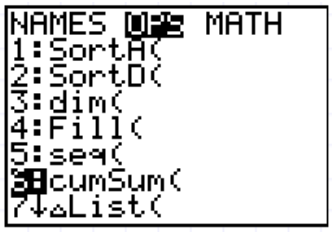
From a Frequency Table:

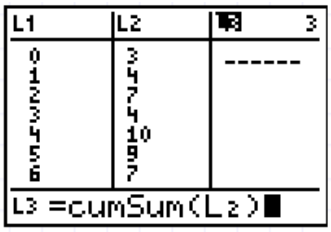
X	0	1	2	3	4	5	6	7	8	9	10
f	3	4	7	4	10	9	7	3	6	2	4

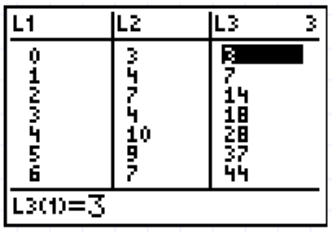
prepare a cumulative frequency histogram representing this data.

1. Enter the data values in **L1.** Enter their frequencies in **L2,** being careful that each data value and its frequency are entered on the same horizontal line. (See Basic Commands for entering data.)

2. Have the calculator prepare a cumulative sum of the values in **L2** and place the answers in **L3.** Move your cursor to **L3** and arrow up **onto L3.** Now, go to **LIST (2nd STAT),** arrow to the right to **OPS** and choose **#6 cumSum(.** Indicate that **L2** is the list that will create the cumulative sum. Press **ENTER.** The cumulative sum now appears in **L3.**

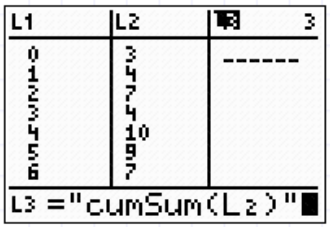




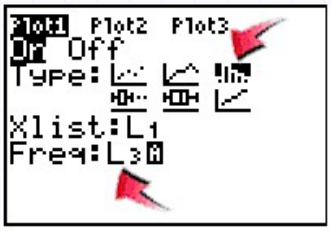


Hint: If you enter **cumSum(L_2)** in quotes, the sum will automatically update wherever **L_2** changes.

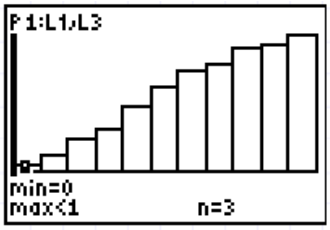
The quotes are found by pressing **ALPHA** and then the + key.



3. Activate the histogram. Press **2nd STATPLOT** and choose **#1 PLOT 1.** You will see the screen at the right. Be sure the plot is **ON,** the histogram icon is highlighted, and that the list you will be using is indicated next to **Xlist.** When preparing a Cumulative Frequency Histogram set **Freq: L3** so that the cumulative sum values for the data values will be determined by the numbers from **L3.**



4. To see the histogram, press **ZOOM** and **#9 ZoomStat**. Press the **TRACE** key to see on-screen data about the histogram.



Mean, Mode, Median with Grouped Data

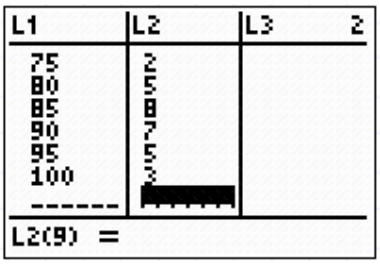
Find the mean, median and mode for this grouped data of test scores.

Scores	Frequency
65	2
70	3
75	2
80	5
85	8
90	7
95	5
100	3

This problem could be solved by entering ALL 35 scores into one list, with the score 65 appearing twice, 70 appearing 3 times, and so on. But if we deal with the data as we deal with a frequency histogram, we can accomplish our task more quickly.

Enter the scores into L_1 and the frequencies into L_2.

(See Basic Commands for entering data.)



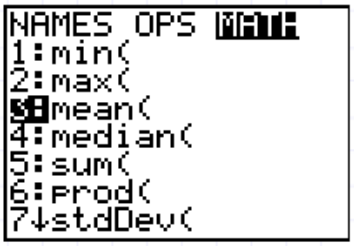
Find the Mean and Median:

Method 1: *(fast and easy)*

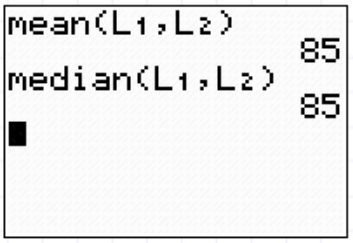
Press **2nd MODE (QUIT)** to return to the home screen.

Press **2nd STAT (LIST).** Arrow to the right to **MATH.**

Choose option **#3: mean(** if you want the mean.
Choose option **#4: median(** if you want the median.



Your choice will appear on the home screen waiting for you to tell it which list to use. This time we will tell the calculator the list containing the scores **AND** the list containing the frequencies. Notice the comma separating the lists.



Remember the List names appear on the face of the calculator above the number keys 1-6.

Find the Mean and Median:

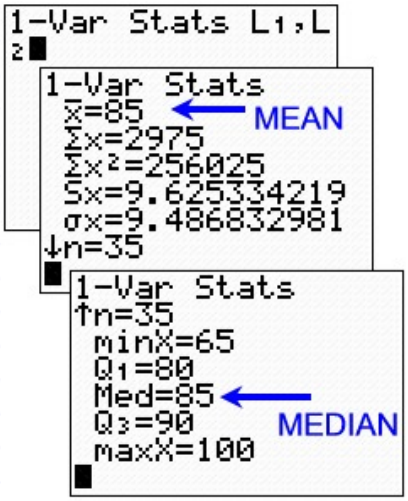
Method 2: *(a bit more sophisticated)*

Press **STAT.** Arrow to the right to **CALC.** Now choose option **#1: 1-Var Stats**.

When **1-Var Stats** appears on the home screen, tell the calculator the name of the list containing the scores **AND** the name of the list containing the frequency
(such as: **1-Var Stats L1, L2)**

Press **ENTER.**

Arrow up and down the screen to see the statistical information about the data.



Find the Mode: The mode can be quickly determined by examining the chart to see which score occurred most often. No calculator work needed.

The mode is 85.

Mean Absolute Deviation

The Mean Absolute Deviation (MAD) of a set of data
is the average distance between each data value and the mean

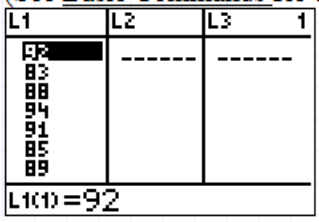
The steps to find the MAD include:
1. find the mean (average)
2. find the difference between each data value and the mean
3. take the absolute value of each difference
4. find the mean (average) of these differences

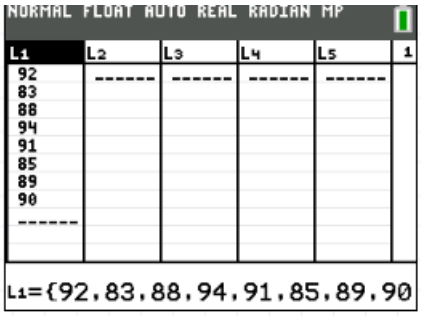
While we could work through each of these steps on the calculator's home screen, let's, instead, try to utilize the calculator's features to minimize our work. We will look at two different approaches to find the population MAD.

$$\text{Population MAD} = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$$

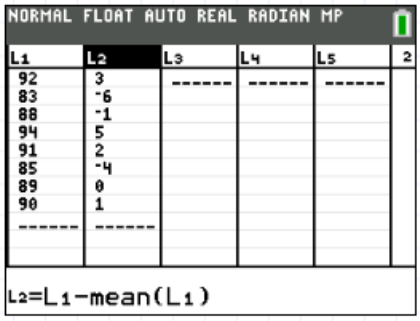
Data Set: {92, 83, 88, 94, 91, 85, 89, 90}.
Find the Mean Absolute Deviation.

Method 1: *(doing one step at a time with the lists = easy)*
Enter the data:
Step 1: Enter the data into a list **(L1)**
(See Basic Commands for entering data.)





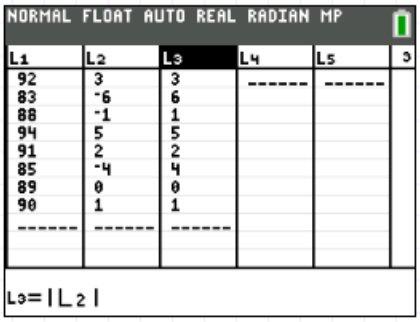
Calculate the differences:
Step 2: Tell **L2** to calculate the values from **L1** minus the mean of **L1**.
Arrow up onto **L2**.
Enter **L1 - mean(L1)** (you can find "mean" in the Catalog)
[**mean(** is also found in **2nd LIST, MATH, #3 mean(**]
Hit **ENTER.**
Notice that if you add the sum of the positive differences and the negative differences, you will get zero.



Take the absolute value of the differences:

Step 3:
Arrow up onto **L3.**
Enter **abs(L1)** (you can find "abs" in the Catalog)
Hit **ENTER.**

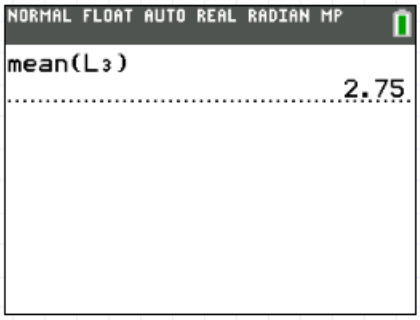
(This step could have been combined with Step 2.)



Find the mean (average) of the absolute values:

Step 3:
Go to the **HOME** screen. (**2nd QUIT**)
Enter **mean(L3).**
Hit **ENTER.**

This value is the Mean Absolute Deviation (MAD).

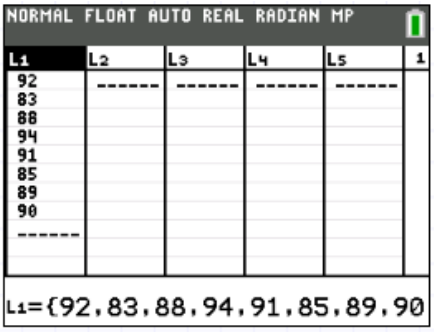


Since the MAD is small, it implies that the mean of 89 is indicative of the other values within the data set.

Method 2: *(doing most work in one statement - harder to remember the locations of the functions)*

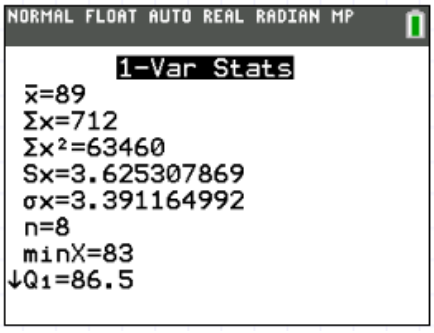
Enter the data:

Step 1: Enter the data into a list **(L1)**
(See Basic Commands for entering data.)



Have the calculator compute the mean and the count of entries.

Step 2: Hit **STAT → CALC**
Choose **#1 1-Var Stats**
On the home screen: **#1 1-Var Stats L1**
Hit **ENTER.**
*The calculator now has stored the value for the mean, $\bar{x}$, and it has the count of the number of entries in the list, **n**.*



Compute the answer using one formula entry:

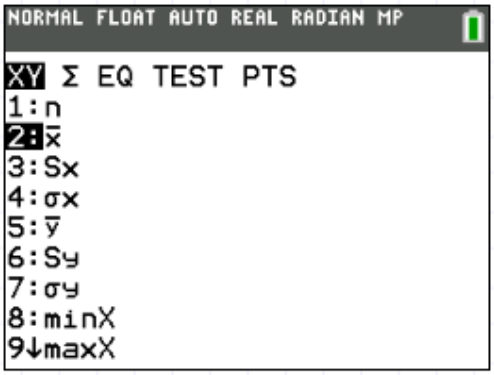
Step 3:
- Go to the **HOME** screen.
- left parenthesis
- **2nd LIST → MATH #5 sum(**
- from Catalog, **abs(**

- Enter **L1** (**2nd** and the key for "**1**")
- Subtraction symbol
- **VARS, #5 Statistics, #2** $\bar{x}$
- Division symbol
- **VARS, #5 Statistics, #1 n**

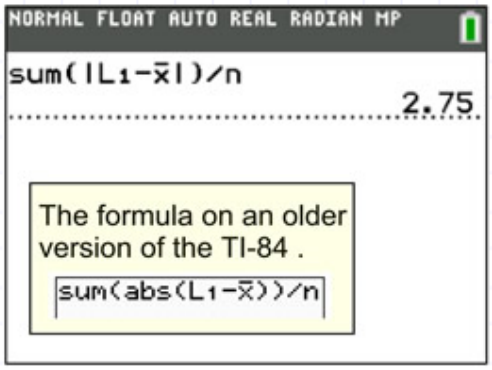
Hit **ENTER.**



2nd LIST → MATH #5 sum(



VARS, #5 Statistics, #2 $\bar{x}$



Hit **ENTER**

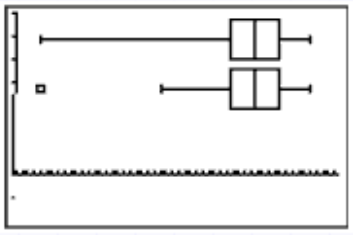
NOTE: When adding the entries in a list, be sure to use the **sum(** function which appears under **LIST - MATH.** Do not use the sigma notation ∑ from **ALPHA-WINDOW**, as it does not understand how to access numbers from a list.

Box and Whisker Plots

Given the data set:{85, 100, 97, 84, 73, 89, 73, 65, 50, 83, 79, 92, 78, 10}.
Create a box and whisker plot to represent this data.

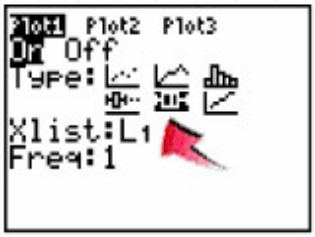
1. **CLEAR** out the graphs under **y =** (or turn them off).

2. Enter the data into the calculator lists.
Choose **STAT, #1 EDIT** and type in entries.
(See Basic Commands for entering data.)



3. Two icons for Box-and-Whisker Plots:

Choose the **second icon** for beginning level work.



Press **2nd STATPLOT** and choose **#1 PLOT 1.** You should see the screen at the right. Be sure the plot is ON, the second box-and-whisker icon is highlighted, and that the list you will be using is indicated next to **Xlist.** **Freq: 1** means that each piece of data will be counted one time.

What about that other icon?

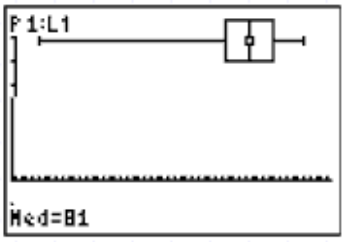
The first box-and-whisker icon is the modified box plot dealing with **outliers**. This modified version will not plot points that are 1.5*IQR beyond the quartiles. These points, called **outliers**, are plotted as individual points beyond the whisker in an attempt to give a more accurate picture of the dispersion of the data. Notice the two plots displayed at the top of this page representing the same set of data.
NOTE: IQR stands for the Interquartile Range which is Q3 – Q1.



Used in more advanced statistics.

4. Seeing the graph:

To see the box-and-whisker plot, press **ZOOM** and **#9 ZoomStat.** Press the **TRACE** key to see on-screen data about the box-and-whisker plot. The whiskers extend from the minimum data point in the set to the first quartile, and from the third quartile to the maximum point. The box itself is defined by Q1, the median and Q3. The spider will jump from the minimum value to Q1, to median, to Q3 and to the maximum value.



5. Placement of the graph:

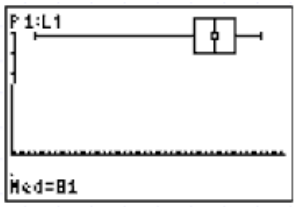
The calculator ignores y-values when plotting box and whiskers. You can plot up to 3 box-and-whisker plots on one screen display. The first will be at the top of the screen, the second in the middle and the third at the bottom.

Five Numbers Summary

The "**five numbers summary**", or **five statistical summaries**", consists of (1) the minimum, (2) the maximum, (3) the median, (4) the first quartile and (5) the third quartile.

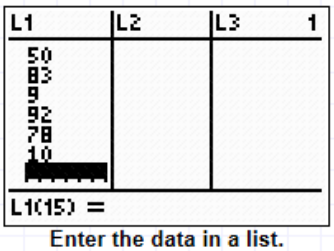
Find the "five numbers summary" for the data set:
{85, 100, 97, 84, 73, 89, 73, 65, 50, 83, 79, 92, 78, 10}.

Method 1. A box and whisker plot will clearly show a five number summary. See box-and-whiskers for information on creating these graphs.

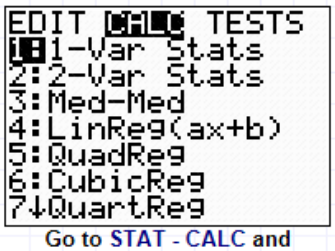


Pressing the **TRACE** key will display the values. The whiskers extend from the minimum data point in the set to the first quartile, and from the third quartile to the maximum point. The box itself is defined by Q1, the median and Q3. The spider will jump from the minimum value to Q1, to median, to Q3 and to the maximum value.

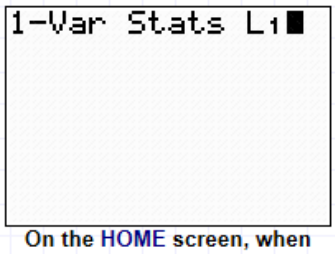
Method 2. A complete five number summary is displayed on the lower portion of the **1-Var Stats** screen.



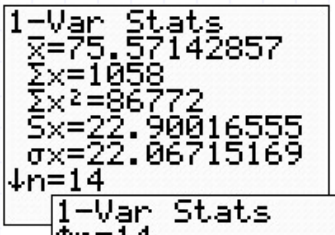
Enter the data in a list.

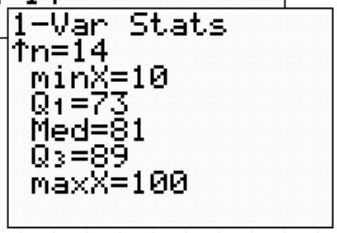


Go to **STAT - CALC** and choose **1-Var Stats**



On the **HOME** screen, when **1-Var Stats** appears, type the list containing the data.





Hit **ENTER**.

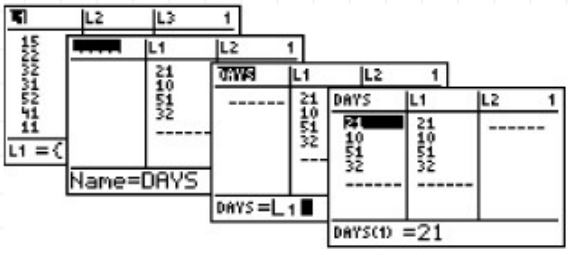
When the **1-Var Stats** information appears, notice that there is an arrow pointing downward at the bottom of the screen. Arrow down.

The **five number summary** is the listed as the last 5 items on this screen.

Additional Tidbits of Information

Naming a List:

You may create your own names for lists. Highlight **L1**. Choose **INS (2nd DEL)**. Enter the name up to 5 letters. Press **ENTER**. **L1** will not be lost, a new list will be created. To transfer the data from **L1** to the new list, highlight **DAYS**, enter **L1**, press **ENTER**.



To delete this new list – highlight **DAYS** and press **DEL**. This will remove the column listing but not the list from memory. To delete the list from memory, choose **2nd MEM, #2 Mem Mgmt/Del, #4 List**, arrow down to **DAYS**, and press **DEL**.

If L1 - L6 disappear from your listings:

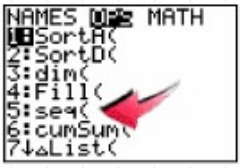
If any of your lists, **L1** through **L6**, should disappear from your columns, choose **STAT, #5 SetUpEditor.** This will restore all lists **L1** through **L6**. It will not remove data in residence.



To Automatically Fill a List:

(when the entries are the result of the evaluation of an expression)

Highlight **L1**. Choose **LIST (2nd STAT)**. Choose **OPS** from the top. Choose **#5 seq(** . Type **(x, x, 0, 10, 1)** to automatically generate numbers from 0 to 10. The parameters are (expression, variable, begin, end, increment).



Other Valuable Options from the OPS menu:

#4 Fill(- replaces each value in a list with a constant value - **Fill (8,L3)** will fill **L3** with 8s. If **L3** is empty you will need to dimension the list first so that the calculator knows how many constants to create.

5→dim(L3) tells the calculator that **L3** will contain 5 values.

#6 cumSum(- returns cumulative sums of the elements in the list, starting with the first element - If **L1** contains **{1,2,3,4,5}**, then **cumSum(L1)** will return **{1,3,6,10,15}**

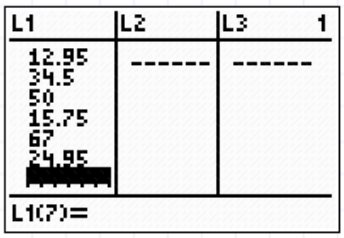
To Add the Entire List:

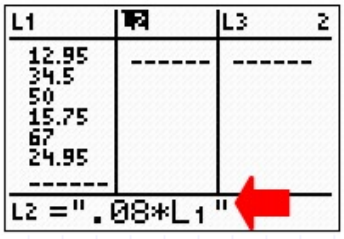
From the home screen, choose **LIST (2nd STAT)**. Choose **MATH** at the top. Option **#5, sum(**, will add all of the elements in the list.

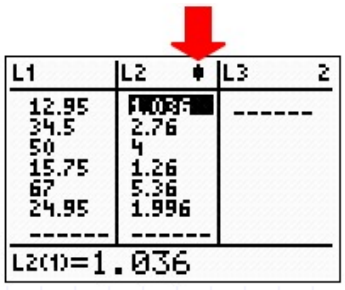
sum(L1) will add all of the elements in **L1**.

To Create a Dynamic List: (a list in L2, based upon L1, that will change automatically when L1 is updated.)

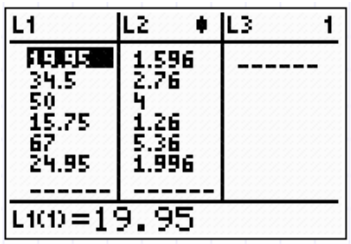
Create **L1** to contain the prices {12.95, 34.50, 50, 15.75, 67, 24.95}.
Create **L2** to contain the 8% sales tax on each of these prices.







Using the **quotes** (ALPHA +) when entering the formula for **L2** will create a dynamic situation where **L2** will automatically update when **L1** changes. Notice the marker placed at the top of **L2** designating that this list is dynamically created.

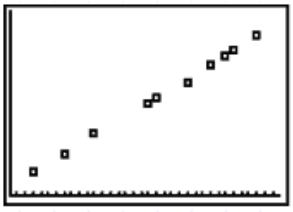


Now, if 12.95 in **L1** is changed to 19.95, the sales tax in **L2** is automatically updated.

Scatter Plots

A scatter plot is a graph used to determine whether there is a relationship between paired data.

In many real-life situations, scatter plots follow patterns that are approximately linear. If y tends to increase as x increases, then the paired data are said to be a **positive correlation**. If y tends to decrease as x increases, the paired data are said to be a **negative correlation**. If the points show no linear pattern, the paired data are said to have **relatively no correlation.**



To set up a scatter plot:

Clear (or deactivate) any entries in "Y=" before you begin.

1. Enter the X data values in **L1**. Enter the Y data values in **L2**, being careful that each X data value and its matching Y data value are entered on the same horizontal line. (See Basic Commands for entering data.)



2. Activate the scatter plot. Press **2nd STATPLOT** and choose **#1 PLOT 1.** You will see the screen at the right. Be sure the plot is **ON**, the scatter plot icon is highlighted, and that the list of the X data values are next to **Xlist**, and the list of the Y data values are next to **Ylist.** Choose any of the three marks.

3. To see the scatter plot, press **ZOOM** and **#9 ZoomStat**. Hitting **TRACE** and right arrow will move along the data points.

4. To turn the scatter plot off, when you are finished with this problem:

Method 1: Go to the **Y=** screen. Arrow up onto the **PLOT** highlighted at the top of the screen. Press **ENTER** to turn it off.

Method 2: Go to **STAT PLOT** (above **Y=**). Choose your **PLOT** location. Arrow to **OFF**. Press **ENTER** to turn it off.

Follow-up:

* At this point, the graph may be observed for the existence of a positive, negative or no correlation between the data.

* A line of best fit can be calculated "manually".
  1. Select two points that you feel would give a line that fits the data.
  2. Using your knowledge of equations of lines and slope, write the equation of your line.
  3. Enter this equation into Y1 and graph.
  4. How well does the line "fit" the data?
  5. Use your line to make predictions.

* Or a line of best fit can be calculated "using the calculator".
  See Line of Best Fit.

Line of the Best Fit

A **line of best fit** (or "trend" line) is a straight line that best represents the data on a scatter plot. This line may pass through some of the points, none of the points, or all of the points.

You can examine lines of best fit with:
1. paper and pencil only,
2. a combination of graphing calculator and paper and pencil,
3. or solely with the graphing calculator.

Example: Is there a relationship between the fat grams and the total calories in fast food?

Sandwich	Total Fat (g)	Total Calories
Hamburger	9	260
Cheeseburger	13	320
Quarter Pounder	21	420
Quarter Pounder with Cheese	30	530
Big Mac	31	560
Arch Sandwich Special	31	550
Arch Special with Bacon	34	590
Crispy Chicken	25	500
Fish Fillet	28	560
Grilled Chicken	20	440
Grilled Chicken Light	5	300

Paper and Pencil Solution:

Can we predict the number of total calories based upon the total fat grams?

1. Prepare a scatter plot of the data on graph paper.
2. Using a strand of spaghetti, position the spaghetti so that the plotted points are as close to the strand as possible.
3. Find two points that you think will be on the "best-fit" line.
4. We are choosing the points (9, 260) and (30, 530). You may choose different points
5. Calculate the slope of the line through your two points (rounded to three decimal places).

$$\frac{530-260}{30-9}=\frac{270}{21}\approx 12.857$$

6. Write the equation of the line.

$$y - y_1 = m(x - x_1)$$
$$y - 260 = 12.857(x - 9)$$
$$y = 12.857(x - 9) + 260$$

7. This equation can now be used to predict information that was not plotted in the scatter plot.
Question: Predict the total calories based upon 22 grams of fat.

$$y = 12.857(22 - 9) + 260$$
$$y = 12.857(13) + 260$$
$$y = 427.141$$

ANS: 427.141 calories

Predicting:
- If you are looking for values that fall within the plotted values, you are ***interpolating.***
- If you are looking for values that fall outside the plotted values, you are ***extrapolating.*** **Be careful** when extrapolating. The further away from the plotted values you go, the less reliable is your prediction.

In step 4 above, we chose two points to form our line-of-best-fit. It is possible, however, that someone else will choose a different set of points, and their equation will be slightly different.

Your answer will be considered CORRECT, as long as your calculations are correct for the two points that you chose. So, if each answer may be slightly different, which answer is the REAL "line-of-best-fit?

So who has the REAL "line-of-best-fit"?

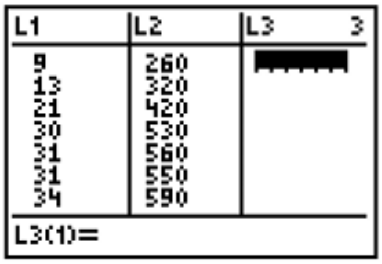
To answer this question, we need the assistance of a graphing calculator. We saw that different people may choose different points and arrive at slightly different equations for their lines of best fit. All of them are "correct", but which one is actually the "best"? Simply stated, the graphing calculator has the capability of determining which line will "actually" represent the REAL line-of-best-fit.

Graphing Calculator Solution:

Can we predict the number of total calories based upon the total fat grams?

1. Enter the data in the calculator lists. Place the data in $\mathbf{L_1}$ and $\mathbf{L_2}$.

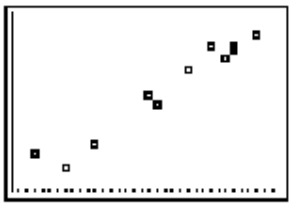
STAT, #1Edit, type values into the lists

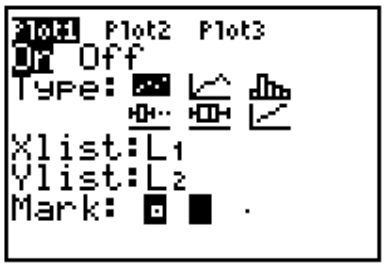


2. Prepare a scatter plot of the data. Set up for the scatterplot.

2nd StatPlot - choices shown at right.

Choose **ZOOM #9 ZoomStat**. Graph shown below.



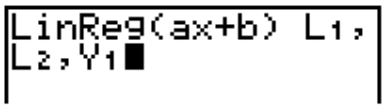


3. Have the calculator determine the line of best fit.

STAT → CALC #4 LinReg(ax+b)

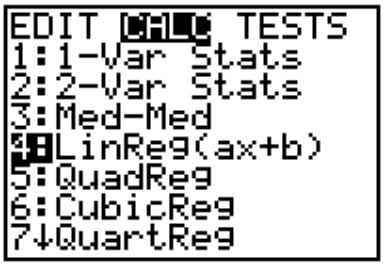
Include the parameters **L_1, L_2, Y_1.**

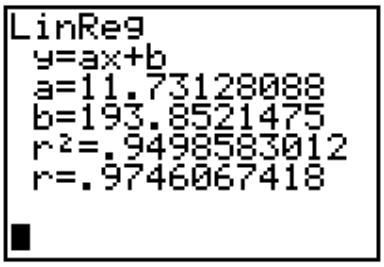
(**Y_1** comes from **VARS → YVARS, #Function, Y_1**)



You now have the values of *a* and *b* needed to write the equation of the **line of best fit.** See values at the right.

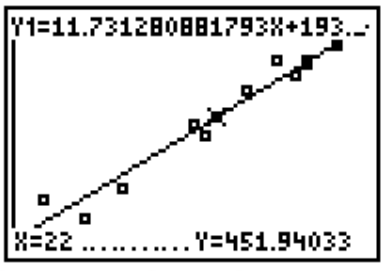
$y = 11.73128088x + 193.8521475$



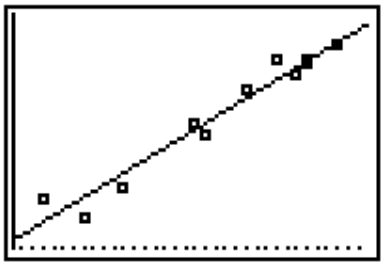


4. Graph the line of best fit. Simply hit **GRAPH**.

To get a predicted value **within the window**, hit TRACE, up arrow, and type the desired value.



The screen above shows $x = 22$.



Question: Predict the total calories based upon 22 grams of fat.

ANS: 451.940 calories

Compare this answer with the answer we got by hand.

Calculate Regression Models

A mathematical model is an equation that best describes a particular set of paired data.
These mathematical models are referred to as regression models and are used to predict one variable based upon another variable.

The linear based regression models on the graphing calculator:

• Linear (LinReg)	$y = ax + b$	The graph of x versus y is linear.

Fits Linear by Transformations:		
• Logarithmic (LnReg)	$y = a + b\ \ln(x)$	The graph of $\ln(x)$ versus y is linear. Calculates a and b using linear least squares on lists of $\ln(x)$ and y instead of x and y.
• Exponential (ExpReg)	$y = a\ (b^{x})$	The graph of x versus $\ln(y)$ is linear. Calculates A and B using linear least squares on lists of x and $\ln(y)$ instead of x and y, and then $a = e^{A}$ and $b = e^{B}$.
• Power (PwrReg)	$y = a\ (x^{b})$	The graph of $\ln(x)$ versus $\ln(y)$ is linear. Calculates A and b using liner least squares on list of $\ln(x)$ and $\ln(y)$ instead of x and y, and then $a = e^{A}$.

Other models available on the graphing calculator:

• Quadratic (QuadReg)	$y = ax^2 + bx + c$	For three points, fits a polynomial to the data. For more than three points, fits a polynomial regression.
• Cubic (CubicReg)	$y = ax^3 + bx^2 + cx + d$	For four points, fits a polynomial to the data. For more than four points, fits a polynomial regression.
• Quartic (QuartReg)	$y = ax^4 + bx^3 + cx^2 + dx + e$	For five points, fits a polynomial to the data. For more than five points, fits a polynomial regression.
• Logistic (Logistic)	$y = \frac{c}{(1 + ae^{-bx})}$	Fits equation to data using iterative least-squares fit.
• Sinusoidal (SinReg)	$y = a\sin(bx + c) + d$	Fits sine wave to data using iterative least-squares fit.

Rules for Developing a Model

Since mathematical models (regression models) are often used to predict the relationship between paired data elements, it is important to understand how to choose a model that will be a "good fit" for the particular data set.

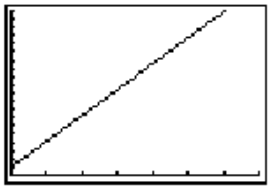
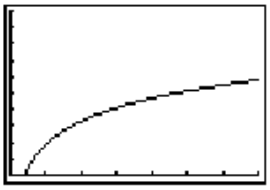
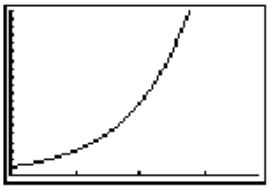
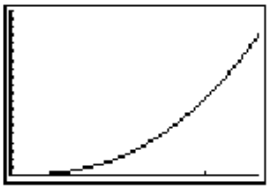
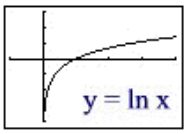
There are several things to keep in mind when attempting to develop a model that will be a "good fit":

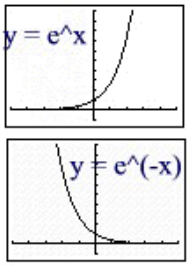
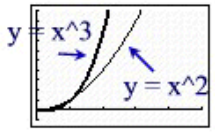
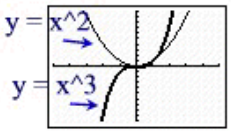
1. Visually compare the graph of the data to the graph of the model. (Look for a pattern from the graph.)

Prepare a scatter plot and examine the graph. Look to see which regression model appears to best represent the scatter plot graph. Know the general shapes of the regression models. When trying to select a model, choose only those models that appear to fit the observed points reasonably well. Extend the WINDOW to see how the regression equation behaves at higher x-values.

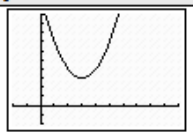
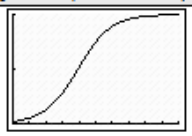
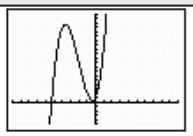
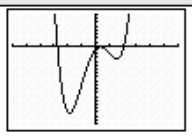
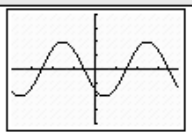
Linear based regression models:

(Other representations of these shapes may also exist due to the different natures of the data.)

Linear $y = ax + b$ (or $y = a + bx$)	Logarithmic $y = a + b \ln x$	Exponential $y = ab^x$	Power $y = ax^b$
			
Does the plotted data resemble a straight line? • The slope may be either positive or negative. • Linear associations are the most popular because they are easy to read and interpret. *NOTE:* See LinReg($ax+b$) versus LinReg($a+bx$) for an explanation of the	Does the plotted data ascend rapidly at the left but level off toward the right? Remember the shape of the natural logarithmic function crossing the x-axis at one and domain $x > 0$.  	Does the plotted data appear to grow (or decline) by percentage increases (decreases)? • Useful for values that grow by percentage increases. • Often deals with growth of populations, bacteria, radio-active decay, etc. Remember the shape of	Does the plotted data possess characteristics not seen in the first three models? Not a straight line, but a more gradual change than exponential? • Power functions are of the form $y = ax^b$. Remember the nature of such graphs when the exponent is odd and even.

"differences" between these two choices on the graphing calculator.		the exponential function, crossing the y-axis at one and range $y > 0$ 	First quadrant:  Outside first quadrant: 

Other regressions:

Quadratic $y = ax^2 + bx + c$	Logistic $y = c/(1 + ae^{-bx})$	Cubic	Quartic	Sinusoidal $y = a\sin(bx + c) + d$
 Modified version of power model.	 S-shaped	 Modified version of power model.	 Modified version of power model.	 Remember the periodic nature of such graphs.

2. Calculate a correlation coefficient, r (for some models).

The correlation coefficient measures the strength and the direction of a linear relationship between two variables. A value of $|r|$ near one may indicate a "good fit".

3. Calculate a coefficient of determination, r^2 (R^2).

The coefficient of determination represents the percent of the data that is the closest to the line of best fit. For example, if $r = 0.922$, then $r^2 = 0.850$, which means that 85% of the total variation in y can be explained by the linear relationship between x and y (as described by the regression equation). The other 15% of the total variation in y remains unexplained.

Do not place too much importance on small differences between r^2 values, such as $r^2 = 0.987$ and $r^2 = 0.984$. Also, keep in mind that r, r^2 and R^2 values cannot be directly compared when calculating certain regression models.

4. Examine the residuals.

Examine the scatter plot of the residuals, which depicts the measure of the signed distances between the actual data values and the outputs predicted by the model. A good linear model has residuals that are near zero and are randomly distributed.

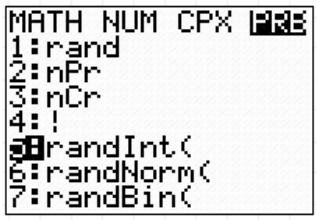
5. Think about your answer.
Is your choice realistic? Don't use a model that will lead to predicted values that are totally unrealistic.

Each time you compute a new regression model, the previously computed model is lost. If you compute a linear model and then compute an exponential model, your information pertaining to the linear model is lost. You will need to re-compute the linear information if you wish to investigate that model further.

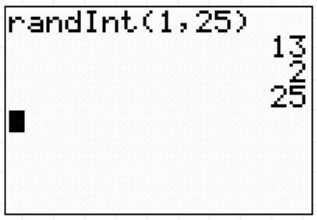
Generating Random Numbers

Random numbers can be quickly and easily generated by using the graphing calculator. Let's start our investigation by looking at generating random integers.

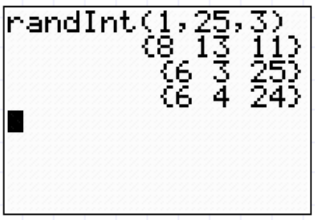
Generating Random Integers on the Home Screen: (good for games)



Go to **MATH → PRB**
Choose **#5 randInt(**

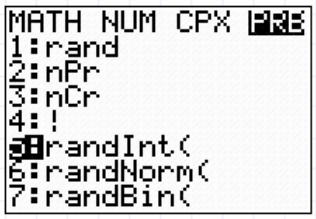


From the home screen, enter the smallest value needed, followed by the largest value. Hitting **ENTER** will generate the random integers. (Random values may repeat.)
This example generates random numbers from 1 to 25 (good for Bingo).

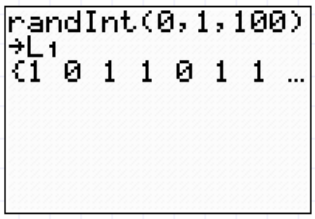


Adding a third parameter indicates the number of random integers that will appear on the screen at one time.

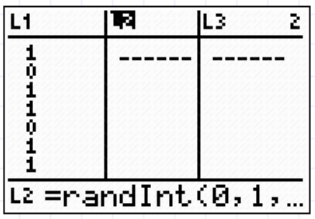
Generating Random Integers in Lists: (good for statistical studies)



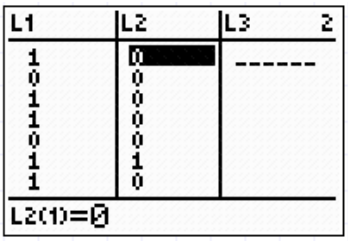
Go to **MATH → PRB**
Choose **#5 randInt(**



From the home screen, enter the **randInt** followed by the smallest value in the desired range, the largest value, and the number of terms needed. The results are stored (**STO**) into List 1.
This example stores 100 random integers from 0 to 1 in L1 to simulate the toss of a coin.



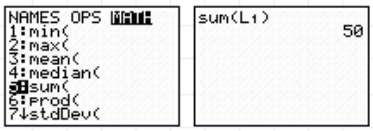
OR, from the list screen, arrow up onto **L2**, and type **randInt(0,1,100)**. Hit **ENTER**. Be sure to enter the third parameter so the calculator will know "how many" numbers to place in the list.



Such lists can be used to simulate the toss of one (or more) fair coin(s). The number of entries represents the number of tosses. An even random number represents heads, while an odd number represents tails.

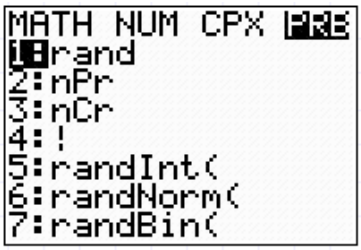
If tossing one coin, use **sum** command to count the number of heads, where heads are 1, and tails are 0.

2nd STAT - MATH - #5 sum

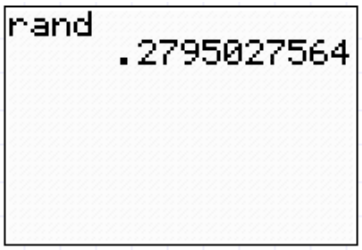


Using the rand command: (not integers)

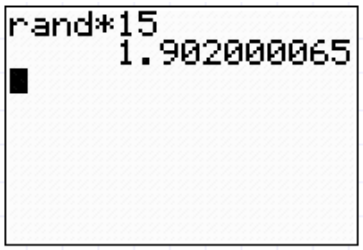
The **rand** (**MATH → PRB #1 rand**) command will generate random values, **not integers.**



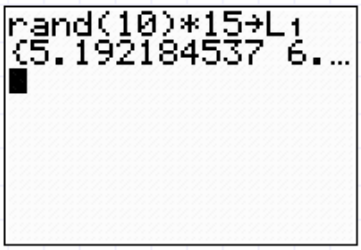
MATH → PRB #1 rand



The **rand** command will create a random number **between 0 and 1**.



To generate a random number between 0 and 15, enter **rand*15**.

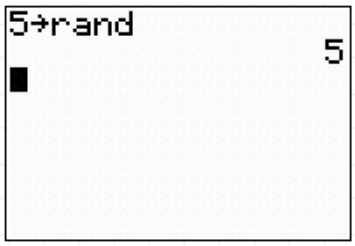


This last entry shows how to generate a list of 10 random numbers between 0 and 15 and store them in List 1.

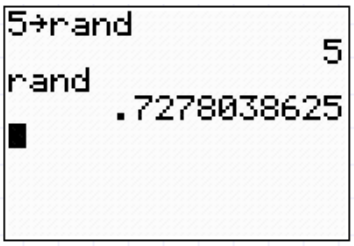
Re-Seeding the Random Number Generator:

Calculators (and computers) are not capable of creating "truly random" numbers. They create what are called "pseudo-random" numbers, meaning they use a formula to create the values. To engage this formula, the calculator uses a starting value, called a "seed", and then creates the random numbers based upon this seed. If two calculators start with the same seed value, they will generate the same sequence of random values.

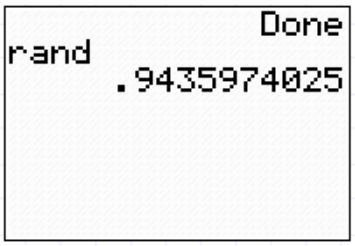
If you wish to ensure that each student in the class has a different set of random numbers, assign a different number to each student as their seed value. You could also have the students enter their birth date as the random seed (04021990), assuming no two students have the same birth date.



If you wish, you can control the starting "seed" value.
To seed the random generator, choose a seed value and store it into the rand command.



Now, start generating your random values.
If you feed two calculators the same seed value, they will each produce the same result when rand is entered.



After running a **RESET (DEFAULTS)** the calculator will return to using its default seed value. Engaging **rand** will always produce the value seen above. This same value will appear on all TI-84+ calculators after a reset.

Turn on the Diagnostics Flag

The Diagnostic flag must be "on" for you to see your r and r^2 values when examining a regression equation.

Note: When your calculator is reset (or the default is set), your Diagnostic flag will be turned off. You will need to turn your Diagnostics back on.



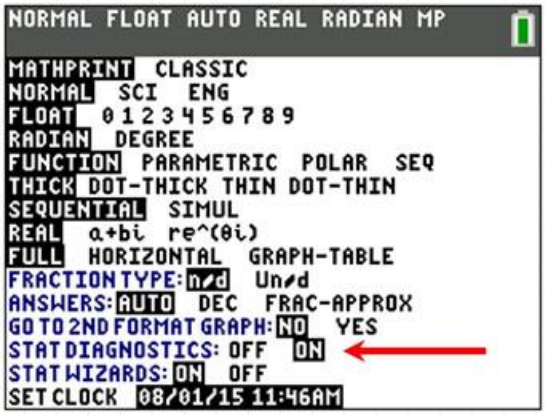
To turn the Diagnostics on:

1. Press 2nd CATALOG (above the numeral zero) to display the Catalog in alpha mode (note the A in the upper right hand corner).

2. Press D (to fast forward to the D's) and use the down arrow ▼ to move the pointer to DiagnosticOn.

3. Press ENTER. DiagnosticOn will appear on the home screen. Press ENTER and "Done" will appear.

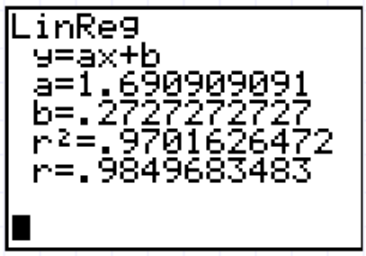
With the newer operating systems, you can control the Diagnostics Flag under MODE:



Correlation Coefficient

How well does your regression equation truly represent your set of data?

One of the ways to determine the answer to this question is to exam the *correlation coefficient* and the *coefficient of determination.*



The correlation coefficient, r, and the coefficient of determination, r^2, will appear on the screen that shows the regression equation information (be sure the Diagnostics are turned on --- 2nd Catalog (above 0), arrow down to DiagnosticOn, press ENTER twice.)

In addition to appearing with the regression information, the values r and r^2 can be found under VARS, #5 Statistics → EQ #7 r and #8 r^2.

Correlation Coefficient, *r* :

- The quantity r, called the *linear correlation coefficient*, measures the strength and the direction of a linear relationship between two variables. The linear correlation coefficient is sometimes referred to as the *Pearson product moment correlation coefficient* in honor of its developer Karl Pearson.
- The mathematical formula for computing ***r*** is:

$$r = \frac{n\sum xy - \left(\sum x\right)\left(\sum y\right)}{\sqrt{n\left(\sum x^2\right) - \left(\sum x\right)^2}\sqrt{n\left(\sum y^2\right) - \left(\sum y\right)^2}}$$

where n is the number of pairs of data.

(Aren't you glad you have a graphing calculator that computes this formula?)

- The value of r is such that $-1 \le r \le +1$. The + and – signs are used for positive linear correlations and negative linear correlations, respectively.
- *Positive correlation:* If x and y have a strong positive linear correlation, r is close to +1. An r value of exactly +1 indicates a perfect positive fit. Positive values indicate a relationship between x and y variables such that as values for x increases, values for y also increase.
- *Negative correlation:* If x and y have a strong negative linear correlation, r is close to -1. An r value of exactly -1 indicates a perfect negative fit. Negative values indicate a relationship between x and y such that as values for x increase, values for y decrease.

- *No correlation:* If there is no linear correlation or a weak linear correlation, r is close to 0. A value near zero means that there is a random, nonlinear relationship between the two variables
- Note that r is a dimensionless quantity; that is, it does not depend on the units employed.
- A *perfect* correlation of ± 1 occurs only when the data points all lie exactly on a straight line. If $r = +1$, the slope of this line is positive. If $r = -1$, the slope of this line is negative.
- A correlation greater than 0.8 is generally described as *strong*, whereas a correlation less than 0.5 is generally described as *weak*. These values can vary based upon the "type" of data being examined. A study utilizing scientific data may require a stronger correlation than a study using social science data.

Coefficient of Determination, r^2 or R^2 :

- The *coefficient of determination,* r^2, is useful because it gives the proportion of the variance (fluctuation) of one variable that is predictable from the other variable. It is a measure that allows us to determine how certain one can be in making predictions from a certain model/graph.
- The *coefficient of determination* is the ratio of the explained variation to the total variation.
- The *coefficient of determination* is such that $0 \leq r^2 \leq 1$, and denotes the strength of the linear association between x and y.
- The *coefficient of determination* represents the percent of the data that is the closest to the line of best fit. For example, if $r = 0.922$, then $r^2 = 0.850$, which means that 85% of the total variation in y can be explained by the linear relationship between x and y (as described by the regression equation). The other 15% of the total variation in y remains unexplained.
- The *coefficient of determination* is a measure of how well the regression line represents the data. If the regression line passes exactly through every point on the scatter plot, it would be able to explain all of the variation. The further the line is away from the points, the less it is able to explain.

Correlation Does Not Equal Causation!

You were given the data at the right comparing the weight of cars in pounds with their highway gas mileage. You found a linear regression equation and determined that your model was a good fit.

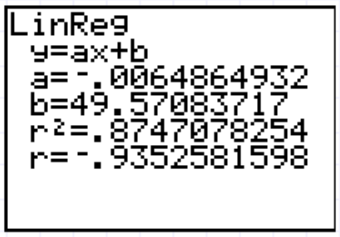
So, you now state for the whole world to hear that heavier cars get less gas mileage. Right???

Not necessarily. Your statement may be correct for this particular set of data, but it may not be a universal truth.

It may also be true that the weight of the car has nothing to do with the gas mileage. Perhaps some other factor is affecting the gas mileage.

Just because a correlation exists does not guarantee that the change in one of your variables is causing the change in the other variable.

Car Weight in Pounds	Gas Mileage MPG (highway)
3489	28
3955	25
3345	27
3085	29
4915	18
4159	21
4289	20
3992	26



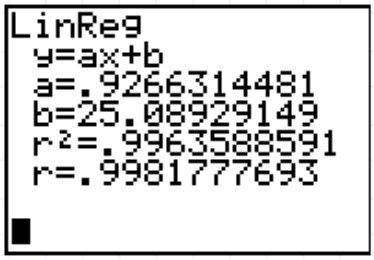
Consider this situation:

During the months of March and April, the weekly weight increases of a puppy in New York were collected. For the same time frame, the retail price increases of snowshoes in Alaska were collected. The data was examined and was found to have a very strong linear correlation.

So, this must mean that the weight increase of a puppy in New York is causing snowshoe prices in Alaska to increase. Of course this is not true!

The moral of this example is: "be careful what you infer from your statistical analyses." Be sure your relationship makes sense. Also keep in mind that other factors may be involved in a cause-effect relationship.

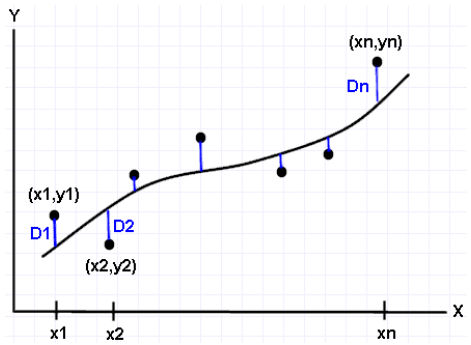
Weekly Data Collection	
The weight of a growing puppy in New York	The retail price of snowshoes in Alaska
8 pounds	$32.45
8.5	$32.95
9	$33.45
9.6	$34.00
10.1	$34.50
10.7	$35.10
11.5	$35.63



Residuals and Least Squares

Developing a Model by Observation:
A simple type of regression equation is a straight line. A scatter plot of the data is drawn, two points are chosen that "appear" to lie on the line of best fit, the slope is determined and an equation is written. This is known as a freehand method of curve fitting. Unfortunately, different observers, who choose different points, may obtain different equations.

Developing a Model by Least Squares:
To avoid individual judgment in curve fitting, it is necessary to agree on a definition of a "best-fitting line" or curve. Consider the following set of points:



For a given value of *x*, say *x*1, there will be a difference between the value *y*1 and the corresponding value as determined by the "best fitting" curve. This distance, D1, is referred to as a *residual*.

A *residual* is the difference from the actual *y*-value and the value obtained by plugging the *x*-value (that goes with the *y*-value) into the regression equation.

Using these residuals, the following definition has been developed:

Definition:

Of all curves approximating a given set of data points, the curve having the property that

$$D_1^2 + D_2^2 + \ldots + D_N^2$$

is a minimum is called a *best-fitting curve*.

A curve having this property is said to fit the data in the least-squares sense and is called a *least-squares curve*.

The graphing calculator uses this least squares process to determine regression models. When regression models are computed, residuals are automatically stored in a list called **RESID**.

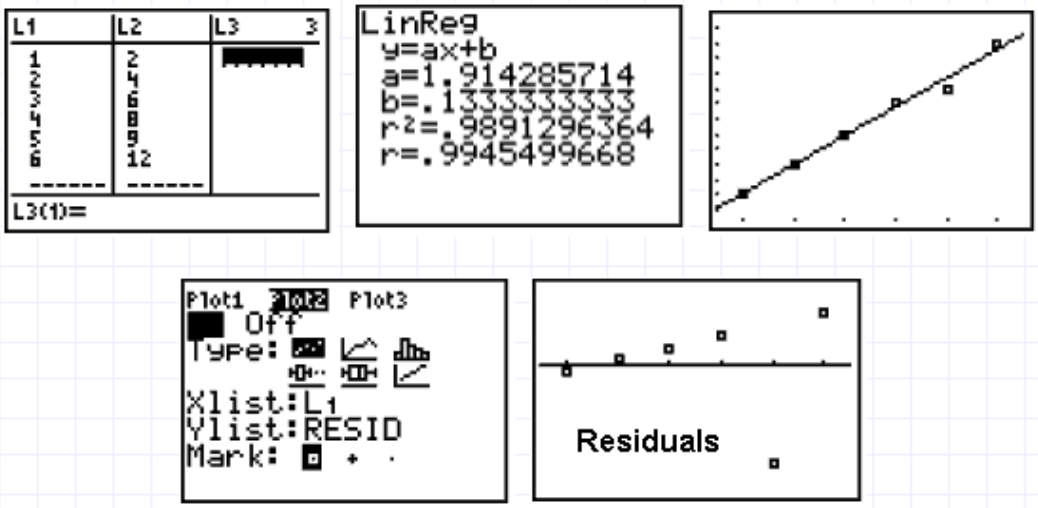
Note: For a perfect fit, the residuals will be all zero and ZOOM 9: ZoomStat will result in a WINDOW RANGE error since Ymin = 0 and Ymax = 0. If you still wish to see the plot, change Ymin = -1 and Ymax = 1 and then press GRAPH.

Graphing Residuals

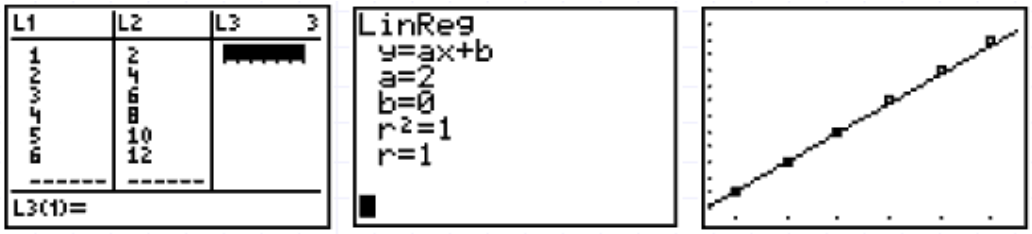
The graphing calculator uses a least squares regression equation to determine regression models. When regression models are computed, residuals are automatically stored in a list called **RESID**.

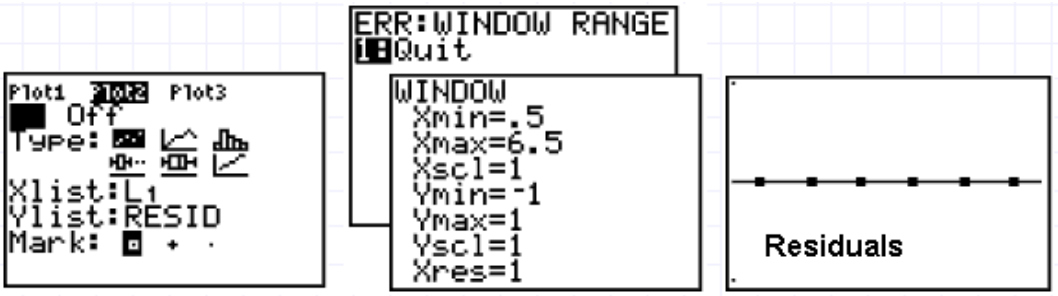
If you want to see the **RESID** list, in the column list section of the calculator, you can place the values in **L3** (for example). Go to **STAT** and choose **EDIT**. Place the cursor on the heading for **L3**. Press **LIST (2nd STAT)** and choose **#7 RESID**. Press **ENTER**. The residual values are now in **L3** for easy viewing.

Consider this set of data (which is "almost" a perfect linear equation)



If the data had been a perfect linear equation, you would have:

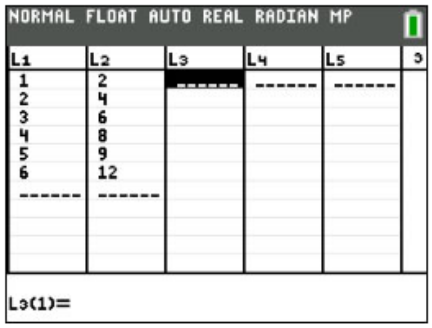




After adjusting the window, just hit **GRAPH** (not **Zoom**).

Step-by-step Instructions for Producing a Residual Plot:

1. Store your data in lists.
Enter the *x*-values in L1 and the *y*-values in L2.

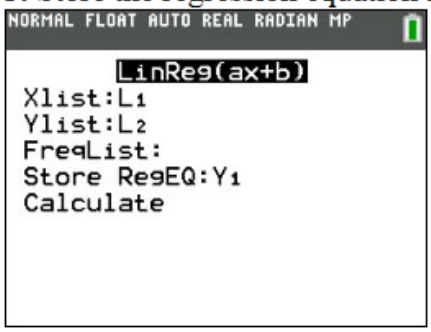


2. Perform a linear regression.

STAT, arrow right to CALC, choose either
4:LinReg(ax+b) or 8:LinReg(a+bx)

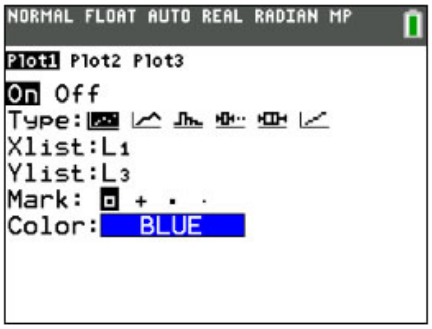
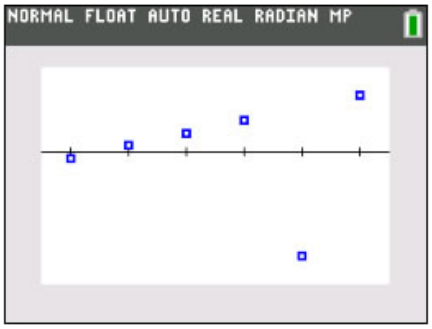
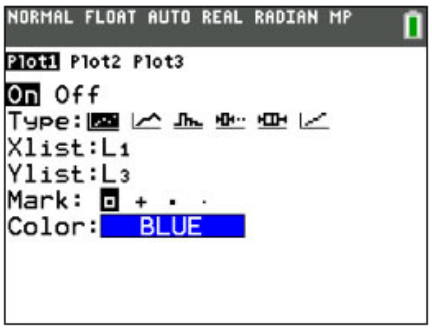
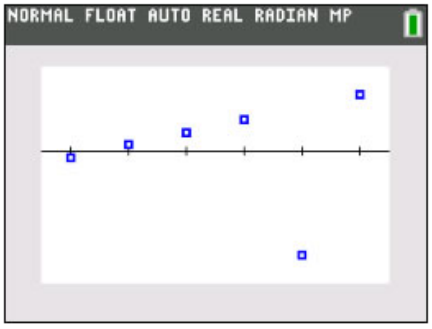


3. Store the regression equation in Y1.





4. Go to the top of L3 and press ENTER.
The cursor is now flashing at the bottom of the screen after L3 =.
Type in L2 - Y1(L1).

5. Go to the STAT PLOT menu (2nd Y=). Choose Plot1, hit ENTER. Turn the Plot ON (by hitting ENTER). Arrow down to Type: choose the first option (scatter plot). Make XList: L1 (the *x*-values) Make YList: L3 (the residuals)	
6. Go to ZOOM. Choose ZOOM 9:ZoomStat You should see your Residual Plot.	
5. Go to the STAT PLOT menu (2nd Y=). Choose Plot1, hit ENTER. Turn the Plot ON (by hitting ENTER). Arrow down to Type: choose the first option (scatter plot). Make XList: L1 (the *x*-values) Make YList: L3 (the residuals)	
6. Go to ZOOM. Choose ZOOM 9:ZoomStat You should see your Residual Plot.	

Linear Regression

A linear regression is also known as the "line of best fit".

Side note: Although commonly used when dealing with "sets" of data, the linear regression can also be used to simply find the equation of the line between two points.

Example: Find the equation of the line passing through (-1, 1) and (-4,7).

Entering the information as shown in the screens below, we arrive at the equation of the line:



The equation is $y = -2x - 1$.

The correlation coefficient is -1 since both point are "on" the line and the line slopes negatively.

Linear Regression Model Example

Data: Is there a relationship between Math SAT scores and the number of hours spent studying for the test? A study was conducted involving 20 students as they prepared for and took the Math section of the SAT Examination.

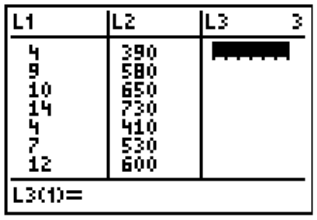
Task:
a) Determine a linear regression model equationation to represent this data
b) Graph the new equation.
c) Decide whether the new equation is a "good fit" to represent this data.
d) Interpolate data: If a student obtained a Math SAT score of 720, based upon this study, how many hours did the student most likely spend studying?
e) Interpolate data: If a student obtained a Math SAT score of 720, based upon this study, how many hours did the student most likely spend studying?
f) Extrapolate data: If a student spent 100 hours studying, what would be the expected Math SAT score? Discuss this answer.

Hours Spent Studying	Math SAT Score
4	390
9	580
10	650
14	730
4	410
7	530
12	600
22	790
1	350
3	400
8	590
11	640
5	450
6	520
10	690
11	690
16	770
13	700
13	730
10	640

Any answers in relation to this problem are to be rounded to the *nearest tenth.*
If rounding is not indicated in a problem, leave the full calculator entries as answers.

Step 1. Enter the data into the lists.

For basic entry of data, see Basic Commands.

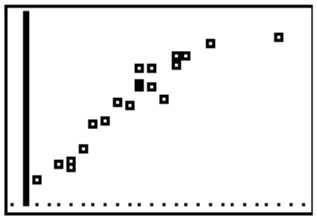


Step 2. Create a scatter plot of the data.

Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.



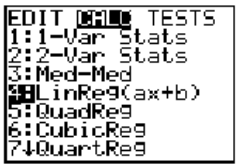




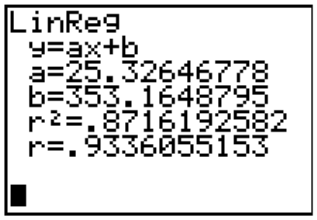
Step 3. Choose Linear Regression Model.

Press STAT, arrow right to CALC, and arrow down to 4: LinReg (ax+b). Hit ENTER. When LinReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation into Y= for you.

(Y_1 comes from **VARS → YVARS, #Function, Y_1**)







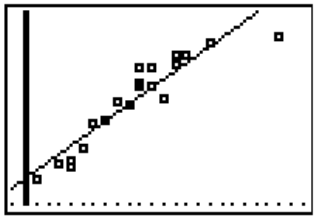
The linear regression equation is

$y = 25.3x + 353.2$

(answer to part a)

Step 4. Graph the Linear Regression Equation from Y1.

ZOOM #9 ZoomStat to see the graph.



(answer to part b)

Step 5. Is this model a "good fit"?

The correlation coefficient, r, is .9336055153 which places the correlation into the "strong" category. (0.8 or greater is a "strong" correlation)

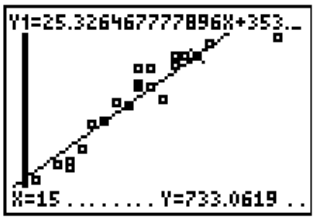
The coefficient of determination, r^2, is .8716192582 which means that 87% of the total variation in y can be explained by the relationship between x and y. The other 13% remains unexplained.

Yes, it is a "good fit".
(answer to part c)

Step 6. Interpolate: (*within the data set*)

If a student studied for 15 hours, based upon this study, what would be the expected Math SAT score?

From the graph screen, hit TRACE, arrow up to obtain the linear equation at the top of the screen, type 15, hit ENTER, and the answer will appear at the bottom of the screen.



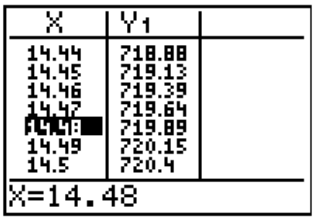
(answer to part d --
Math SAT score of 733.1)

Step 7. Interpolate: (*within the data set*)

If a student obtained a Math SAT score of 720, based upon this study, how many hours did the student most likely spend studying?

Go to TBLSET (above WINDOW) and set the TblStart to 13 (since 13 hours gives a score of 700). Set the delta Tbl to a decimal setting of your choice. Go to TABLE (above GRAPH) and arrow up or down to find your desired score of 720, in the Y1 column.





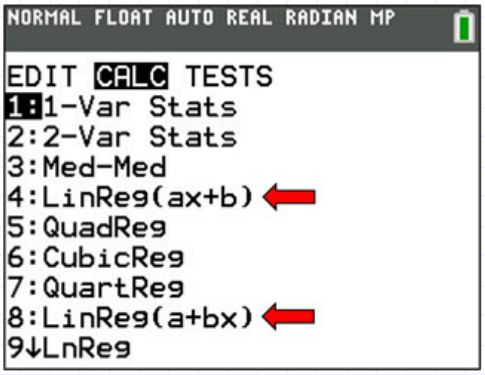
(answer to part e -- approx. 14.5 hours)

LinReg($ax + b$) versus LinReg($a + bx$)

Why are there TWO choices for linear regression equations on the graphing calculator?

First: There is no mathematical difference between the answers produced by these two forms. There is only a difference in the order in which the answer is written.

The two equations represent a difference in philosophy held by different disciplines in the mathematical community.



A linear equation can be written as $y = mx + b$, $y = ax + b$ or even $y = a + bx$.
These equations can all represent the same graphs, assuming a horizontal x-axis and a vertical y-axis.

Algebraic Philosophy: LinReg(ax + b)	**Statistical Philosophy:** LinReg(a + bx)
In Algebra, **the equation of a line** is represented by $y = mx + b$, where m is the slope and b is the y-intercept. Thus, algebraists prefer to maintain this format by using the form LinReg($ax + b$), where a is the slope and b in the y-intercept.	In Statistics, the preferred **equation of a line** is represented by $y = a + bx$, where b is the slope and a is the y-intercept. (The preferred form is actually $y = b_0 + b_1x$.) Thus, statisticians prefer to maintain this format by using the form LinReg($a + bx$), where a is the y-intercept and b is the slope.
The format $y = mx + b$ is a common algebraic format for the equation of a line, appearing in both high school and college texts.	In advanced multi-variable statistics, equations "get richer" as terms are added at the right (that is, the powers increase moving to the right.)
In Algebra, **the standard form of a quadratic equation** is $y = ax^2 + bx + c$.	In Statistics, **the standard form of a quadratic equation** is $y = a + bx + cx^2$.
Algebraists prefer this method as it more closely follows the **standard form of a polynomial:** $P(x) = a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0$.	Statisticians prefer this method as it more closely follows the **general form of the regression equation:** $y = b_0 + b_1x + b_2x^2 + \ldots + b_nx^n$. Statistician will also say that they prefer this method because the variables represent the same context in each formula

It can be argued that there are various reasons for choosing one of these linear regression equation forms over the other. Unfortunately, it is often seen that one form is chosen to the "exclusion" of the other, with statements such as "DO NOT USE (the other form)". It is more important to understand that there are alternative ways to represent the same concepts depending upon the context in which they are used. While one form may be "preferred", both forms are beneficial.

In today's interdisciplinary world, a more flexible attitude is needed. Students need to understand that parts of linear equations are commutative (such as $ax + b = b + ax$) and that they need to find the x-term to examine the slope (when the graph has a horizontal x-axis and a vertical y-axis). Students also need to understand that using different letters for constants and coefficients will not change the meaning of the parts of the equation (such as $y = ax + b$ and $y = a + bx$, where a is the slope in the first, and b is the slope in the second).

Bottom line: In high school Algebra 1, Geometry, Algebra 2, PreCalculus, Advanced Placement Calculus, and in college Algebra courses (and other college courses), LinReg($ax+b$) remains the most widely used calculator form for the linear regression equation. In Advanced Placement Statistics and in College Statistics courses, LinReg($a+bx$) is the more widely preferred form.

If you understand "one" of these forms, you will understand the other as well.

Both work!

Linear Regression with a Biological Data

(or the realities of working with real-life data)

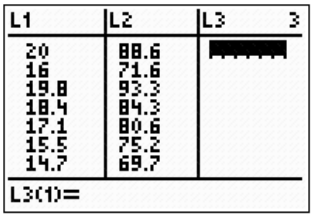
Data: Is there a relationship between Math SAT scores and the number of hours spent studying for the test? A study was conducted involving 20 students as they prepared for and took the Math section of the SAT Examination.

Task: a) Determine a linear regression model equationation to represent this data
b) Graph the new equation.
c) Decide whether the new equation is a "good fit" to represent this data.
d) Interpolate data: If a student obtained a Math SAT score of 720, based upon this study, how many hours did the student most likely spend studying?
e) Interpolate data: If a student obtained a Math SAT score of 720, based upon this study, how many hours did the student most likely spend studying?
f) Extrapolate data: If a student spent 100 hours studying, what would be the expected Math SAT score? Discuss this answer.

Hours Spent Studying	Math SAT Score
4	390
9	580
10	650
14	730
4	410
7	530
12	600
22	790
1	350
3	400
8	590
11	640
5	450
6	520
10	690
11	690
16	770
13	700
13	730
10	640

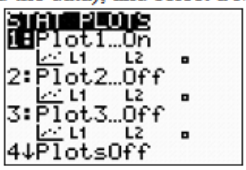
Answers in this problem are to be rounded to the *nearest hundredth.*

Step 1. Enter the data into the lists.
For basic entry of data, see Basic Commands.

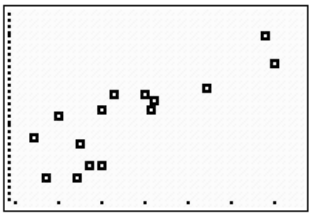


Step 2. Create a scatter plot of the data.

Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.





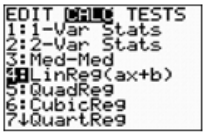


Obviously, there is some scatter to this data. This variability is the norm, rather than the exception, when working with biological data sets. Real life data seldom creates a nice straight line.

Step 3. Choose the Linear Regression Model.

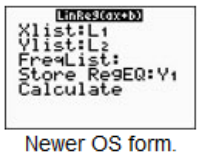
Press STAT, arrow right to CALC, and arrow down to 4: LinReg (ax+b). Hit ENTER. When LinReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation in to Y= for you.

(Y_1 comes from VARS → YVARS, #Function, Y_1)

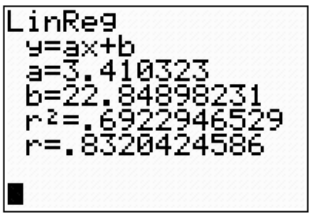




Older OS form.



Newer OS form.



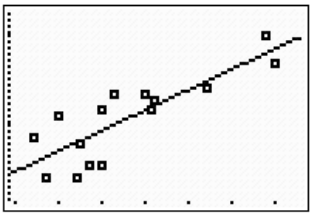
The linear regression equation is

$y = 3.41x + 22.85$

(answer to part a)

Step 4. Graph the Linear Regression Equation from Y1.

ZOOM #9 ZoomStat to see the graph.



(answer to part b)

Step 5. Is this model a "good fit"?

The correlation coefficient, r, is .8320424586 which just barely places the correlation into the "strong" category. (0.8 or greater is a "strong" correlation)

The coefficient of determination, r^2, is .6922946529 which means that 70% of the total variation in y can be explained by the relationship between x and y. The other 30% remains unexplained.

Yes, it is somewhat of a "good fit".

(answer to part c)

Step 6. Extrapolate: *(beyond the data set)*

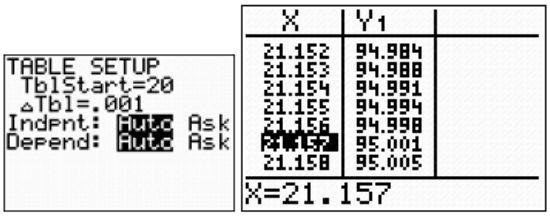
If the ground temperature reached 95°, then at what approximate rate would you expect the crickets to be chirping?

Go to TBLSET (above WINDOW) and set the TblStart to 20+ (since the highest temperature in the data set had 20 chirps/second). Set the delta Tbl to a decimal setting of your choice. Go to TABLE (above GRAPH) and arrow up or down to find your desired temperature, 95°, in the Y1 column.

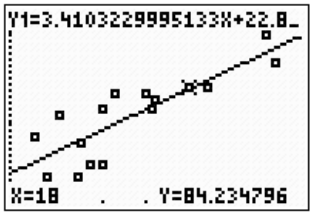
Step 7. Interpolate: *(within the data set)*

With a listening device, you discovered that on a particular morning the crickets were chirping at a rate of 18 chirps per second. What was the approximate ground temperature that morning?

From the graph screen, hit TRACE, arrow up to obtain the power equation, type 18, hit ENTER, and the answer will appear at the bottom of the screen.

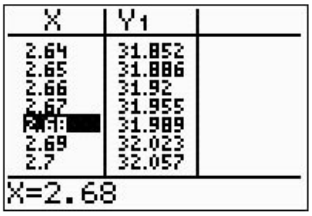


(answer to part d -- approx. 21.16 chirps/second)



(answer to part e -- the ground temperature will be approx. 84.23° F)

Step 8. ***If the ground temperature should drop to freezing (32° F), what happens to the cricket's chirping?***



The TABLE tells us that at 32° F there are 2.68 chirps per second. So, what does this really mean? Are the crickets cold?

These findings are a bit deceiving. At 32° F, the crickets are dead. The lifespan of a cricket in a cold climate is very short. The crickets spend the winter as eggs laid in the soil. These eggs hatch in late spring or early summer, and tiny immature crickets called nymphs emerge. Nymphs develop into adults within approximately 90 days. The adults mate and lay eggs in late summer before succumbing to old age or freezing temperatures in the fall.

Also, remember that the further you extrapolate away from the data set, the less reliable the information will be.

Exponential Regression Model Example

(when doing an exponential regression, the y-values must be greater than 0)

Data: The data at the right shows the cooling temperatures of a freshly brewed cup of coffee after it is poured from the brewing pot into a serving cup. The brewing pot temperature is approximately 180° F.

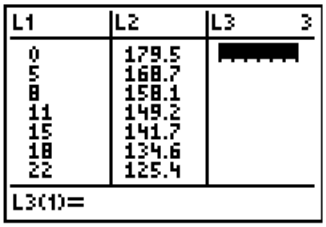
Time (mins)	Temp (° F)
0	179.5
5	168.7
8	158.1
11	149.2
15	141.7
18	134.6
22	125.4
25	123.5
30	116.3
34	113.2
38	109.1
42	105.7
45	102.2
50	100.5

Task:

a.) Determine an exponential regression model equation to represent this data.
b.) Graph the new equation.
c.) Decide whether the new equation is a "good fit" to represent this data.
d.) Based upon the new equation, what was the initial temperature of the coffee?
e.) Interpolate data: When is the coffee at a temperature of 106 degrees?
f.) Extrapolate data: What is the predicted temperature of the coffee after 1 hour?
g.) In 1992, a woman sued McDonald's for serving coffee at a temperature of 180° that caused her to be severely burned when the coffee spilled. An expert witness at the trial testified that liquids at 180° will cause a full thickness burn to human skin in two to seven seconds. It was stated that had the coffee been served at 155°, the liquid would have cooled and avoided the serious burns. The woman was awarded over 2.7 million dollars. As a result of this famous case, many restaurants now serve coffee at a temperature around 155°. How long should restaurants wait (after pouring the coffee from the pot) before serving coffee, to ensure that the coffee is not hotter than 155° ?
h.) If the temperature in the room is 76° F, what will happen to the temperature of the coffee, after being poured from the pot, over an extended period of time?

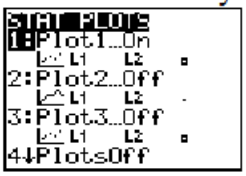
Step 1. Enter the data into the lists.

For basic entry of data, see Basic Commands.

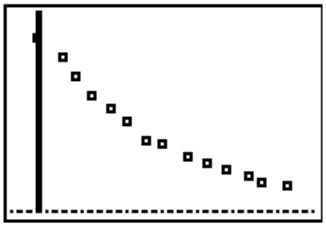


Step 2. Create a scatter plot of the data.

Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.



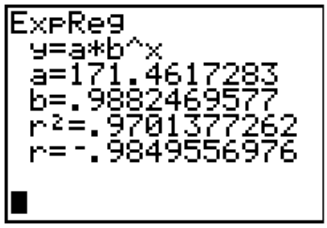


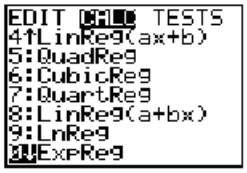


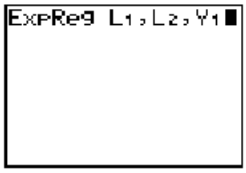
Step 3. Choose Exponential Regression Model.

Press STAT, arrow right to CALC, and arrow down to 0: ExpReg. Hit ENTER. When ExpReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation into Y= for you.

(**Y_1** comes from **VARS → YVARS, #Function, Y_1**)







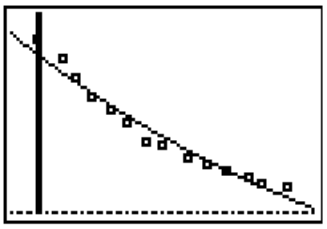
The exponential regression equation is

$y = (171.462) \cdot 0.988^x$

(answer to part a)

Step 4. Graph the Exponential Regression Equation from Y1.

ZOOM #9 ZoomStat to see the graph.



(answer to part b)

Step 5. Is this model a "good fit"?

The correlation coefficient, r, is -.9849556976 which places the correlation into the "strong" category. (0.8 or greater is a "strong" correlation)

The coefficient of determination, r^2, is .9701377262 which means that 97% of the total variation in y can be explained by the relationship between x and y.

Yes, it is a very "good fit".

(answer to part c)

Step 6.

Based upon the new equation, what was the initial temperature of the coffee?

The exponential regression equation is

$y = (171.462) \cdot 0.988^x$

where x stands for time. The initial temperature would occur when the time equals zero. Substituting zero for x gives an initial temperature of 171.462°.

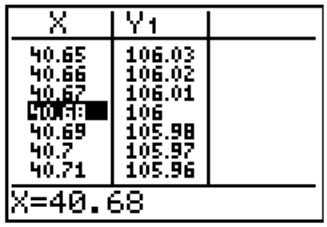
(answer to part d)

Step 7. Interpolate: (*within the data*)

When is the coffee at a temperature of 106 degrees?

Go to TBLSET (above WINDOW) and set the TblStart to 42 (since 42 minutes gives a temperature close to 106°). Set the delta Tbl to a decimal setting of your choice. Go to TABLE (above GRAPH) and arrow up or down to find your desired temp of 106°, in the Y1 column.



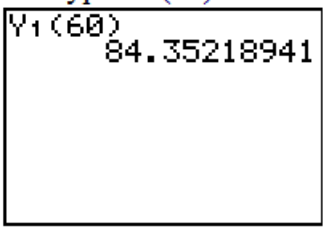


(answer to part e: after approx. 40.7 minutes)

Step 8. **Extrapolate data:** *(beyond the data)*

What is the predicted temperature of the coffee after 1 hour?

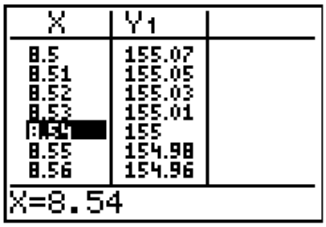
Change 1 hour to 60 minutes. With your exponential equation in Y1, go to the home screen and type Y1(60). Press ENTER.



(answer to part f: 84.4° F)

Step 9. *How long should the restaurants wait (after pouring the coffee from the pot) before serving coffee, to ensure that the coffee is not hotter than 155° ?*

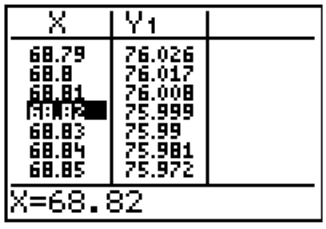
Repeat procedure from Step 7:

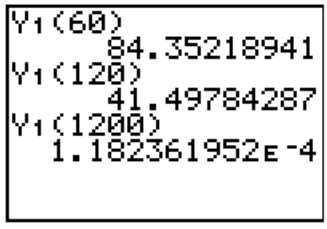


(answer to part g: approx. 8.5 minutes)

Step 10. *If the temperature in the room is 76° F, what will happen to the temperature of the coffee, after being poured from the pot, over an extended period of time?*

The new exponential equation shows that the coffee will reach room temperature after 68.8 minutes. The exponential graph is asymptotic to the *x*-axis, therefore the *y*-values approach zero. This can be seen by observing successively larger values substituted into the new exponential equation (*see home screen substitutions below*). Even though the graph approaches zero asymptotically, the temperature of the coffee will stop cooling once it reaches room temperature of 76°, and will not continue following the curve of the graph. (Also, remember that when extrapolating data, the further away from the data set you go, the less reliable your new data becomes.)





Logarithmic Regression Model Example

Data: The data below show the average growth rates of 12 Weeping Higan cherry trees planted in Washington, D.C. At the time of planting, the trees were one year old and were all 6 feet in height.

Task: a) Determine a logarithmic regression model equation to represent this data.

b) Graph the new equation.

c) Decide whether the new equation is a "good fit" to represent this data.

d) Interpolate: What was the average height of the trees at one and one-half years of age? (to the *nearest tenth* of a foot)

e) Extrapolate: What is the predicted average height of the trees at 20 years of age? Is this prediction realistic? (round the answer to the *nearest tenth* of a foot).

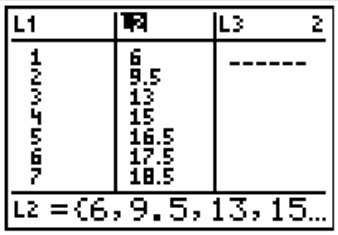
f) Based upon your observations of this data, what would you predict to be the average height of a mature Higan cherry tree, to the *nearest foot*?

g) If the average height of the trees is 10 feet, what is the age of the trees to the *nearest tenth* of a year?

Age of Tree (in years)	Height (in feet)
1	6
2	9.5
3	13
4	15
5	16.5
6	17.5
7	18.5
8	19
9	19.5
10	19.7
11	19.8

Step 1. Enter the data into the lists.

For basic entry of data, see Basic Commands.

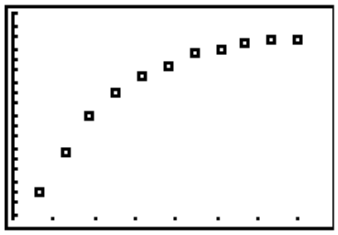


Step 2. Create a scatter plot of the data.

Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.





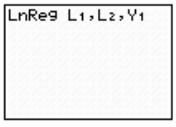


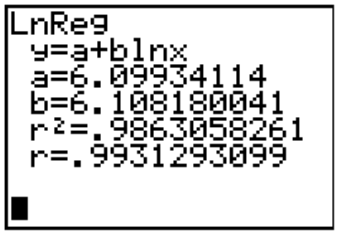
Step 3. Choose Logarithmic Regression Model.

Press STAT, arrow right to CALC, and arrow down to 9: LnReg. Hit ENTER. When LnReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation into Y= for you.

(**Y_1** comes from **VARS → YVARS, #Function, Y_1**)







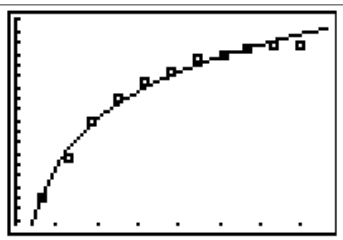
The logarithmic regression equation is

$y = 6.099 + (6.108)\ln x$

(answer to part a)

Step 4. Graph the Exponential Regression Equation from Y1.

ZOOM #9 ZoomStat to see the graph.



(answer to part b)

Step 5. Is this model a "good fit"?

The correlation coefficient, r, is .9931293099 which places the correlation into the "strong" category. (0.8 or greater is a "strong" correlation)

The coefficient of determination, r^2, is .9863058261 which means that 98.6% of the total variation in y can be explained by the relationship between x and y.

Yes, it is a very "good fit".

(answer to part c)

Step 6. Interpolate: *(within the data set)*

What was the average height of the trees at one and one-half years of age? (to the nearest tenth of a foot)

The logarithmic regression equation is

$$y = 6.099 + (6.108)\ln x$$

where x stands for time. Substituting 1.5 for x gives an average height of 8.57558088 feet or 8.6 feet.

(answer to part d)

Step 7. Extrapolate data: *(beyond data set)*

What is the predicted average height of the trees at 20 years of age? Is this prediction realistic? (answer to the nearest tenth of a foot)

The logarithmic regression equation is

$$y = 6.099 + (6.108)\ln x$$

where x stands for time. Substituting 20 for x gives an average height of 24.39693273 feet or 24.4 feet.

Extrapolations far from the stated data are often inaccurate and unreliable. Nine years away from the data set is a large span of time and the reading of 24.4 feet may be "high" based upon the observed leveling nature of the last data entries.

(answer to part e)

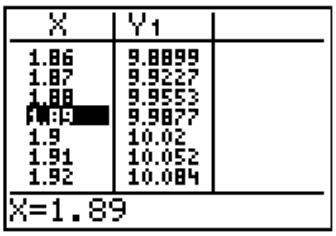
Step 8.

Based upon your observations of this data, what would you predict to be the average height of a mature Higan cherry tree, to the nearest foot?

It can be seen from the graph that the growth rate is slowing down (leveling off). Such a slowing could be interpreted that the trees are reaching their mature height. The twenty year prediction shows a height of 24 feet. Any answer from 20 feet to 24 feet would be an acceptable approximation. While extrapolations far from the

Step 9. *If the average height of the tree is 10 feet, what is the age of the tree to the nearest tenth of a year?*

Use your table to find the age:



X	Y1
1.86	9.8899
1.87	9.9227
1.88	9.9553
1.89	9.9877
1.9	10.02
1.91	10.052
1.92	10.084

X=1.89

(answer to part g -- approx. 1.9 years of age)

stated data are often inaccurate, a reading closer to 20 feet may be more accurate.
(In reality, Higan cherry trees, depending upon the specific specie, reach a mature height anywhere from 15 to 30 feet.)
(answer to part f)

Quadratic Regression Model Example

Data: The table at the right shows the horizontal distance (in feet) traveled by a baseball hit at various angles. The initial speed of the ball at the bat is constant.

Angle (degrees)	Distance (feet
10°	115.6
15°	157.2
20°	189.2
24°	220.8
30°	253.8
34°	269.2
40°	284.8
45°	285.0
48°	277.4
50°	269.2
58°	244.2
60°	231.4
64°	180.4

Task: Express answers to *3 decimal places* unless told otherwise.

a) Prepare a scatter plot of the data.

b) Determine a quadratic regression model equation to represent this data and graph the new equation.

c) Decide whether the new equation is a "good fit" to represent this data.

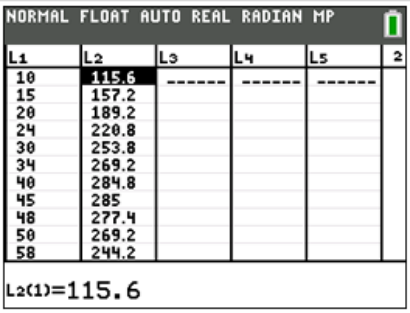
d) *Extrapolate data:* What distance will correspond to an angle of a 5°?

e) *Interpolate data:* What angle(s) will correspond to a distance of 270 feet, to the *nearest degree*?

f) The first baseman is positioned 100 feet from home plate and the right fielder is positioned 180 feet from home plate. The batter wants to hit the ball half way between these players. What angle, to the *nearest degree*, should be used to accomplish this hit? Round to 3 decimal places.

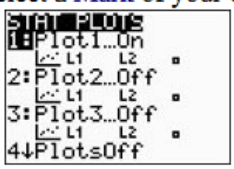
g) The left field fence is 280 feet from home plate. At what angle, or angles, to the *nearest degree*, will the ball be hit past the left field fence?

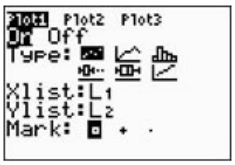
Step 1. Enter the data into the lists.
For basic entry of data, see Basic Commands.

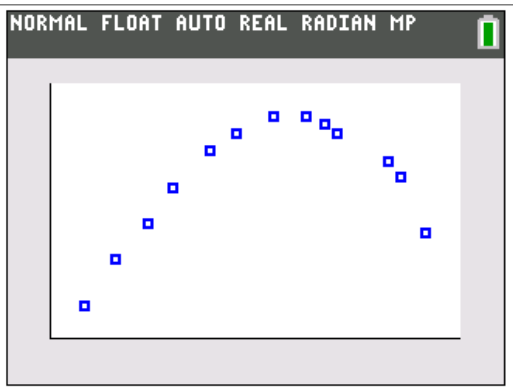


Step 2. Create a scatter plot of the data.

Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice. ZoomStat.





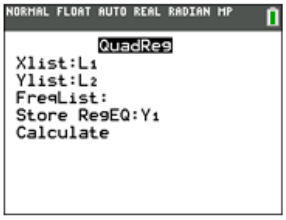


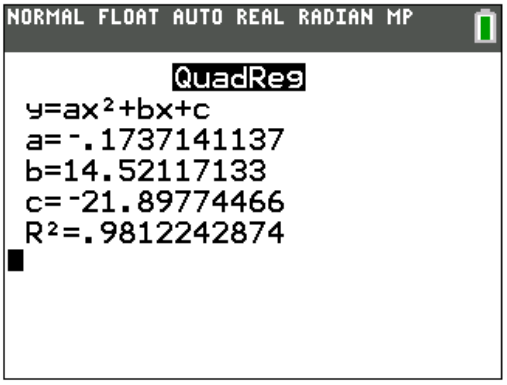
(answer to part a)

Step 3. Choose the Quadratic Regression Model.

Press STAT, arrow right to CALC, and arrow down to 5: QuadReg. Hit ENTER. When the template for QuadReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation into Y= for you. (**Y_1** comes from **VARS → YVARS, #Function, Y_1** or from **ALPHA F4**)





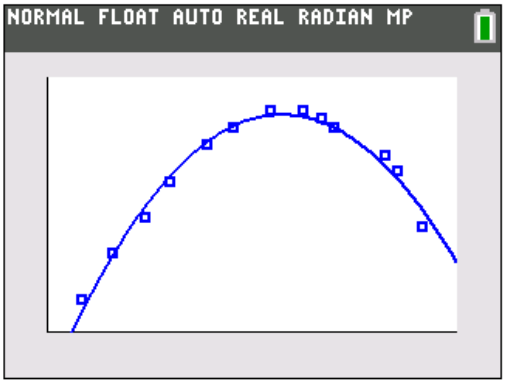


The quadratic regression equation is

$y = -0.173x^2 + 14.521x - 21.898$

(answer to part b)

Step 4. Graph the Quadratic Regression Equation from Y1.



(answer to part b)

Step 5. Is this model a "good fit"?

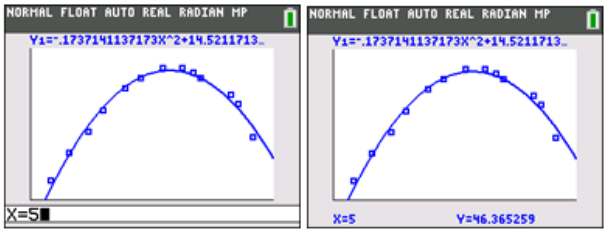
The coefficient of determination, R^2, is .9812242874 which means that 98% of the total variation in y can be explained by the relationship between x and y.

Yes, it is a very "good fit".

(answer to part c)

Step 6. Extrapolate: *(beyond the data set)*

Get the answer from the graph of the regression equation. Check the WINDOW to be sure that 5Â° falls within the domain (it does).) Hit TRACE. Arrow up so the regression equation will show at the top of the screen. Type 5 and hit ENTER.



(answer to part d -- At 5°, the distance will be 46.365 feet.)

Step 7. Interpolate: *(within the data set)*

From TABLE, find the value in L2 closest to 270. Adjust the TBLSET to narrow in on the answer.

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
49.954	270.01
49.955	270
49.956	270
49.957	270
49.958	269.99
49.959	269.99
49.96	269.99
49.961	269.99
49.962	269.98
49.963	269.98
49.964	269.98

X=49.954

(answer to part e -- angle will be 50° or 34Â°)

Step 8. The first baseman is positioned 100 feet from home plate and the right fielder is positioned 180 feet from home plate. The batter wants to hit the ball half way between these players. What angle, to the *nearest degree*, should be used to accomplish this hit?

Half way between 100 feet away and 180 feet away is 140 feet. Find the angle within the scatter plot domain that corresponds with 140 feet. 13 degrees (answer to part f).

NORMAL FLOAT AUTO REAL RADIAN MP
PRESS + FOR △Tbl

X	Y1
13.243	139.94
13.244	139.95
13.245	139.96
13.246	139.97
13.247	139.98
13.248	139.99
13.249	140
13.25	140.01
13.251	140.02
13.252	140.03
13.253	140.04

X=13.249

Step 9. The left field fence is 280 feet from home plate. At what angle, or angles, will the ball be hit past the left field fence?

Use the TABLE to find values greater than 280. Since the graph is parabolic, you are looking for two locations. (answer to part g) any angle between 39° and 44°.

Power Regression Model Example

Side note: Power regressions will not allow an independent variable value of zero. This example does not utilize an independent variable value of zero.

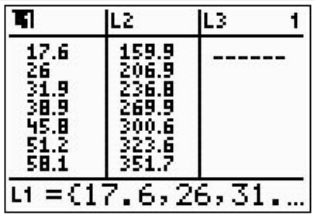
Data: The data below represents the length and mid-shaft diameters of the humerus bones of African Antelopes.

Diameter (mm)	Length (mm)
17.6	159.9
26.0	206.9
31.9	236.8
38.9	269.9
45.8	300.6
51.2	323.6
58.1	351.7
64.7	377.6
66.7	384.1
80.8	437.2
82.9	444.7

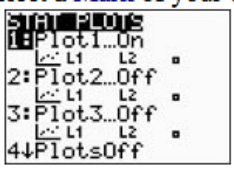
Task: Express answers to the *nearest tenth*.

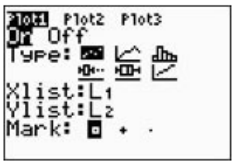
a) Prepare a scatter plot of the data.
b) Determine a power regression model equation to represent this data.
c) Graph the new equation.
d) Decide whether the new equation is a "good fit" to represent this data.
e) *Extrapolate data:* What length will correspond to a diameter of 84 mm?
f) *Interpolate data:* What length will correspond to a diameter of 47 mm?
g) What mid-shaft diameter will correspond to a length of 305.7 mm?

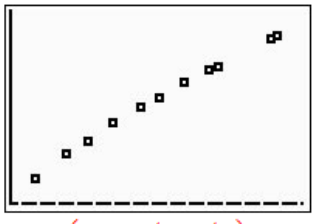
Step 1. Enter the data into the lists.
For basic entry of data, see Basic Commands.



Step 2. Create a scatter plot of the data.
Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice.





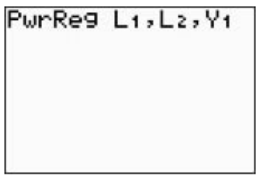


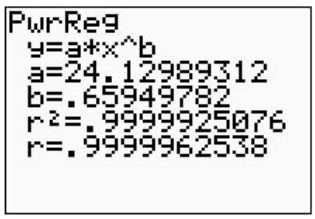
(answer to part a)

Step 3. Choose the Power Regression Model.

Press STAT, arrow right to CALC, and arrow down to A: PwrReg. Hit ENTER. When PwrReg appears on the home screen, type the parameters L1, L2, Y1. The Y1 will put the equation into Y= for you. (Y1 comes from **VARS → YVARS, #Function, Y1**)







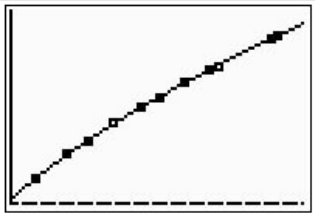
The power regression equation is

$y = 24.1x^{0.7}$

(answer to part b)

Step 4. Graph the Power Regression Equation from Y1.

ZOOM #9 ZoomStat to see the graph.



(answer to part c)

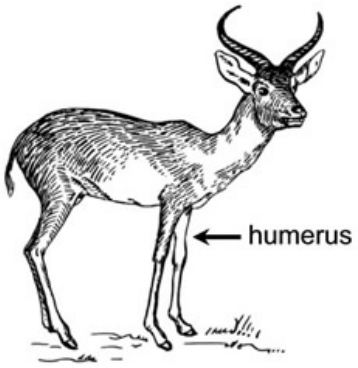
Step 5. Is this model a "good fit"?

The correlation coefficient, r, is .9999937121 which indicates a very strong correlation since it is close to 1.

The coefficient of determination, r^2, is .99999250766 which means that 99% of the total variation in y can be explained by the relationship between x and y.

Yes, it is a very "good fit".

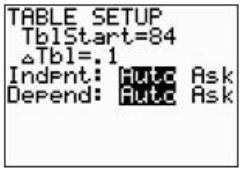
(answer to part d)

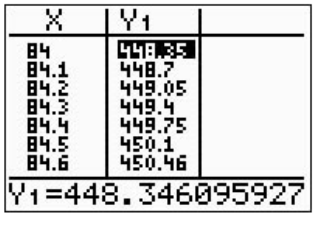


Step 6. Extrapolate: *(beyond the data set)*

Go to TBLSET (above WINDOW) and set the TblStart to 84. Notice how setting the increment (the deltaTbl = 0.1) displays certain values to two decimal places. The calculator is trying to tell you that this value has been "rounded" to the nearest hundredth and should not be now rounded to the nearest tenth. Always check the "full" Y1 value, as seen at the bottom of the screen, before rounding.

(See an alternate method in Step 7.)



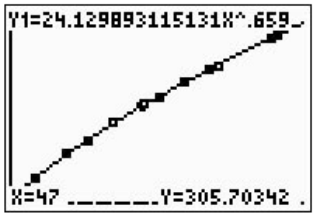


(answer to part e - the length will be 448.3 mm)

Step 7. Interpolate:

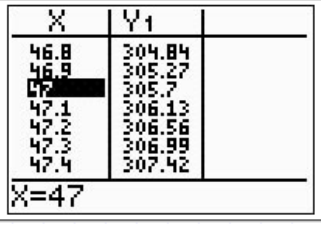
(within the data set)

From the graph screen, hit TRACE, arrow up to obtain the power equation, type 47, hit ENTER, and the answer will appear at the bottom of the screen.



(answer to part f - length will be 305.7 mm)

Step 8. What mid-shaft diameter will correspond to a length of 305.7 mm?
In reference to the equation, go to TABLE (above GRAPH) and arrow down until you find a Y1 value equal to (or close to) the desired value. The answer will appear in the X column. 47 mm.



Sinusoidal Regression Model Example

Data: The table below shows the highest daily temperatures (in degrees Fahrenheit) averaged over the month for the cities of Syracuse, NY; Washington, DC; and Austin, TX

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
	1	2	3	4	5	6	7	8	9	10	11	12
Syracuse, NY	32	34	43	57	69	78	82	80	72	60	48	36
Washington, DC	43	47	56	67	75	84	88	87	80	68	58	47
Austin, TX	62	65	72	80	87	92	96	97	91	82	71	63

Be sure your calculator is in radian mode!

The calculator will give the regression equation in the form:

$$y = a\sin(bx + c) + d$$

where $|a|$ is the amplitude, b is the frequency (where $b > 0$),
$\frac{2\pi}{b}$ is the period, $\frac{|c|}{b}$ is the horizontal shift (to the right if $c < 0$ and to the left if $c > 0$)
and d is the vertical shift (up if $d > 0$ and down if $d < 0$).

Task: Express answers to 3 decimal places

a) Determine a sinusoidal regression model equation to represent the temperatures for each of the three cities.

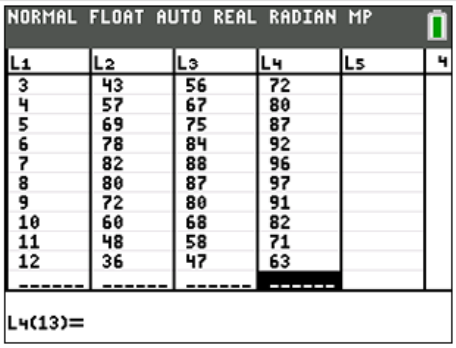
b) Graph the three new equation on the same grid.

c) Do the graphs intersect? Are these results representative of what was observed in the data? Explain.

The direction below will show the complete process for finding the sinusoidal regression equation for Syracuse, NY. The equations for the other cities will be done in the same manner.

Step 1. Enter the data into the lists.

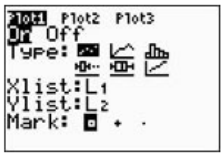
For basic entry of data, see Basic Commands.
The month numbers are in L1.
Syracuse temperatures are in L2.
Washington temperatures are in L3.
Austin temperatures are in L4.

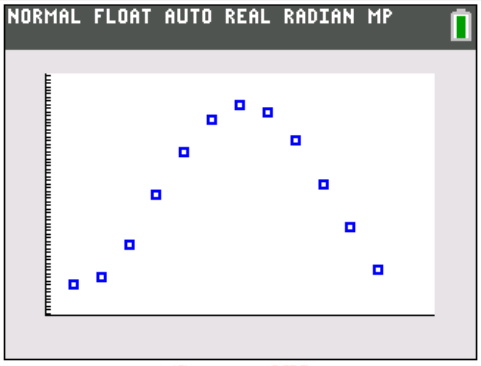


Step 2. Create a scatter plot of the data.

Go to STATPLOT (2nd Y=) and choose the first plot. Turn the plot ON, set the icon to Scatter Plot (the first one), set Xlist to L1 and Ylist to L2 (assuming that is where you stored the data), and select a Mark of your choice. Zoom #9.







(Syracuse, NY)

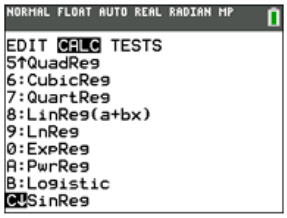
Step 3. Choose the Sinusoidal Regression Model.

Press STAT, arrow right to CALC, and arrow down to C: SinReg. Hit ENTER. When SinReg appears on the home screen, type the parameters 16, L1, L2, 12, Y1. The Y1 will put the equation into Y= for you. (Y_1 comes from **VARS → YVARS, #Function, Y_1** OR **ALPHA F4**)

PARAMETERS:

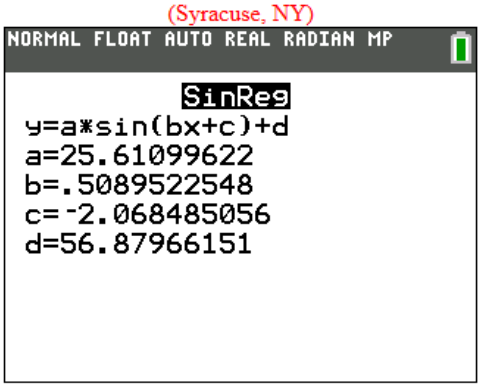
â€¢ SinReg(Iternations, Xlist, Ylist, Period, Store)
â€¢ In a sinusoidal regression, the "iterations" is the maximum number of times the SinReg command will iterate to find the equation. Any integer from1 to 16 can be used, but 16 generally gives a more accurate result than smaller integers. Default = 3. Higher values may take more time to compute.
â€¢ The "period" is the horizontal distance between two maximums or two minimum points. If you leave the period empty, the calculator will take over.
â€¢ The "store" is the location where the equation will be stored. Default is Y1.

Notice that there is no correlation coefficient or coefficient of determination.





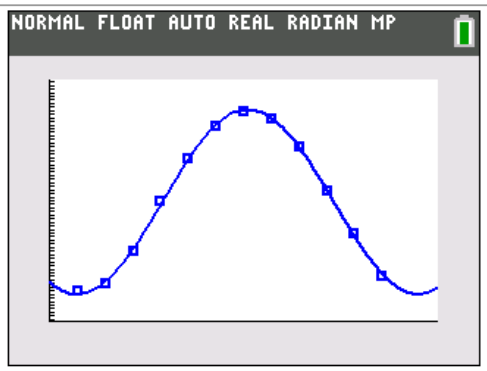
(Syracuse, NY)



The sinusoidal regression equation is
$y = 25.611 \sin(0.509x + (-2.068)) + 56.880$

Be sure you notice when working with the calculator's sinusoidal equation, that the b has NOT been factored out. It is not the same as the equation
$y = A\sin(B(x - C)) + D$.

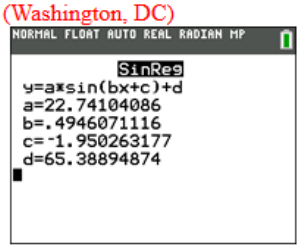
Step 4. Graph the Power Regression Equation from Y1.



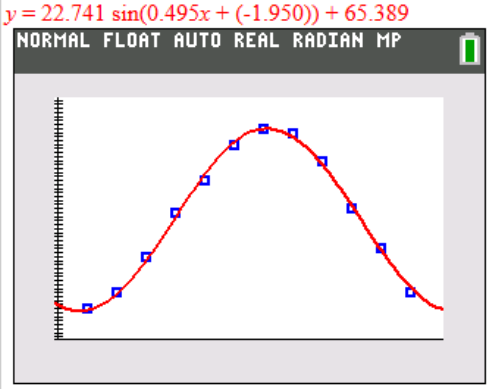
(Syracuse, NY)

Step 5. Get the sinusoidal equations for the other cities.

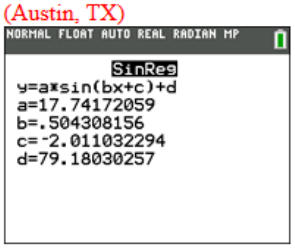
(Washington, DC)



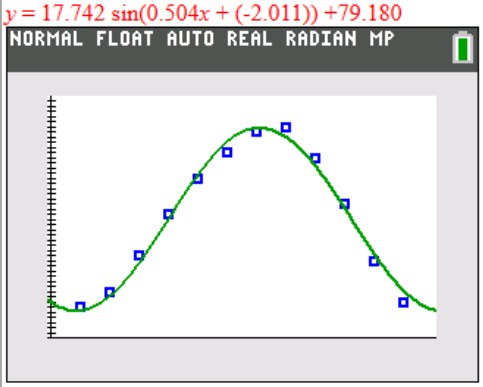
$y = 22.741 \sin(0.495x + (-1.950)) + 65.389$



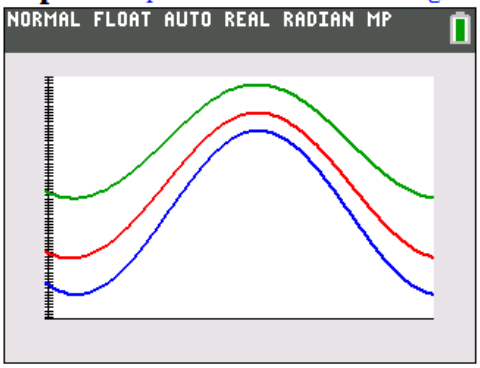
(Austin, TX)



$y = 17.742 \sin(0.504x + (-2.011)) + 79.180$



Step 6. Graph all three cities on same grid.



Do the graphs intersect? NO

Are these graphs representative of what was observed in the data? YES

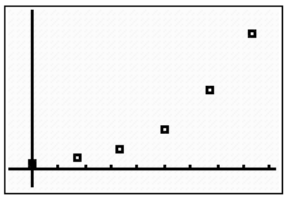
Explain. If you examine the data, you notice that the cities are arranged such that Syracuse is "cold", Washington is "warm" and Austin is "hot" for every month. There will never be a point of intersection.

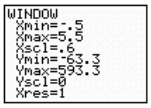
Comparing an Exponential and a Power Regression Models

A rapidly growing bacteria has been discovered. Its growth rate is shown in the chart.

Hours since observation began	Number of bacteria in the sample
0	20
1	40
2	75
3	150
4	297
5	510

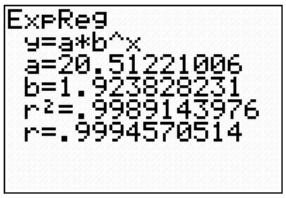
a) Prepare a scatter plot of the data with hours as the independent variable and the number of bacteria as the dependent variable.

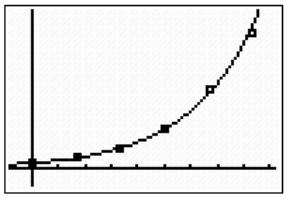




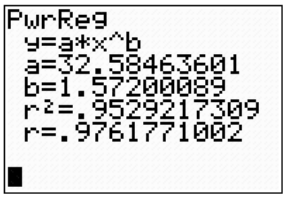
b) Determine which regression model will best approximate your data.

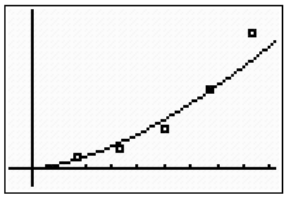
We will limit our choices to linear, logarithmic, exponential, and power as possible regression models. The scatter plot of the data clearly shows a "curve" to the data, so we will eliminate the linear model at this time. The positioning of the plots appears to be compatible with an exponential model, or possibly a power model since the plots might be the right hand side of a parabola. Let's examine both.





The exponential model is a "good fit", as it passes through most of the plotted points and appears to follow the increasing rate of the data.

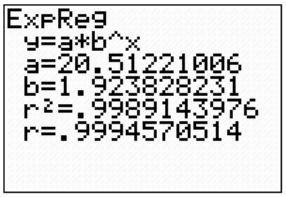




The power model hits only a few of the points and does not seem to follow the degree of increase as well as the exponential model. (NOTE: Power regressions on the calculator will not allow the independent variable to be zero. For that reason, the zero time and corresponding number of bacteria had to be eliminated from the data set for this plot.)

Choose the exponential model. It makes sense that this model would best represent the data, since exponential models are often used with population growth (even when the population is bacteria).

c) Write the regression equation for your model, rounding values to three decimal places.



ANSWER: $y = 20.512(1.924)^x$

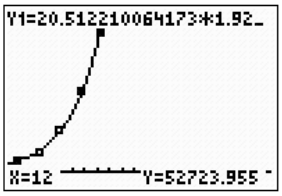
d) What is the correlation coefficient for this data and what does it tell you?

The correlation coefficient is $r = .9994570514$.
The closer this value is to 1, the more accurate your model will be when used for predictions. This model will be a good predictor.

Notice that both the exponential and the power regression models showed high correlation coefficients, but examination of the graph showed that the exponential model was the better fit.

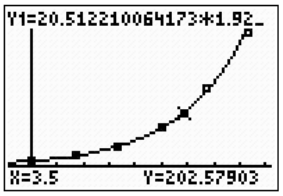
e) Using your regression equation, determine how many bacteria, to the nearest integer, will be present in 12 hours.

Substituting 12 into the equation, we arrive at an answer of 52,724 bacteria, to the nearest integer. Looking for values that fall outside the plotted data is called ***extrapolating.*** **Be careful** when extrapolating. The further away from the plotted data you go, the less reliable is your prediction.



f) Using your regression equation, determine how many bacteria, to the nearest integer, will be present in 3.5 hours.

Substituting 3.5 into the equation, we arrive at an answer of 203 bacteria, to the nearest integer. Looking for values that fall within the plotted data is called ***interpolating.***



Measures of Dispersion

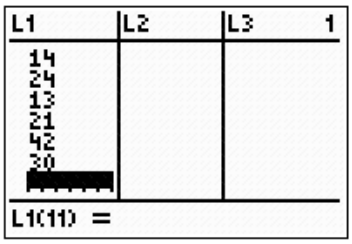
Range, Standard Deviation, Variance, Mean Absolute Deviation

Problem: For the data set {10, 12, 40, 35, 14, 24, 13, 21, 42, 30}, find the range, the standard deviation, the variance, and the mean absolute deviation to the *nearest hundredth.*

A quick reminder before we begin the solution:
In statistics, the population form is used when the data being analyzed includes the entire set of possible data.
The sample form is used when the data is a random sample taken from the entire set of data.
Check the "Standard Deviation" and "Variance" sections below for more information on how the calculator presents population versus sample forms.

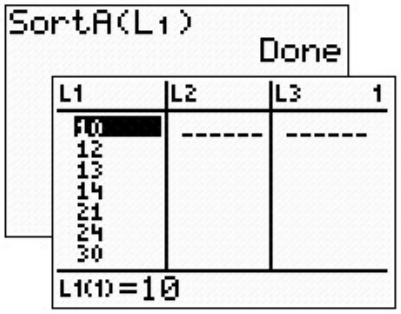
To find the range:

To find the range:
Enter the data, as is, into L_1. You can enter the list on the home screen and "store" to L_1, or you can go directly to L_1 (2nd STAT, #1 Edit).

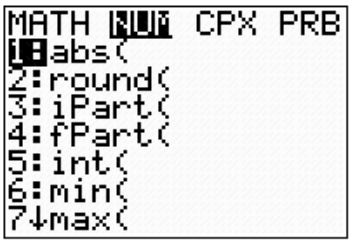


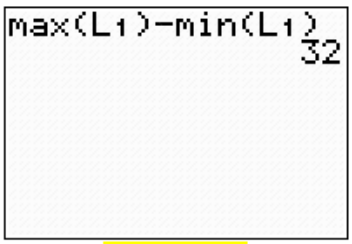
Sort the list to quickly retrieve the highest and lowest values for the range. (2nd STAT, #2 SortA). You can choose ascending or descending.

Read the high and low values from L_1 for computing the range.
Range = 42 - 10 = 32.



OR: To find the range: Do not sort. Simply type on the home screen using the min and max functions found under MATH → NUM #6 min and #7 max.



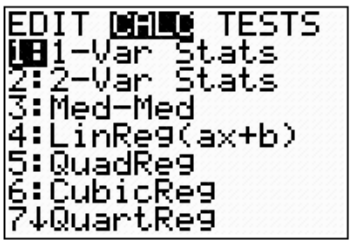


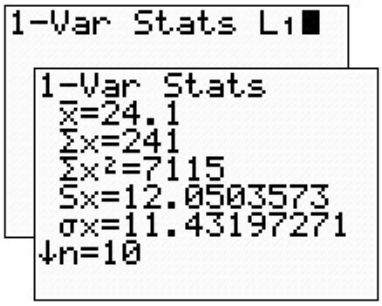
Range = 32

To find standard deviation:

To find standard deviation: *Since this question deals with the complete set, we will be using "population" form, not sample form.*

Go to one-variable stats for "population" standard deviation. STAT → CALC #1 1-Var Stats





$\bar{x}$ = mean

$\sum x$ = sum of the data

$\sum x^2$ = sum of squares of the data

Sx = sample standard deviation

σx = population standard deviation

NOTE! The standard deviations found in the CATALOG, stdDev, and also found by 2nd LIST → MATH #7 stdDev are both **Sample** standard deviations.



n = sample size (# of pieces of data)

$\min X$ = smallest data entry

Q_1 = first quartile

med = median (second quartile)

Q_3 = third quartile

$\max X$ = largest data entry

Population Standard Deviation = 11.43

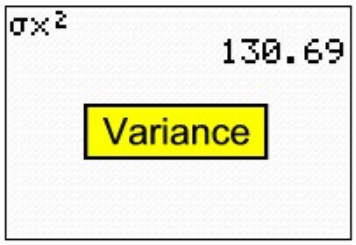
To find variance:

To find variance: The "population" variance is the square of the population standard deviation.

The σx symbol is under VARS - #5 Statistics



NOTE! The variance found in the CATALOG and also found by 2nd List → MATH #8 variance are both **Sample** variances.



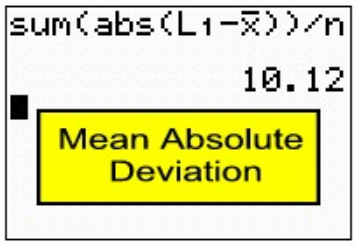
To find mean absolute deviation:

To find mean absolute deviation:

To calculate the mean absolute deviation you will have to enter the formula.

$$\text{Population MAD} = \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$$

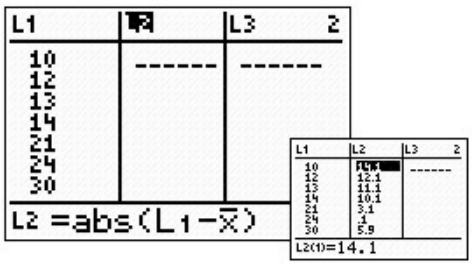
NOTE! Be sure that you have run 1-Var Stats (under STAT - CALC #1) first, so that the calculator will have computed $\bar{x}$. Otherwise, you will get an error from this formula.



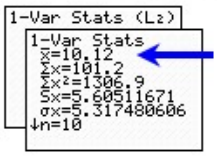
$\bar{x}$ and n are found under VARS #5 Statistics. Sum and abs are quickly found in CATALOG. Sum is also under 2nd LIST - MATH #5 sum. abs is also under MATH - NUM #1abs.

OR: To find mean absolute deviation:

A longer, but workable, solution can also be accomplished using the lists. As stated above, run 1-Var Stats so the calculator will compute $\bar{x}$. Now, go to L_2 (STAT #1 EDIT) and move *UP* onto L_2. Type, at the bottom of the window, the



portion of the formula that finds the difference between each data entry and the mean, using absolute value to make these distances positive. Now, find the mean, $\bar{x}$, of L_2 by using 1-Var Stats on L_2, and read the answer of 10.12.



Finding Standard Deviation "By-Hand"

$$\text{Population standard deviation} = \sigma x = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$\text{Sample standard deviation} = Sx = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

Problem: For the data set {10, 12, 40, 35, 14, 24, 13, 21, 42, 30}, find the range, the standard deviation, the variance, and the mean absolute deviation to the *nearest hundredth.*

Spreadsheet-style solution for standard deviation:

1.

L1	L2	L3 1
10	------	------
12		
40		
35		
14		
24		
13		

L1(1)=10

Enter the data in L_1.

2.

L1	L2	L3 2
10	------	------
12		
40		
35		
14		
24		
13		

L2 ="L1-mean(L1)"

Create L_2 to be the deviations, $x_i - \bar{x}$.

2nd LIST → MATH #3 mean

Using the quotes (ALPHA +) will create a dynamic situation where L_2 will automatically update when L_1 changes.

3.

L1	L2	L3 3
10	-14.1	198.81
12	-12.1	146.41
40	15.9	252.81
35	10.9	118.81
14	-10.1	102.01
24	-.1	.01
13	-11.1	123.21

L3 ="L2²"

Create L_3 to be the deviations squared $(x_i - \bar{x})^2$.

Again, using the quotes (ALPHA +) will create a dynamic situation where L_3 will automatically update when L_2 changes.

4.

√(sum(L3)/dim(L3))
11.43197271

From the HOME screen.

Compute the sum of the squared deviations, divide by the number of pieces of data (for a population form), and take the square root.

2nd LIST → MATH #5 sum

2nd LIST → OPS #3 dim gives the dimension (size) of the list.

ANSWER:

population standard deviation is **11.43**

Measures of Dispersion on Grouped Data

Problem:

Data Entry	Frequency
100	8
150	15
200	21
250	14
300	5

For the data set shown in this table, find the range, the standard deviation, and the variance to the *nearest hundredth.*

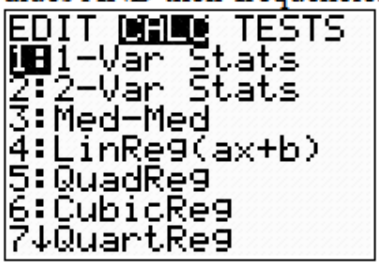
Since this question deals with the complete set, we will be using "population" form, not sample form.

Solution:

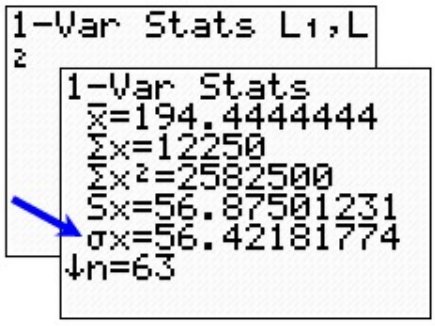
To find the range: No need for calculator work for the range. It is easily observed from the table.
Range = 300 - 100 = 200.

To find standard deviation: *Remember, we are looking for "population" form which will be found using 1-Var Stats.*

Enter the "Data Entry" into L1 and the "Frequency" into L2. Go to one-variable stats to find "population" standard deviation.
STAT → CALC #1 1-Var Stats
Be sure to use parameters L1, L2 to indicate both the values AND their frequencies.



NOTE! The standard deviation found in the CATALOG, stdDev, and also found by 2nd LIST → MATH #7 stdDev are both **Sample** standard deviations.



$\bar{x}$ = mean

$\sum x$ = sum of the data

$\sum x^2$ = sum of squares of the data

Sx = sample standard deviation

σx = population standard deviation

n = sample size (# of pieces of data)

$\min X$ = smallest data entry

Q_1 = first quartile

med = median (second quartile)

Q_3 = third quartile

$\max X$ = largest data entry



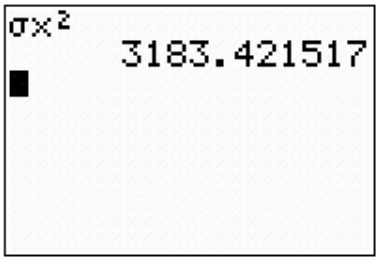
Population Standard Deviation = 56.42

To find variance: The "population" variance is the square of the population standard deviation.

The σx symbol is under VARS - #5 Statistics

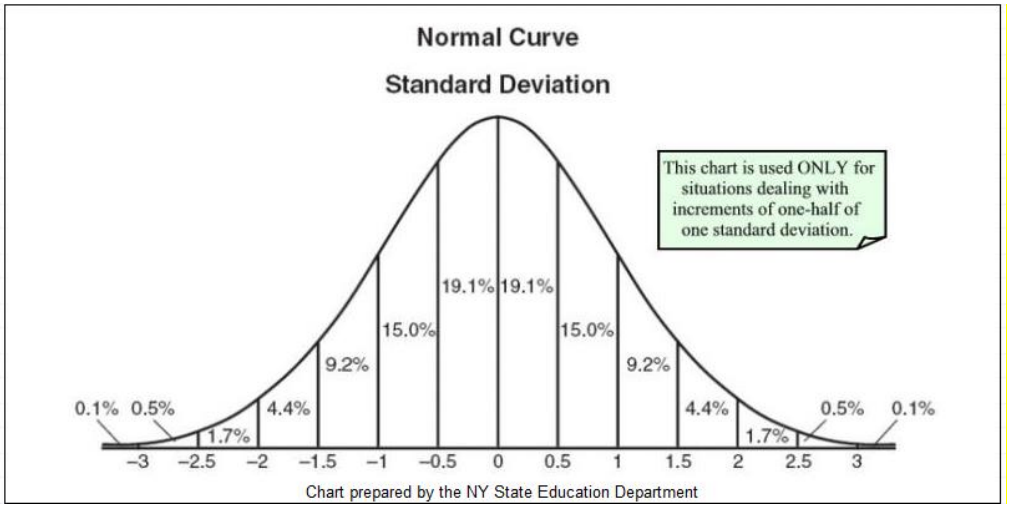


NOTE! The variance found in the CATALOG and also found by 2nd List → MATH #8 variance are both **Sample** variances.



Population Variance = 3183.42

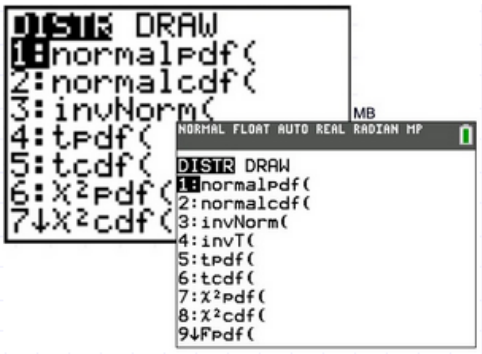
Normal Probability Distribution



A chart, such as that seen above, is often used when dealing with normal distribution questions. Understand that this chart shows only percentages that correspond to subdivisions up to one-half of one standard deviation. Percentages for other subdivisions require a statistical mathematical table or a graphing calculator. (See example 4)

The Normal Probability Distribution menu for the TI-83+/84+ is found under **DISTR** (2nd VARS).

NOTE: A mean of zero and a standard deviation of one are considered to be the default values for a normal distribution on the calculator, if you choose not to set these values.



The Normal Distribution functions:

#1: normalpdf *pdf = Probability Density Function*

This function returns the probability of a single value of the random variable *x*. Use this to graph a normal curve. Using this function returns the *y*-coordinates of the normal curve.

Syntax: *normalpdf (x, mean, standard deviation)*

#2: normalcdf *cdf = Cumulative Distribution Function*

This function returns the cumulative probability from zero up to some input value of the random variable *x*. Technically, it returns the percentage of area under a continuous distribution curve from negative infinity to the *x*. You can, however, set the lower bound.

Syntax: *normalcdf (lower bound, upper bound, mean, standard deviation)*

#3: invNorm(*inv = Inverse Normal Probability Distribution Function*

This function returns the *x*-value given the probability region to the left of the *x*-value. ($0 \leq$ area ≤ 1 must be true.) The inverse normal probability distribution function will find the precise value at a given percent based upon the mean and standard deviation.

Syntax: *invNorm (probability, mean, standard deviation)*

Example 1:

Given a normal distribution of values for which the mean is 70 and the standard deviation is 4.5. Find:

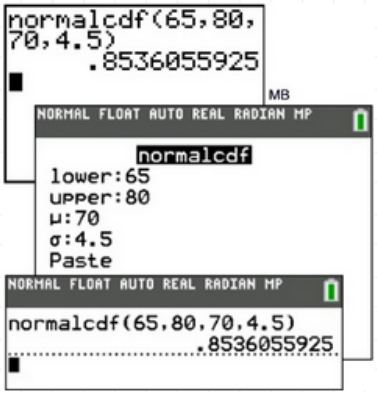
a) the probability that a value is between 65 and 80, inclusive.
b) the probability that a value is greater than or equal to 75.
c) the probability that a value is less than 62.
d) the 90th percentile for this distribution.

(answers will be rounded to the nearest thousandth)

1a: Find the probability that a value is between 65 and 80, inclusive. (This is accomplished by finding the probability of the cumulative interval from 65 to 80.)
Syntax: *normalcdf(lower bound, upper bound, mean, standard deviation)*

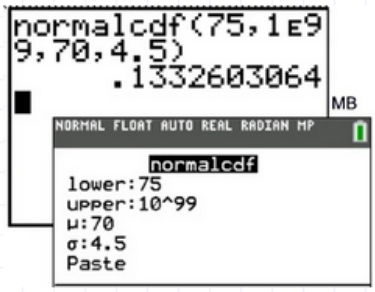
Answer: The probability is 85.361%.

(The "PASTE" command simply means that the values that you typed after the template prompts will be "pasted" into the normalcdf() function and will appear on the home screen, as shown.)



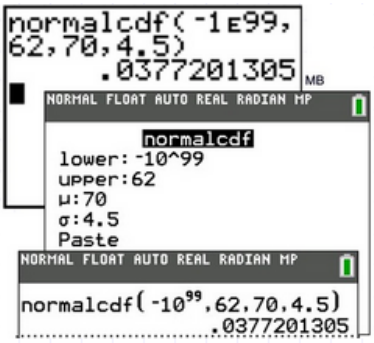
1b: Find the probability that a value is greater than or equal to 75.
(The upper boundary in this problem will be positive infinity. The largest value the calculator can handle is 1 x 10^{99} . Type 10^99 to represent postive infinity, [or type 1 EE 99. Enter the EE by pressing 2nd, comma -- only one E will show on the screen.]

Answer: The probability is 13.326%.



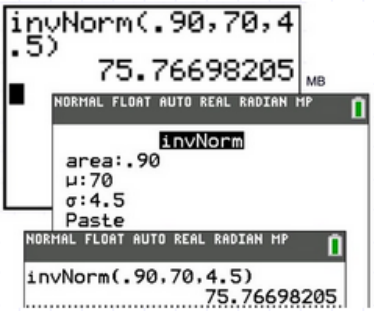
1c: Find the probability that a value is less than 62.
(The lower boundary in this problem will be negative infinity. The smallest value the calculator can handle is -1 x 10^{99} . Type -10 ^ 99 [or type -1 EE 99. Enter the EE by pressing 2nd, comma -- only one E will show on the screen.]

Answer: The probability is 3.772%.



1d: Find the 90th percentile for this distribution.
(Given a probability region to the left of a value (i.e., a percentile), determine the value using invNorm.)

Answer: The *x*-value is 75.767.

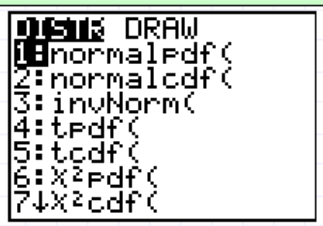


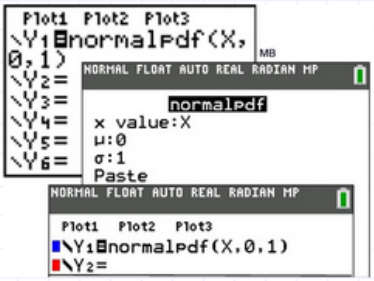
Example 2:

Graph and investigate the normal distribution curve where the mean is 0 and the standard deviation is 1.

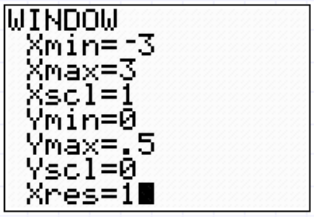
For graphing the normal distribution, choose normalpdf.

The **normalpdf** (normal probability density function) is found under DISTR (2nd VARS) #1normalpdf(.

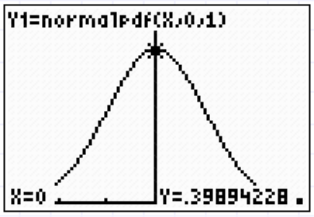




Go to the Y = menu.
The parameters will be (*variable, mean, standard deviation*).

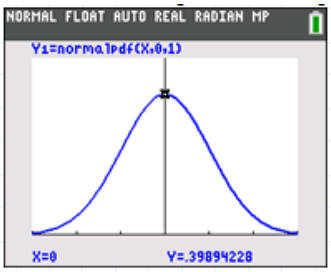


Adjust the WINDOW.
You will have to set your own window.

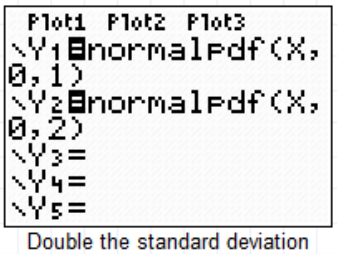


GRAPH. Using **TRACE**, simply type the desired *x* value and the point will be plotted.

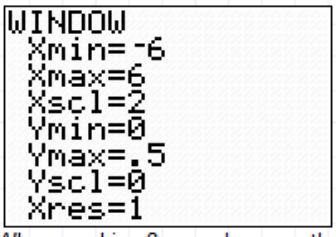
Guideline is:
Xmin = mean - 3 SD
Xmax = mean + 3 SD
Xscl = SD
Ymin = 0
Ymax = 1/(2 SD)
Yscl = 0



Investigate: What happens to the curve as the standard deviation increases?

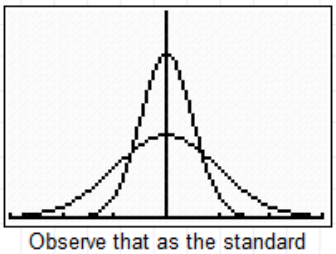


Double the standard deviation and see what happens to the graph.



When graphing 2 normal curves, the window will need to be adjusted.

Xmin = mean - 3 (largest SD)
Xmax = mean + 3 (largest SD)
Xscl = largest SD
Ymin = 0
Ymax = 1/(2 smallest SD)
Yscl = 0



Observe that as the standard deviation increases, the more spread out the graph becomes.

Now, the *area under the curve* between particular values represents the probabilities of events occurring within that specific range. This area can be seen using the command ShadeNorm(.

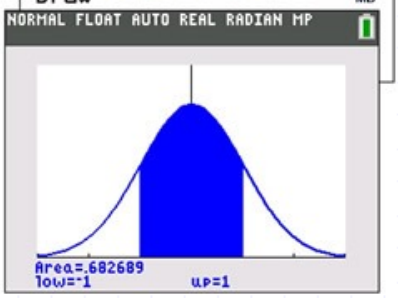
Before attempting ShadeNorm, be sure that Y1 = normalpdf(*x*, mean, standard deviation) is active and that the appropriate window has been set as indicated in the Window Guidelines in the previous section.

To find ShadeNorm(, go to DISTR and right arrow to DRAW. Choose #1:ShadeNorm(.

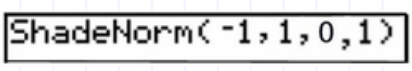
ShadeNorm (lower bound, upperbound, mean, standard deviation)

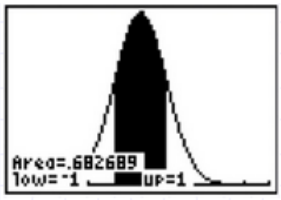






Notice that the calculator defaults to a mean of 0 and a standard deviation of 1 unless changed by the user. Remember that there is approximately a 68% probability of a score falling within 1 standard deviation from the mean in a normally distributed set of values. The "area" value in this graph is indicating 0.682689 or approximately 68%.

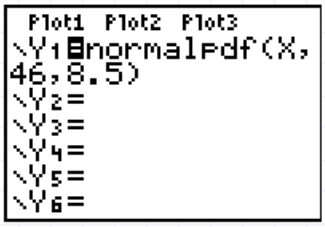




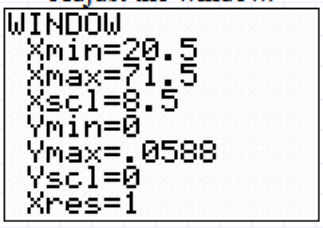
Notice how this answer supports the percentage listed in the chart at the top of this page.

Example 3: Graph and examine a situation where the mean score is 46 and the standard deviation is 8.5 for a normally distributed set of data.

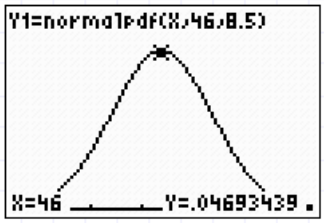
Go to Y= .



Adjust the window.

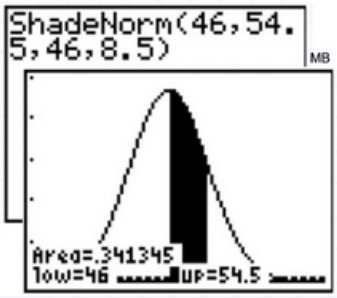


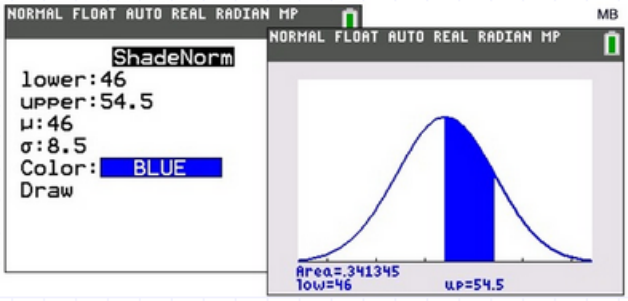
GRAPH.



Examine: What is the probability of a value falling between the mean and the first standard deviation to the right? *Answer: approximately 34%*

Notice how this percentage supports the information found in the chart at the top of this page for the percentage of information falling within one standard deviation above the mean.





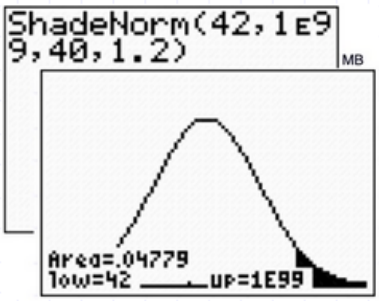
Example 4: The lifetime of a battery is normally distributed with a mean life of 40 hours and a standard deviation of 1.2 hours. Find the probability that a randomly selected battery lasts longer than 42 hours.

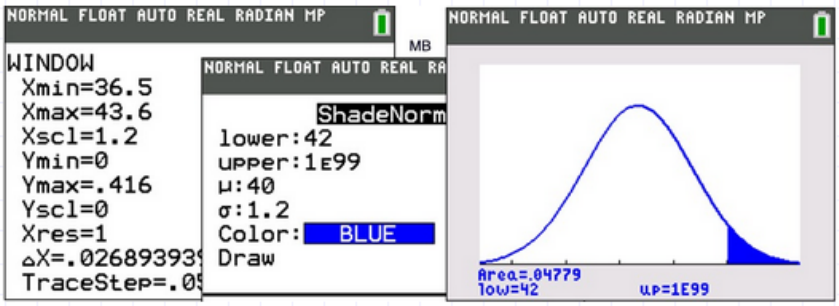
36

The most accurate answer to this problem cannot be obtained by using the chart at the top of this page. One standard deviation above the mean would be located at 41.2 hours, 2 standard deviations would be at 42.4, and one and one-half standard deviations would be at 41.8 standard deviations. None of these locations corresponds to the needed 42 hours. We need more power than we have in the chart. Calculator to the rescue!!

Now: What is the probability of a value falling to the right of 42 hours (between 42 hours and infinity)?

Answer: approximately 4.8%



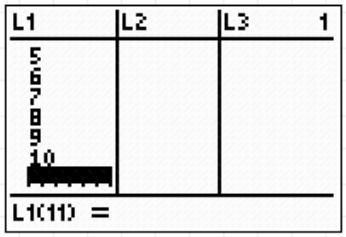


ShadeNorm(go to DISTR and right arrow to DRAW. Choose #1:ShadeNorm(.
ShadeNorm (lower bound, upperbound, mean, standard deviation)
ENTER. The percentage is read from the Area =.

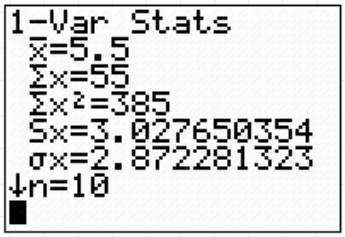
Changes in Statistical Information

What effect does changing the data have upon statistical information?
Let's examine a few situations with the aid of the graphing calculator

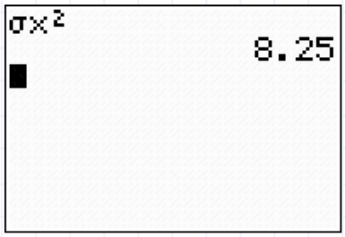
Situation 1: If all of the data in a set were multiplied by 8, by what factor would the variance of the data change?



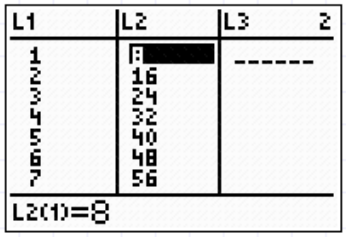
Create a simple test situation.
Pick 5 or 10 pieces of data at random and enter them in a list.
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}



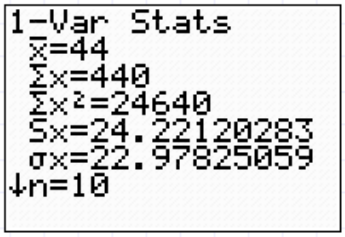
Calculate **1-Var Stats** to obtain the population standard deviation.



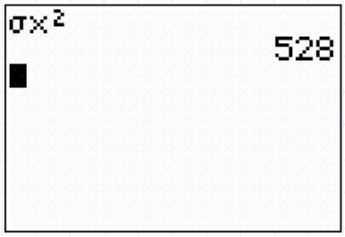
Compute the variance.



Multiply the data times 8.



Re-calculate the **1-Var Stats** for the new list.



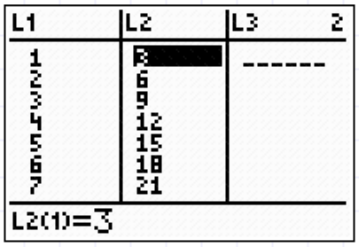
Re-calculate the variance.

Now, compare the two variances to ascertain what is happening when the data is multiplied by a factor of 8.

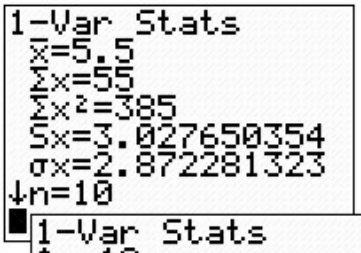
We notice that 8.25 x 64 = 528

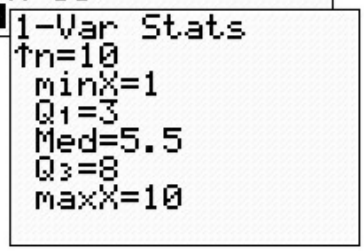
The variance is multiplied by a factor of 64, when the data is multiplied by a factor of 8.

Situation 2: If all of the data in a set is tripled, which, if any, of the summary statistics are also tripled?

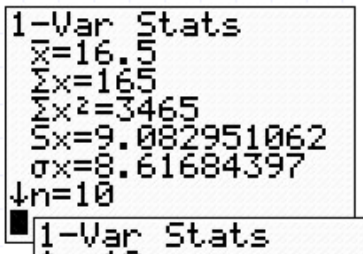


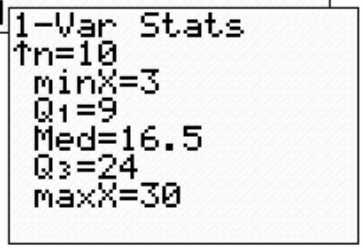
Create a simple test situation.
{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}
Create a second list with the data tripled.





Calculate **1-Var Stats** on the original data.





Calculate **1-Var Stats** on the tripled data.

Now, compare the findings. From examining the calculations, we know that the mean, the standard deviation (both sample and population), the minimum, the first quartile, the median, the third quartile and the maximum are also tripled.

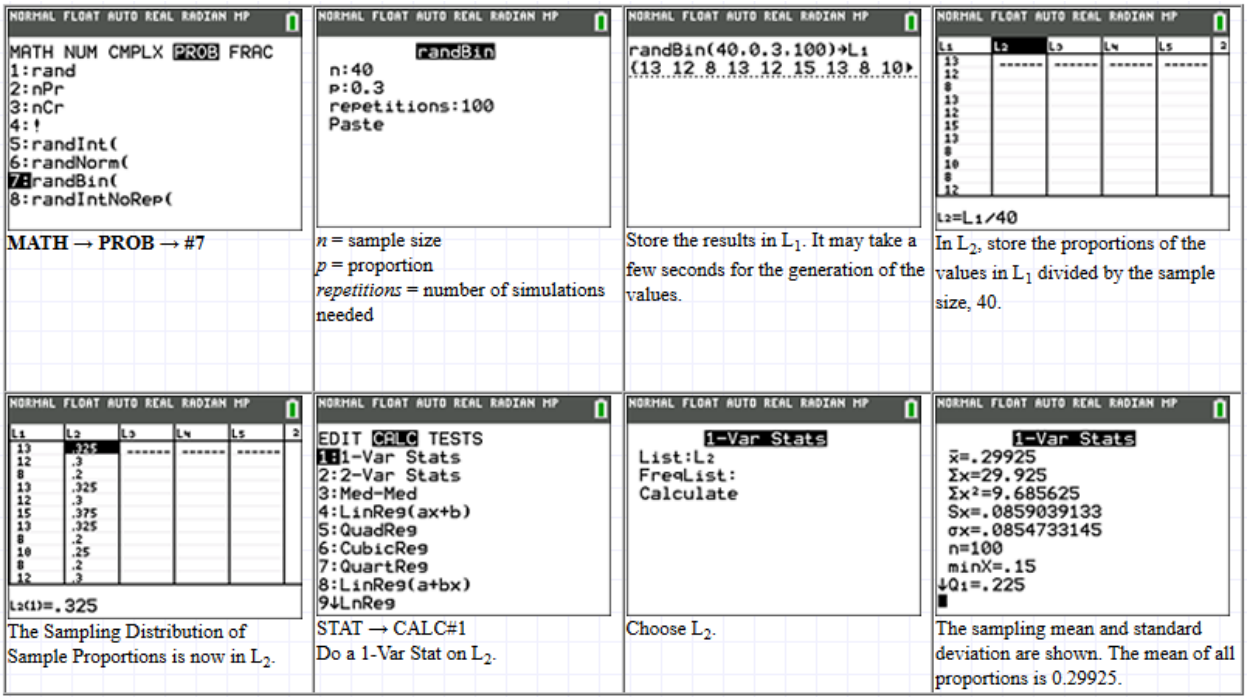
In both of these examples, the **1- Var Stats** allowed us to quickly examine the data to see what was happening when the data was rescaled.

Creating a Sampling Distribution Simulation

Our goal is to simulate the collection of multiple samples. For this task, we are going to use the command **randBin** found under **MATH → PROB → #7**. The parameters are **randBin(*n, p, repetitions*)** where *n* is the sample size, *p* is the proportion, and *repetitions* is the number of times you want to run the simulation.

For this example, the sample size is 40, the proportion is 0.3, and the number of simulation repetitions is 100. From the home screen, set randBin(40, 0.3, 100) and store the results in L_1.
randBin(40,0.3,100) → L_1

Once $L_1$is established, we need the proportion for each L_1 sample in list L_2. Type $L_1/40 \rightarrow L_2$ on the home screen (or from the Lists in STAT #1, move up onto L_2, and type $L_1/40$ at the bottom of the screen, hit ENTER). You now have the Sampling Distribution of the Sample Proportions in L_2. Using L_2 you can find the sampling mean and the sampling standard deviation for the sample proportions.



MATH → PROB → #7

n = sample size
p = proportion
repetitions = number of simulations needed

Store the results in L_1. It may take a few seconds for the generation of the values.

In L_2, store the proportions of the values in L_1 divided by the sample size, 40.

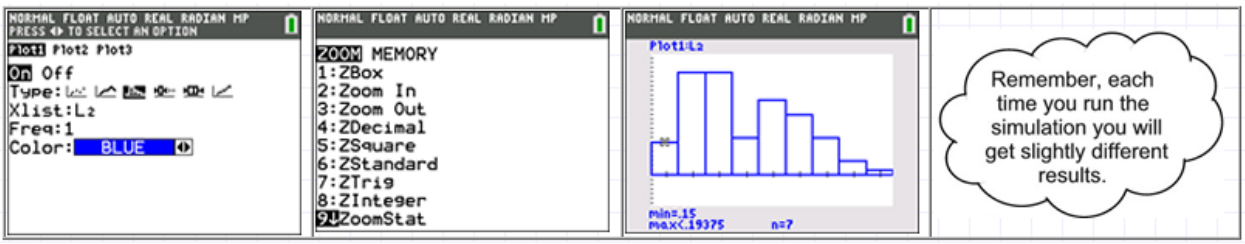
The Sampling Distribution of Sample Proportions is now in L_2.

STAT → CALC#1
Do a 1-Var Stat on L_2.

Choose L_2.

The sampling mean and standard deviation are shown. The mean of all proportions is 0.29925.

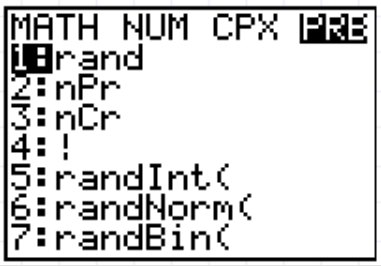
To see this Sampling Distribution from L_2 as a histogram, set STAT PLOT for histogram, ZOOM #9



Factorials, Permutations, Combinations

To use these probability options, access the **Math** key.
Arrow to the right to find the **PRB** (probability) menu.
On this screen you will find:

#2 nPr permutation
#3 nCr combination
#4 ! factorial

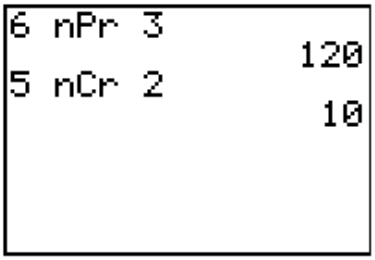


Each of these functions, requires that you **enter a value FIRST** and then access the function. For example:

Permutations and Combinations are handled in the same manner:

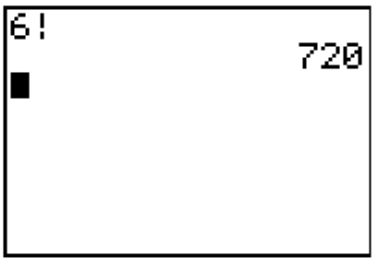
Compute $_6P_3$ and $_5C_2$

Go to the **HOME** screen.
Enter your first value (for this example, 6 or 5)
Hit the **MATH** key, arrow to the right to **PRB**.
Choose **#2, nPr** or **#3 nCr**
Enter your second value (for this example, 3 or 2)
Press **Enter.**



Factorial: **Compute 6!**

Go to the **HOME** screen.
Enter your value first (for this example, 6)
Hit the **MATH** key, arrow to the right to **PRB**.
Choose **#4, !**
Press **Enter.**



Bernoulli Trials - Binomial Probability Distribution

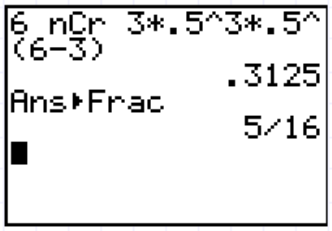
The formula used with Bernoulli trials computes the binomial probability of obtaining exactly "*r*" events in "*n*" trials:

$_nC_r \bullet p^r \bullet q^{n-r}$	n = number of trials r = number of specific events you wish to obtain p = probability that the event will occur q = probability that the event will **not** occur ($q = 1 - p$, the *complement* of the event)

If you enter the formula directly on the home screen, **be careful to use parentheses** when entering the exponent of $n - r$ (or do the subtraction mentally and enter your calculation).

Consider a problem where
$n = 6$, $r = 3$, and $p = 50\%$
(so, $p = .5$ and $q = .5$, where $q = 1 - p$)

(Remember, the function ***nCr*** is found under
MATH → PRB #3 *nCr*
and requires that the first value, ***n***, be entered before the function is called.)

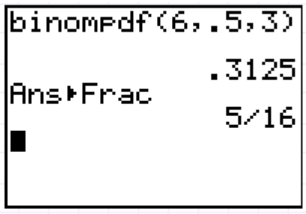


The easiest way to utilize the calculator to solve this formula is to engage the **binompdf** function

binompdf(	binomial distribution probability density function, which is: $f(x) = \binom{n}{r} p^r (1-p)^{n-r}$ where $\binom{n}{r} = {}_nC_r$ (When using this built-in function there is no need to type in the formula - YEA!!)

Consider, again, a problem where
$n = 6$, $r = 3$, and $p = 50\%$

(Remember, the function **binompdf** is found under **DISTR (2nd VARS)**, arrow down to **#0 binompdf** and the parameters are:
binompdf (*number of trials, probability of occurrence, number of specific events*)



Binomial Probability
"Exactly", "At Most", "At Least"

Problem used for demonstration:
A fair coin is tossed 100 times. What is the probability that:
a. heads will appear **exactly** 52 times?
b. there will be **at most** 52 heads?
c. there will be **at least** 48 heads?

Dealing with "Exactly":

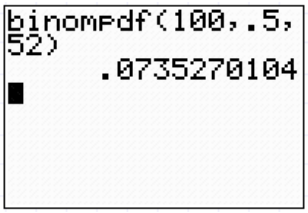
A fair coin is tossed 100 times. What is the probability that:
part a. heads will appear **exactly** 52 times?

We have seen that the formula used with Bernoulli trials (binomial probability) computes the probability of obtaining exactly "r" events in "n" trials: $\binom{n}{r} \cdot p^r \cdot q^{n-r}$

where n = number of trials, r = number of specific events you wish to obtain, p = probability that the event will occur, and q = probability that the event will **not** occur ($q = 1 - p$, the *complement* of the event).

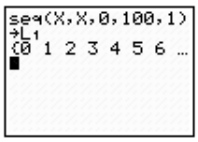
We have also seen that the built-in command binompdf (binomial probability density function) can also be used to quickly determine "exactly".

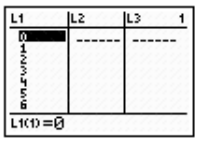
(Remember, the function **binompdf** is found under
DISTR (2nd VARS), arrow down to **#0 binompdf**
and the parameters are:
binompdf (*number of trials, probability of occurrence, number of specific events*)
binompdf (*n, p, r*)
Here is our answer to *part a*



If you want more information at a glance, this command can also be used to produce a list of the exact probabilities

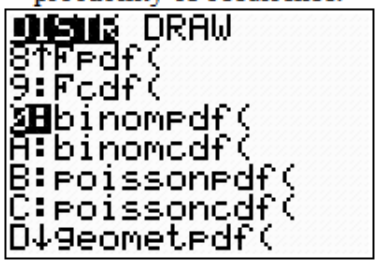
Prepare two lists.
In L_1 place the numbers from 0 to 100 for easy reference. The built-in sequence command will quickly fill L_1 from 0 to 100.

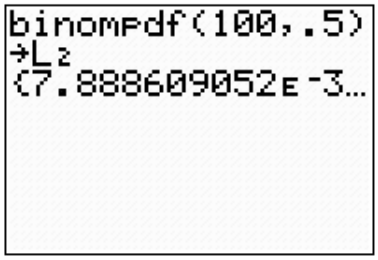




The format for seq: *expression, variable, starting value, ending value, increment.*

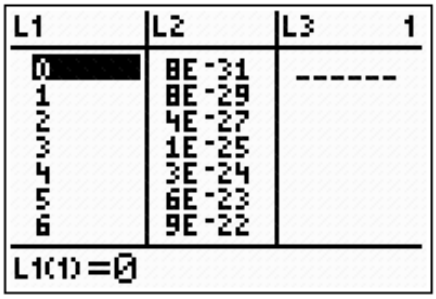
In L_2, prepare the "exact" probabilities using binompdf, using only the number of trials and the probability of occurrence.

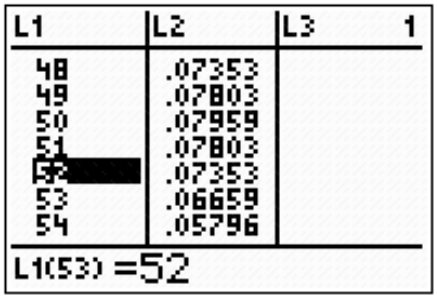




(This is one method of filling a list.)

The lists now contain, for 100 trials, the number of specific events (heads) in L_1 and the exact probability of occurrence in L_2. Notice that our answer for exactly 52 heads remains the same.





Dealing with "At Most":

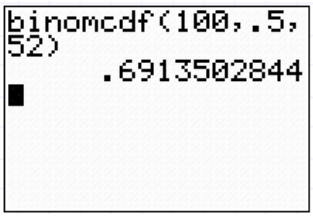
A fair coin is tossed 100 times. What is the probability that:
part b. there will be **at most** 52 heads?

The formula needed for answering *part b* is :

$$P(x \le 52) = \sum_{x=0}^{52} \binom{100}{x} \bullet (.5)^x \bullet (.5)^{100-x}$$

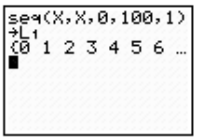
There is a built-in command binomcdf (binomial cumulative density function) that can be used to quickly determine "at most". Because this is a "cumulative" function, it will find the sum of all of the probabilities up to, and including, the given value of 52.

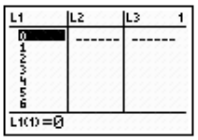
(The function **binomcdf** is found under
DISTR (2nd VARS), arrow down to **#A binomcdf**
and the parameters are:
binomcdf (*number of trials, probability of occurrence, number of specific events*)
binomcdf (*n, p, r*)
Here is our answer to *part b*



Again, if you want more information at a glance regarding cumulative probabilities, this command can be used to produce a list of the "at most" probabilities

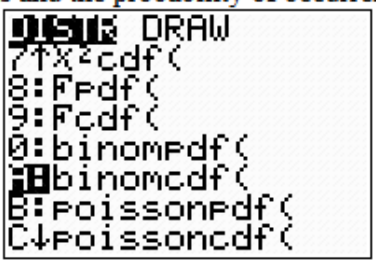
Prepare two lists.
In L_1 place the numbers from 0 to 100 for easy reference. The built-in sequence command will quickly fill L_1 from 0 to 100.



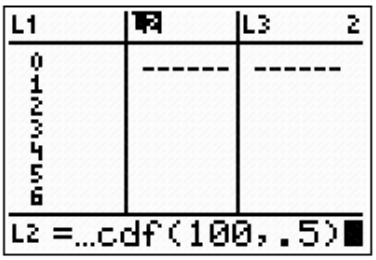


The format for seq: *expression, variable, starting value, ending value, increment.*

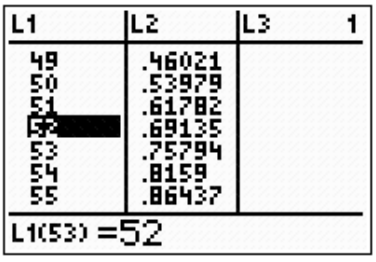
In L_2, prepare the "at most" (cumulative) probabilities using binomcdf, using only the number of trials and the probability of occurrence.



(Method shown below is another method of filling a list.)



Notice that our answer remains the same.



Dealing with "At Least":

A fair coin is tossed 100 times. What is the probability that:
part c. there will be **at least** 48 heads?

The formula needed for answering part b is :

$$P(x \geq 48) = \sum_{x=48}^{100} \binom{100}{x} \bullet (.5)^x \bullet (.5)^{100-x}$$

Keep in mind that "at least" 48 is the complement of "at most" 47.

$$P(x \geq 48) = 1 - P(x \leq 47)$$

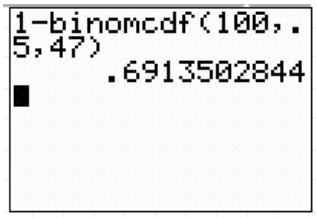
In a binomial distribution, $P(x < 48)$ is really $P(x \leq 47)$.

While there is no built-in command for "at least", you can quickly find the result by creating this complement situation by subtracting from 1. Just remember to adjust the value to 47.

(Remember, the function **binomcdf** is found under
DISTR (2nd VARS), arrow down to **#A binomcdf**.)

The adjusted formula for "at least" is 1 - binomcdf (*n, p, r - 1*).

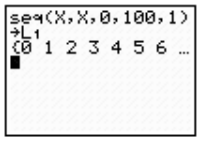
Here is our answer to *part c*

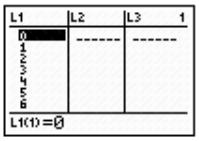


The fact that this answer is the same as the "at most" answer for the number 52, is due to the symmetric nature of the distribution about its mean of 50.

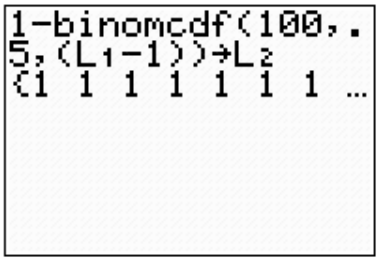
Again, if you want more information at a glance regarding cumulative probabilities, this subtraction process can be used to produce a list of the "at least" probabilities.

Prepare two lists. In L_1 place the numbers from 0 to 100 for easy reference. The built-in sequence command will quickly fill L_1 from 0 to 100.	In L_2, prepare the "at least" (1 - binomcdf) probabilities, using as parameters the number of trials, the probability of occurrence, and **one less than** the value in L_1.

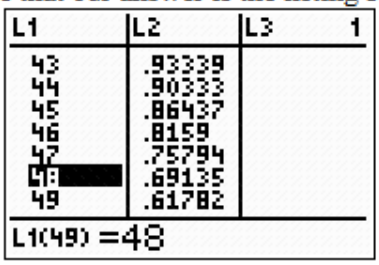




The format for seq: *expression, variable, starting value, ending value, increment.*



Notice that our answer is the listing for 48.



8. CALCULUS

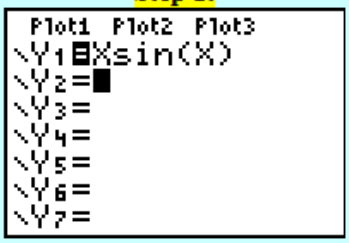
Finding Maxima and Minima (Extrema)

Method 1: (using the CALCULATE Menu)

Locate the relative maximum/minimum for the function

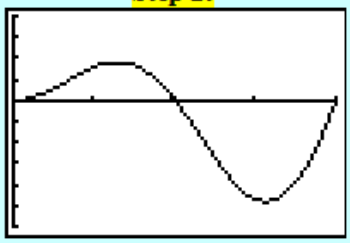
$$f(x) = x\sin x, \text{ where } 0 \le x \le 2\pi.$$

Step 1:



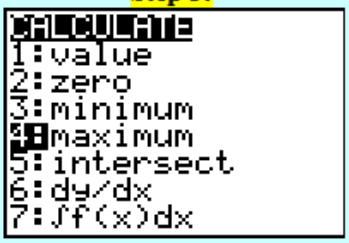
Enter the equation into **Y=**

Step 2:



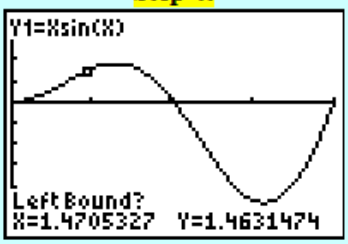
Adjust the **WINDOW** to coincide with the given domain. Hit **GRAPH.** Be sure the graph is viewable in the graphing window. Adjust accordingly.

Step 3:



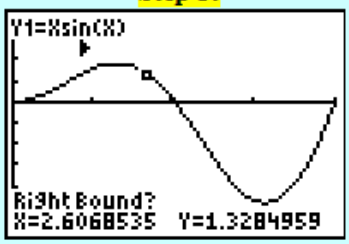
Let's find the maximum value first. Under the **CALC** (**2nd TRACE**) menu, choose **#4 maximum.**
Hit **ENTER.**

Step 4:



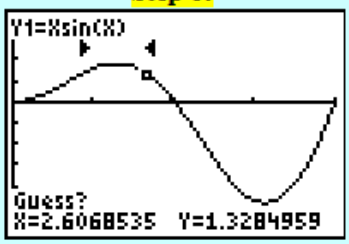
When asked for **Left Bound**, move the cursor (use arrow keys) to the left of the observed maximum location. Hit **ENTER.** You will see a ▶mark indicating that you have "locked" this position.

Step 5:



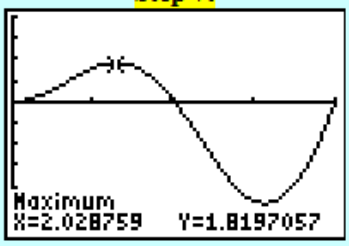
When asked for **Right Bound**, move the cursor (use arrow keys) to the right of the observed maximum location. Hit **ENTER.** You will see a ▶mark indicating that you have "locked" this position.

Step 6:



When asked for **Guess**, simply hit **ENTER.**

Step 7:



The coordinates of the maximumm value (within your marked boundaries) will appear.
ANSWER:
Max (2.029, 1.920)

Note:

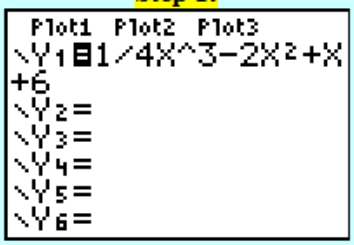
If you are investigating more than one location on a graph, you must start this process from the beginning for each point being investigated. (Return to Step 3.)

This same process is used to examine relative minima.
(#3: minimum)
ANSWER: Min(4.913, -4.814)

Method 2: (using the fMin and fMax functions)

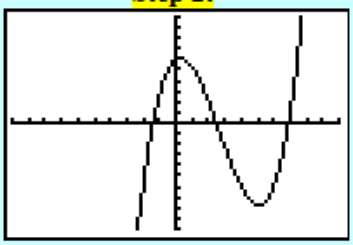
Investigate relative minima/maxima for $y=\frac{1}{4}x^3-2x^2+x+6$.

Step 1:



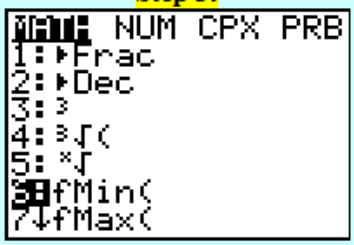
Enter the equation into **Y=**

Step 2:



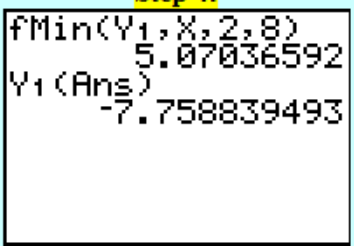
Hit **GRAPH.** Be sure the graph is viewable in the graphing window. Adjust the **WINDOW** if needed.

Step 3:



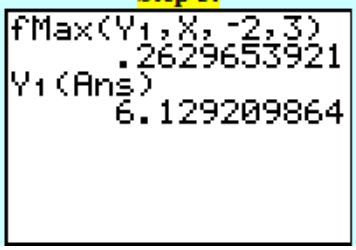
From the **HOME** screen, hit the **MATH** key. Choose either **#6 fMin** or **#7 fMax.** Hit **ENTER**.

Step 4:



The parameters for **fMin** and **fMax** the same:
fMin(*expression, variable, left bound, right bound*)
Be careful: The answer from **fMin** is the ***X*-coordinate where the minimum occurs.** It is not the actual *y*-value minimum. You must then calculate the *y*-value.

Step 5:



Again, remember that the answer from **fMax** is the ***X*-coordinate** where the maximum occurs. You must then calculate the *y*-value.

HINT: To get **Y_1(Ans):**
Y_1: VARS -> Y-VARS - #1Function
Ans: 2nd (-) key

ANSWERS:

Min(5.070, -7.759)

Max(.263, 6.129)

Finding Numerical Derivatives

While the TI-84+ calculators are not symbolic manipulators (they cannot tell you the algebraic expression for a derivative), they can tell you an approximate numerical value for a derivative at a specific location. Several methods may be used.

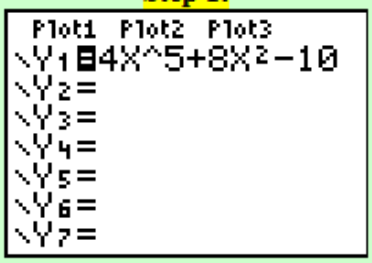
Method 1: using nDeriv(expression, variable, value, [ε])

The **nDeriv** command uses the symmetric difference quotient method, which approximates the numerical derivative value as the slope of the secant line, $f'(x) = (f(x+\varepsilon) - f(x-\varepsilon))/2\varepsilon$. As ε becomes smaller, the answer becomes more accurate.
Be careful!!! The use of this method can yield an incorrect answer at non-differentiable points.
The default value for ε is 1 E -3.

Find the numerical value of the derivative for the function stated below at the indicated location (express answer to three decimal places).

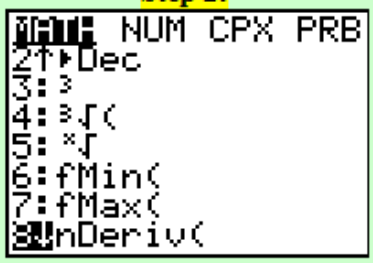
$$f(x) = 4x^5 + 8x^2 - 10 \text{ at } x = 1.2$$

Step 1:



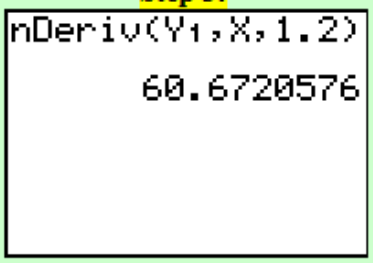
Enter the function into **Y=**

Step 2:



From the **HOME** screen, hit the **MATH** key and arrow down to choice **#8 nDeriv(**.

Step 3:



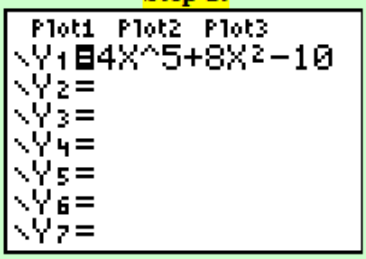
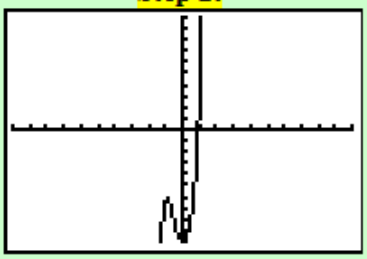
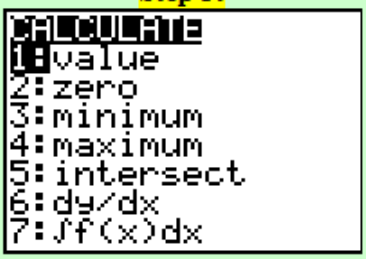
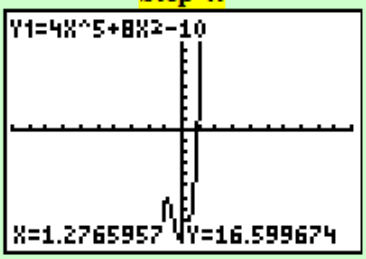
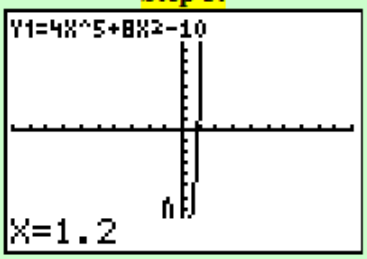
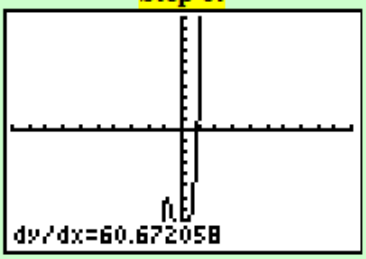
The parameters for **nDeriv** are expression, variable, value. (A fourth parameter exists involving the size of epsilon and is usually left off.)
Hit **ENTER** .for the answer.

Method 2: using the $\frac{dy}{dx}$ feature and the graph

Let's examine that same problem again:

Find the numerical value of the derivative for the function stated below at the indicated location (express answer to three decimal places).

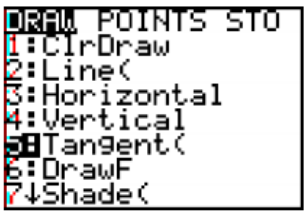
$$f(x) = 4x^5 + 8x^2 - 10 \text{ at } x = 1.2$$

Step 1:	Step 2:	Step 3:
		
Enter the equation into **Y=**	Hit **GRAPH.** Be sure that the **WINDOW** includes the *x* value that you will be investigating. If not, adjust the **WINDOW**.	Hit the **MATH** key and choose **#6 dy/dx.** Hit **ENTER**.
Step 4:	**Step 5:**	**Step 6:**
		
The graph will reappear. (If more than one graph appears, use the up/down arrows to select the desired graph.) Use the left ◄ and right ► arrows to locate the point you wish to investigate, or simply type the point. Hit **ENTER**.	Simply type the desired value on the keypad. The number will appear below the graph.	Hitting **ENTER** will reveal the answer.

Graphing the Tangent Line

If you have graphed your function, you can use

2nd DRAW to draw a tangent line at a given *x*-value.



First graph your function in a window that shows the *x*-value.

While in the graphing window, press 2nd DRAW and select Tangent(.

Select an *x*-value by moving to it or typing a number.

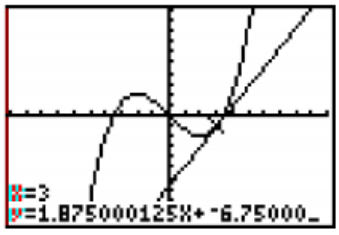
Then press ENTER .

The screen shows the graph of the tangent line and its equation.

Graph the tangent line to $y = \dfrac{x^3 - 12x}{8}$ at $x = 3$.

Find the approximate equation of this tangent line.

Be sure that you are in the graphing window.



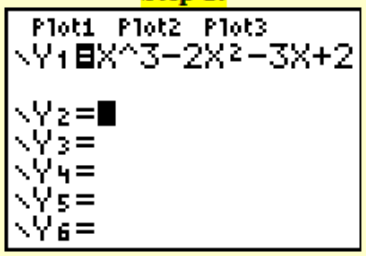
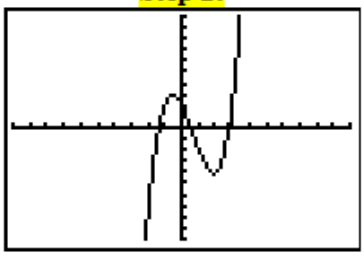
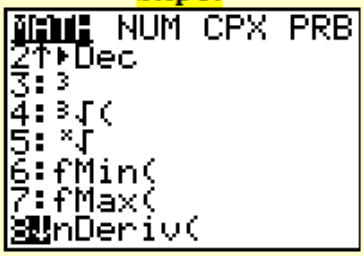
To draw the tangent line at $x = 3$,
press [2nd] DRAW [5] *(for draw tangent),*
[3] [ENTER] *(for the x-value).*

The approximate equation of the tangent line is $y = 1.875x - 6.75$.

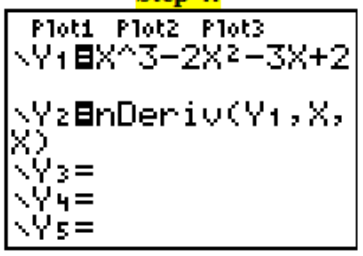
To clear the drawing, press [2nd] DRAW *(above* [PRGM] *)*
and [ENTER] *(to select option 1, for* ClrDraw*) and* [ENTER] .

Graphing Derivatives

Graph the derivative of the function: $f(x) = x^3 - 2x^2 - 3x + 2$

Step 1:	Step 2:	Step 3:
		
Enter the function into **Y1=**. Arrow down so the cursor is in the **Y2=** entry space.	**GRAPH** the original function to see that your viewing window gives a good representation of the graph.	We will be using the **nDeriv** command found under the **MATH** key. Hit the **MATH** key and arrow down to choice **#8 nDeriv(**.

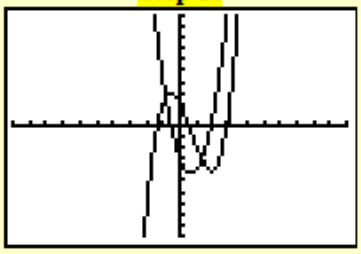
Step 4:



The **nDeriv** command will appear in the **Y2=** entry space, where you left the cursor. We will be using the parameters

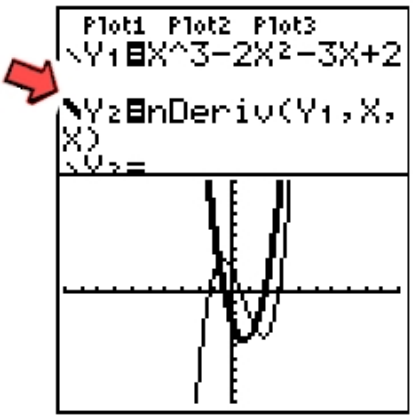
nDeriv(Y1, X, X)

Step 5:



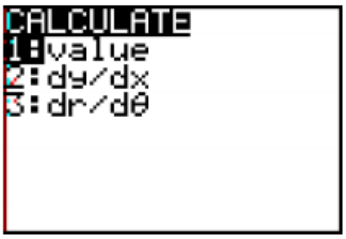
Hit **GRAPH.**

Step 6:



To more clearly distinguish the derivative graph, set the graph style to a thicker line (or graph the derivative separately).

Graphing Derivatives – Polar Functions



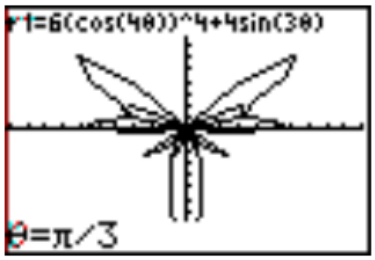
If you have graphed a polar function, you can use the TI-84/83 Plus to find the numeric derivatives $\frac{dy}{dx}$ and $\frac{dr}{d\theta}$ using [2nd] CALC.

Find the approximate value of $\frac{dr}{d\theta}$ at $\theta = \frac{\pi}{3}$ if $r = 6\cos^4 4\theta + 4\sin 3\theta$.

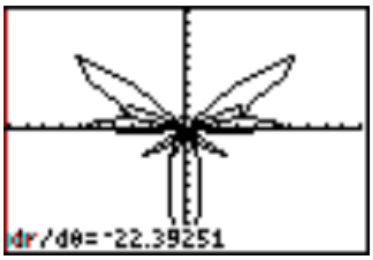
Be sure that your calculator is in polar mode and radian mode. Graph the function in a standard window.

Then press [2nd] CALC [3] *(for dr/dθ)*

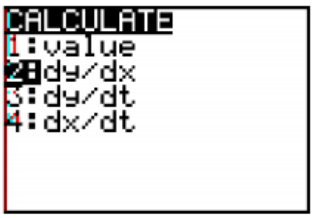
[2nd] π [÷] [3] [ENTER] .



The numeric derivative is about -22.39.



Graphing Derivatives – Parametric Equations

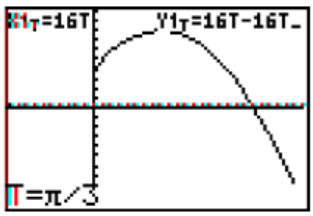


If you have graphed a parametric equation, you can use the TI-84/83 Plus to find the numeric derivatives $\frac{dy}{dx}$, $\frac{dy}{dt}$ and $\frac{dr}{d\theta}$ using [2nd] CALC.

Find the approximate value of $\frac{dy}{dt}$ at $t = \frac{\pi}{3}$ if

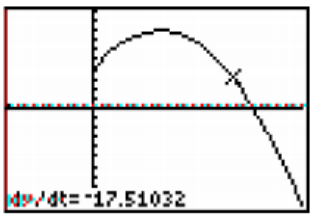
$$\begin{cases} x = 16t \\ y = 16t - 16t^2 + 4 \end{cases}$$

Be sure that your calculator is in parametric mode and radian mode.
Graph the function in a standard window. (See Section 5, Example 3.)



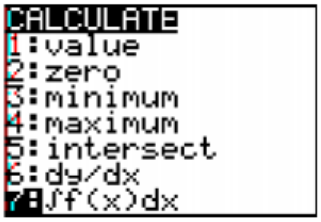
Then press [2nd] CALC [3] *(for dy/dt)*
[2nd] π [÷] [3] [ENTER] .

The numeric derivative is about -17.51.



Finding a Definite Integral

If you have graphed your function, you can use [2nd] CALC (for calculate) to find a definite integral of a function.



First graph your function in a window that shows the lower and upper limits of integration.

The press [2nd] CALC and select $\int f(x)dx$.

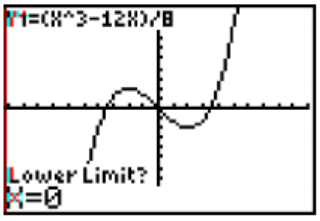
Select a lower limit by moving to it or typing a number and pressing [ENTER].
Select an upper limit by moving to it or typing a number and pressing [ENTER].

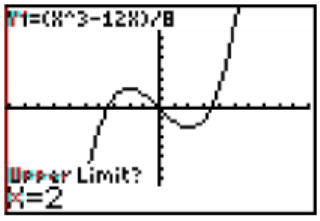
The calculator shades the region and gives the value of the definite integral.

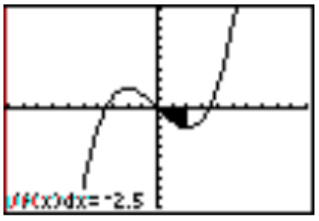
Find $\int_0^2 \frac{x^3 - 12x}{8} dx$.

We have already stored this function as y_1 and graphed it in a standard window.

To find the numeric integral, press [2nd] CALC [7] *(for definite integral),* [0] [ENTER] *(for the lower limit),* [2] [ENTER] *(for the upper limit).*



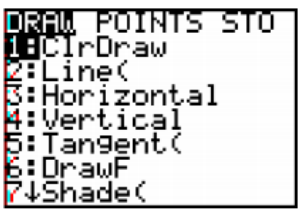


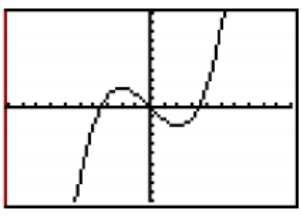


The integral $\int_0^2 \frac{x^3 - 12x}{8} dx$ has an approximate value of -2.5.

Compare this to our result

To clear the drawing, press [2nd] DRAW *(above* [PRGM] *) and* [ENTER] *(to select option 1 for* ClrDraw*) and* [ENTER] .



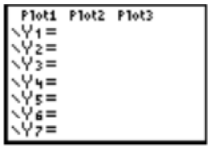


9. INDEX

TI-84+ Menu Map

Top Row Keys

Y= →



STAT PLOT →



PLOT1



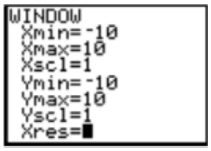
ZOOM →



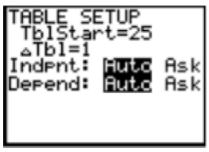
FORMAT



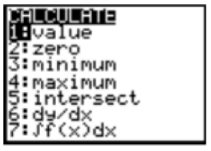
WINDOW →



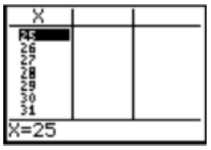
TBLSET



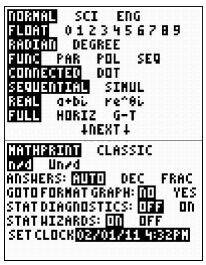
CALC



TABLE



MODE



LINK →



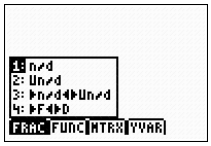
RECEIVE



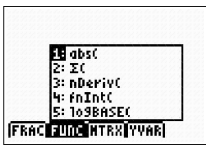
OPENING SCREEN



F1 – FRAC

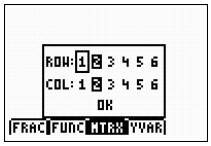


F2 - FUNC

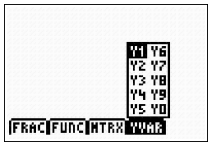




F3 – MTRX



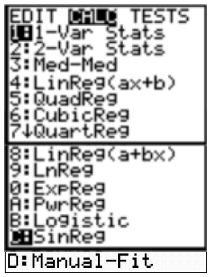
F4 - YVAR



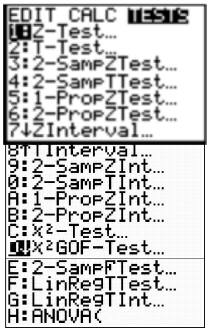
STAT →



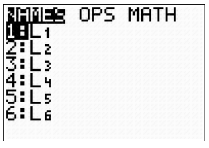
CALC →



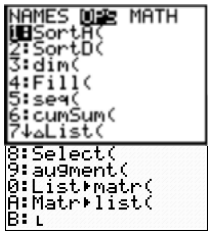
TESTS



LIST →



OPS →



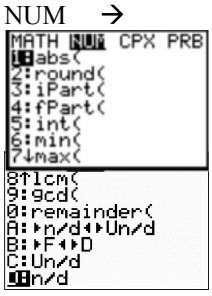
MATH →



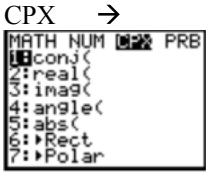
MATH →



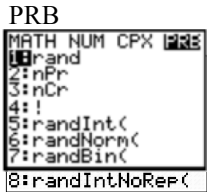
NUM →



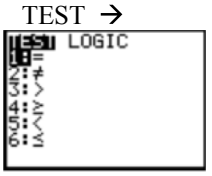
CPX →



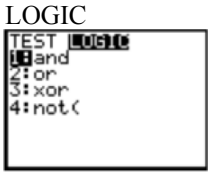
PRB



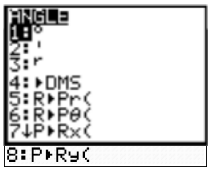
TEST →



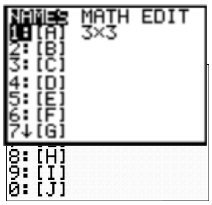
LOGIC



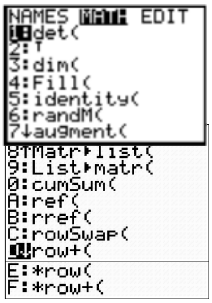
ANGLE



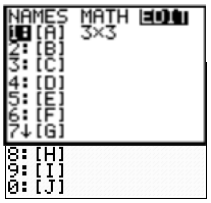
MATRIX →



MATH →



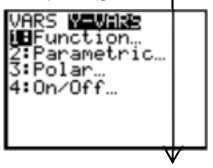
EDIT



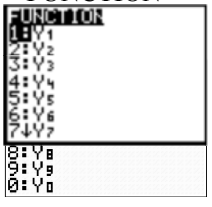
VARS →



Y-VARS



FUNCTION



CATALOG



MEM



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